Simulation and Analysis of Tidal Force on Various Comets using PKDGRAV

Abstract

N-body problem is one of the most fundamental problems in physics system simulation. As finding an analytical solution is usually difficult, computers are widely used to numerically solve and simulate such systems.

This experiment aims to use the technologies of computer science to simulate and study the tidal destruction of comets using PKDGRAV, which is an n-body integrator. With data from the JPL/Horizon project, by properly setting the parameters and creating an appropriate initial condition, the behaviour of SL9 as it passing Jupiter was successfully simulated, with a result that matches the actual observation in 1992, which is also known as a "pearl string".

By rerun the simulation with different parameters, it is found that the behaviour of comets with different densities can roughly be classified into three different types. For small-density comets, they will tend to be completely torn apart. Medium-density comets are expected to be partly ripped, while large-density comets will keep their structure.

This experiment has again verified the theory of Roche limit, and also shows the behaviour of comets with different densities, which will help in understanding the formation of rings of the gas giants.

Introduction

SL9 was a comet that was torn apart by tidal forces in 1992 (NASA, 2019). This astronomical event provided a valuable opportunity for humans to directly observe the phenomenon of tidal destruction and understand its principles.

The theory behind tidal destruction is known as the Roche limit. By modelling the object as a rigid body as well as fluid one, and assume at the critical distance the gravity on surface balance, two different formulas can be determined (Thomas, 2010):

For a rigid body, the Roche limit is:

$$d = R(2\frac{\rho_M}{\rho})^{1/3} \tag{E1}$$

For fluid, the Roche limit is larger:

$$d \approx 2.44R(\frac{\rho_M}{\rho})^{1/3} \tag{E2}$$

where *R* is the radius of the centre star/planet, and ρ_M/ρ is the ratio of the density between the object and the centre star/planet (Thomas, 2010). To verify this theory, specific programs were used.

Early researches on the physical properties of comets have shown that comets can be modelled as “rubble pile” (NASA, 2019), so it is reasonable to consider the simulation of the tidal destruction of comets as an n-body problem. Using the program PKDGRAV, it is possible to model and simulate the physics of SL9 and visualize its behaviour. PKDGRAV is an n-body integrator, which uses the k-D tree data structure to balance the load to different CPUs. Its integrating algorithm (leapfrog) can provide a second order of accuracy on solving of the differential equation (Dobinson, 2011), which should be appropriate for the simulation of second-order dynamic physical systems such as n-body systems.

Because PKDGRAV can only produce rasterized image files at different time steps, FFmpeg, a multimedia framework that provides multiple image, video, audio stream processing functions, was used to merge and convert all images to a single video file, which is easier to visualize (FFmpeg Community, 2019).

Using bash scripts and C# programs, one can efficiently change and rerun the simulation and analyse the outcome. The result has suggested that there are typically three types of behaviour of comets with different densities. Besides, by only considering the small density comets and their destruction distances, a destruction distance vs. comet density graph is obtained, which matches the prediction of the theory.

Methods

1. Preparation of Initial Condition

The Windows Subsystem of Linux was used to run Linux binaries on Windows OS during this experiment.

Each SS file can be considered as a small slice of time in the universe, with all of the numerical physical quantity inside. An initial condition that emulates the comet SL9 on A.D. 1992-Jul-07 was required for the RPG program. The data from real-world can be found on the JPL/Horizon project (Chamberlin, 2021). 4 pm was chosen as the t_0.

The position vector is

$$(-3.612690198669638\text{E}05, 1.847964829204649\text{E}05, -1.057012897678898\text{E}06)$$

And the velocity vector is

$$(3.873710750683169\text{E}03, -6.539554848394263\text{E}03, 1.270894003179407\text{E}04)$$

in km or km/s (Chamberlin, 2021).

The density of SL9 is predicted to be about 500 in km/m^3. With the RPG and RPX program, these conditions can be loaded into one single SS file, where PKDGRAV will start calculate from.

To automatically set position and velocity instead of input them to RPX again and again, a bash script "`rpx-helper.sh`" was created, which can automatically redirect input into RPX. Note that all programs and scripts that were created during this experiment can be found following the link in the appendix.

2. Simulation and Visualisation

To simulate, the ss.par and the ssdraw.par file needs to be edit to get a clear view and integrate with appropriate time accuracy. The dynamical time of our comet is 3875 according to the RPG program, so dDelta should be set to

$$\frac{1}{\sqrt{G\rho}} \approx 0.0009 \tag{E3}$$

The unit is year/2π (G. Winter, et al., 2014).

To merge all frames into a watchable H.264 encoded mp4 file, the following command was used:

```
ffmpeg -i ./output/ras/out_file.%05d.ras -q:v 2 ./output/jpg/out%05d.jpg
```

(C1)

```
ffmpeg -i ./output/jpg/out%05d.jpg -b:v 16M -vcodec h264 -y ./output/output.mp4
```

(C2)

With this method, one can check `./output/jpg/` folder and look at the particle status at every time step, which will be convenient for the determination of the destruction time. The output mp4 file will be written to `./output/output.mp4`, as C2 showing.

Note that these commands are valid only when the names of output rasterised files are of the form "`out_file.xxxxx.ras`", where "`xxxxx`" is a number. If not, the format string needs to be modified to match the file name pattern. Also note that C2 will overwrite the file without asking for any confirmation.

3. Reconstruction of the Data

To automatically run the experiment with different parameters and quantitatively analysis the experiment data, a C# program "RocheeeLimit" was created, which can step the comet density, calculate and change the corresponding dDelta, run the simulation, calculate the median of the particle distance of the comet by time and repeat the process until the max iteration amount is reached. The program will eventually produce a CSV file with all data.

The program is also able to automatically calculate and determine the frame of destruction happened, and extract useful data. In this program, comets with a median of particle distances of larger than a certain pre-set value will be marked as destructed, then this module of the program will append calculate the corresponding distance and append the distance-density relation to a CSV file. By finishing all simulations, the CSV file will contain all the data of destruction distance for all simulated comets.

Results

1. The Simulation of the SL9 Destruction Event

The simulation result has successfully shown a “pearl string” pattern, which matches to the actual observation in 1994. The simulated object is torn apart into a narrow strip, and later the particles group into small collections, which are the “pearls” in the overall pattern.

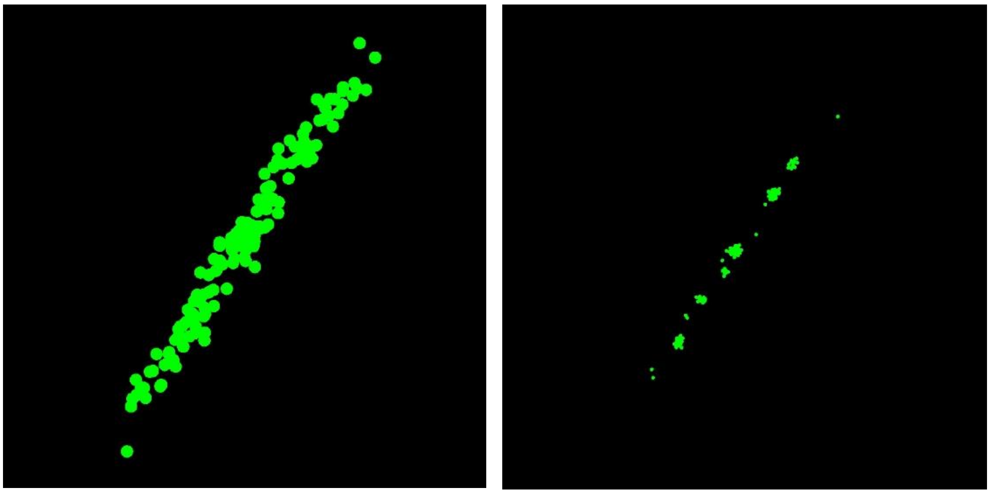

Fig.1

The torn strip of particles at time step 240 and the pearl-like groups forming under gravity at time step 323 respectively

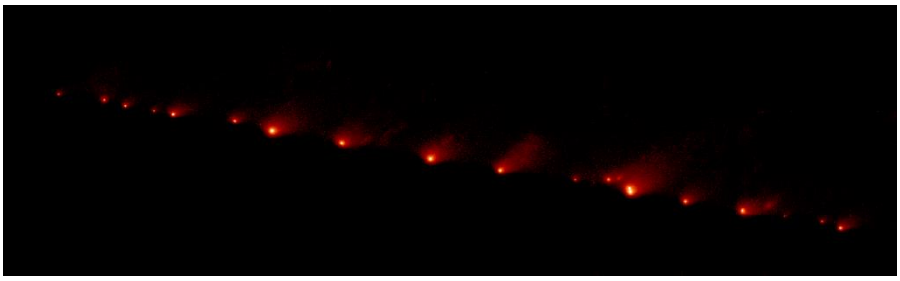

Fig.2 (Hubble Space Telescope, 1994)

Shoemaker-Levy 9 on 1994-05-17, taken by Hubble Space Telescope

2. The behaviour for comets with different densities

When plotting the median of distances vs. time step graphs of different densities, it is found that they can be roughly classified into three classes.

For small density comets with densities of around 0~700 kg/m^3, the gravity is relatively small. As a result, when being torn by tidal force, the resistance from its gravity is ignorable.

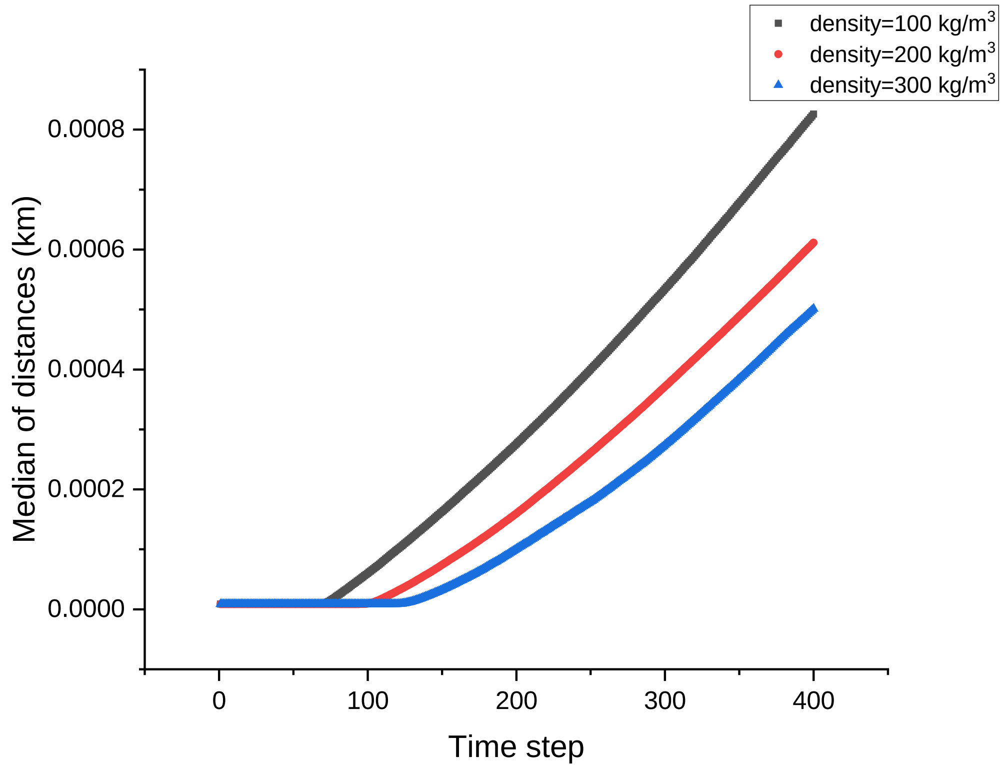

Fig. 3

The median of distances vs. time step graph with three different densities

As can be seen in fig.3, as the time evolving, when the destruction happens (at somewhere between time step 50 and 150 as the graph showing), the median of distances of the particles grow linearly, indicating the comets have disintegrated.

In the case of Jupiter, by simulation, such situation happens on comets that have a density of lower than 700 kg/m^3. For planets or even stars with larger mass, such standard is expected to be higher because the tidal force will be relatively stronger.

Comparing the curves among three different densities in fig.3, it is clear that as the density increase, the gravity increases so the speed of escaped particles decreases, explaining why the slope of the linear part of the curve becomes lower by density.

In addition, the point where curves start to raise delay in time axes as the density increasing, showing that it takes more time for comets with larger densities to be destructed.

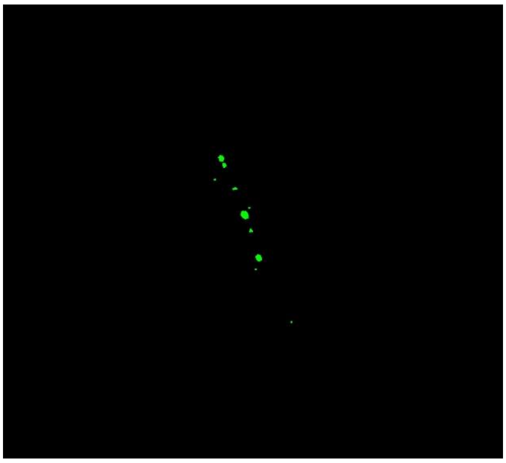

Fig.4

Particle(group)s from comet with a density of 600 kg/m^3 at time step 400

For medium density objects, the situation is relatively different. Most of the time, their gravity is strong enough to compete with the tidal force. Although some part of them may escape forever, their gravity is not ignorable and do have a significant impact on their shape.

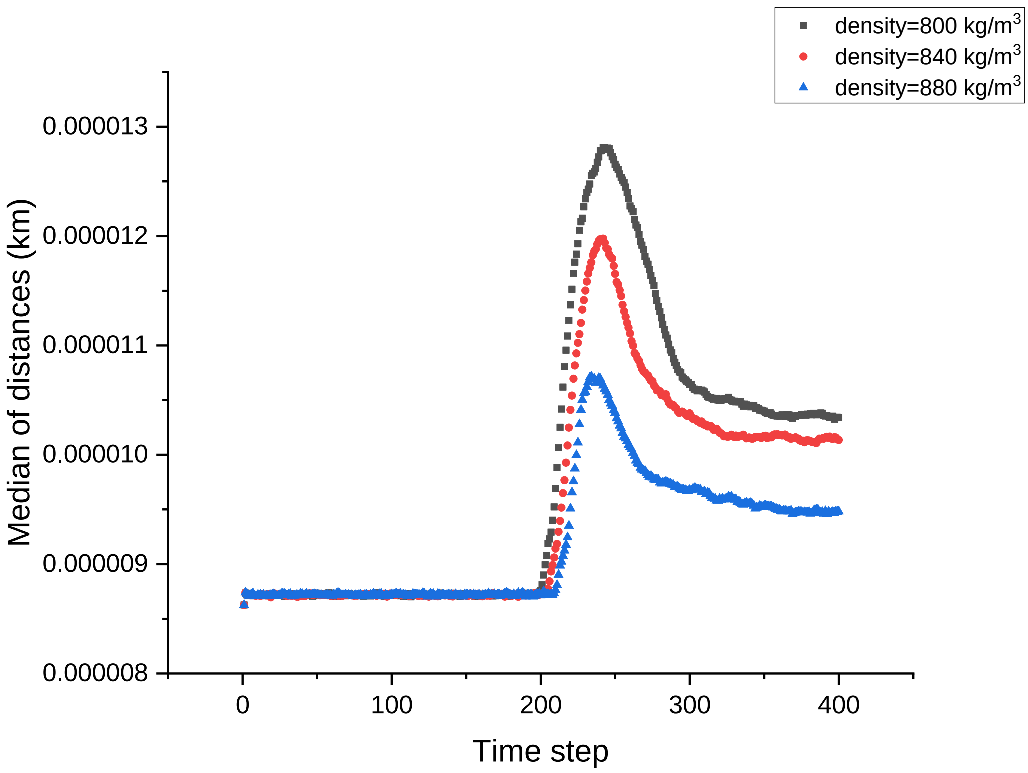

Fig.5

The median of distances vs. time step graph for medium density comets

There is no linear growing part in curves, indicating medium comets were not torn apart. The horizontal tail of the curve is higher than before, and that is because the shape of the comet is dramatically stretched during the reaction of the tidal force, so its overall density is expected to increase.

As can be seen, a few time steps after the destruction happened, the gravity from comets start to dominate again and pulling all particles back together. The reason for this is that SL9 at this time was passing its perigee. As the initial conditions for all simulations are the same as the SL9 on A.D. 1992-Jul-07 at 4 pm, their orbit characteristic will also be identical. As a result, after comets passing the perigee, the tidal force keeps decreasing, while the gravity of comets gains back control. By plotting the distance for comets along their orbit together with the fig.5, it is possible to verify such an explanation:

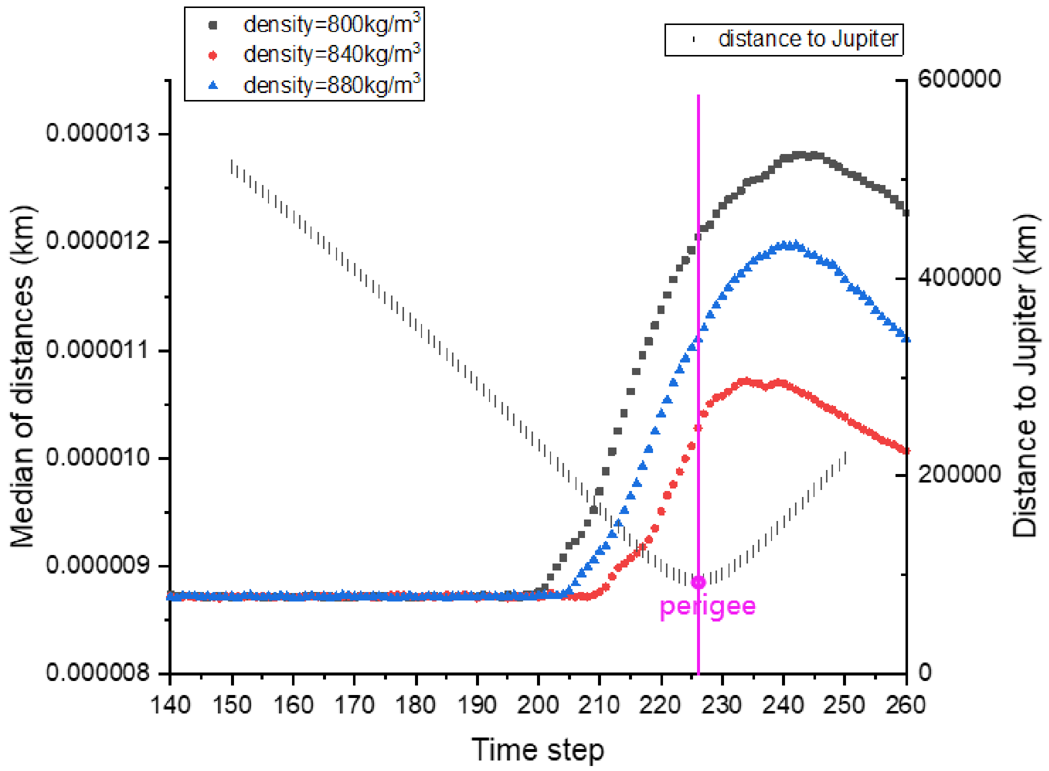

Fig.6

The median of distances vs. time step graph with distance to Jupiter

As fig.6 suggesting, particles are pulled back together just after the distance started to increase again. Also, the positions in time step of the peaks of different curves are indicating that the “pulling-back” event happens sooner for large density comets than small density ones.

Besides, according to JPEG image files such as fig.7, for a comet with medium density during the reaction of the tidal force, a small number of particles will be ripped off, which will escape forever.

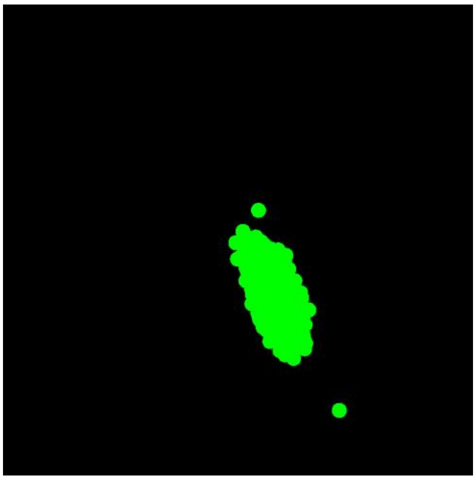

Fig.7

The comet with a density of 850 kg/m^3, with its two little ripped part at time step 400

In fig.7, it is also clear that the comet was dramatically stretched by the tidal force, matching the explanation for higher tails of curves in fig.5.

Finally, for large density objects, the situation is similar to medium density comets. However, their gravity is large enough to bound every single particle and avoid it to escape. As a result, curves generally have the same shapes as medium comets, but JPEG files will not show any escaped particles like fig.7 for medium density comets.

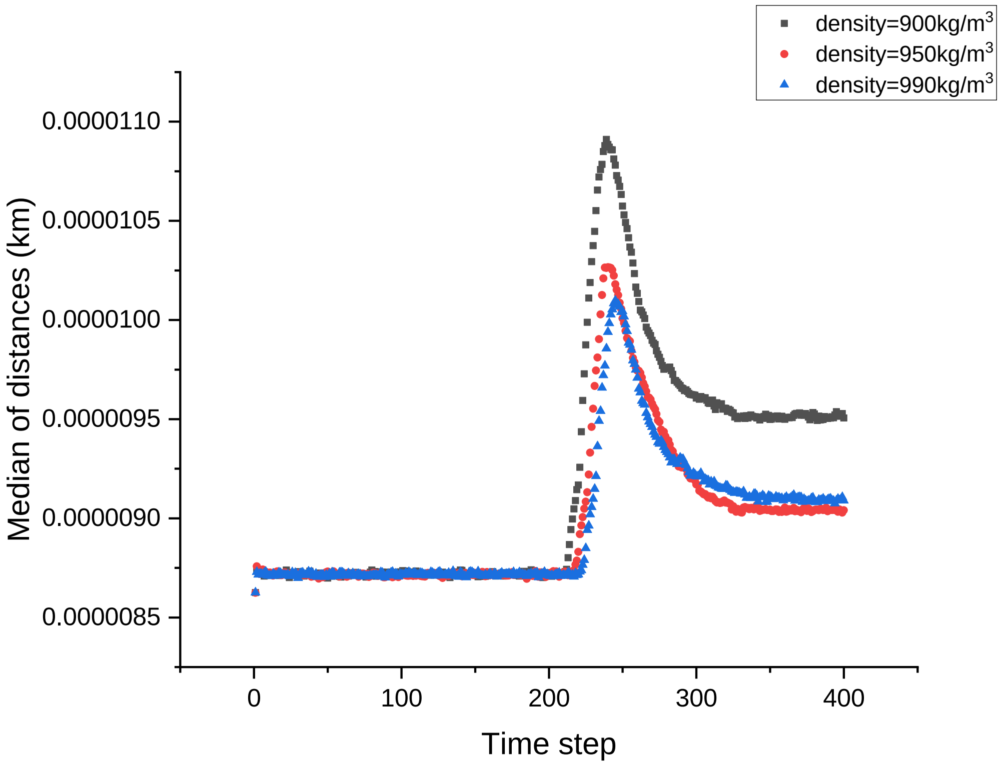

Fig.8

The median of distances vs. time step graph for large density comets

Notice that for density 950 kg/m^3 and density 990 kg/m^3, tails are not as well-separating as small and medium density comets, showing some type of "marginal effect" as the density increase, which means as the density increase to a certain amount, the deformation caused by tidal force will tend to be stable.

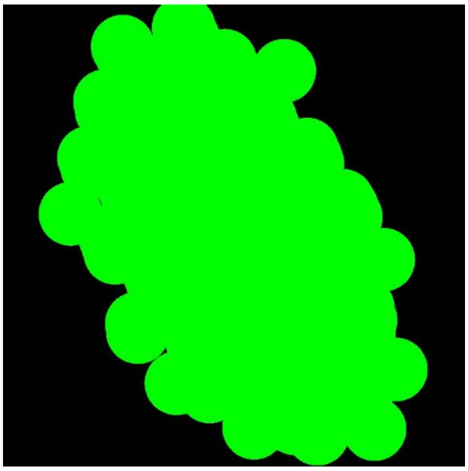

Fig.9
A squeezed comet with density 1000 kg/m^3 at time step 400

Comparing fig.8 to fig.7, large-density comets are significantly less deformed than medium-density comets, which is as expected because the bounding gravity for dense comets is larger.

Generally speaking, all three types of comets will be torn apart when passing Jupiter, but medium-density ones and high-density ones have a gravity that is large enough to restore the main structure before its "too late" after passing the perigee, while low-density ones will be disintegrated permanently.

3. The Relation between Density and Destruction Distance

With the analysis above, it is possible to determine a convincing standard of the "destruction" event.

Notice that in fig.6 and fig.8, medians of distances are around 0.000008 ~ 0.00001 km before the destruction, so it is reasonable to set a standard around this range. By doing so, the frame of destruction will be properly identified. However, some space needs to be reserved for potential fluctuation at the horizontal part, So in this case, 0.00001 km was used as the standard of destruction.

With such standard, a destruction time step can be determined. Hence by using the centre-of-mass position, the destruction position, as well as the distance to Jupiter, can be found.

By properly setting the "setup.par" file for RocheeeLimit program, a destruction distance vs. density table can be generated, from which a scatter plot can be produced:

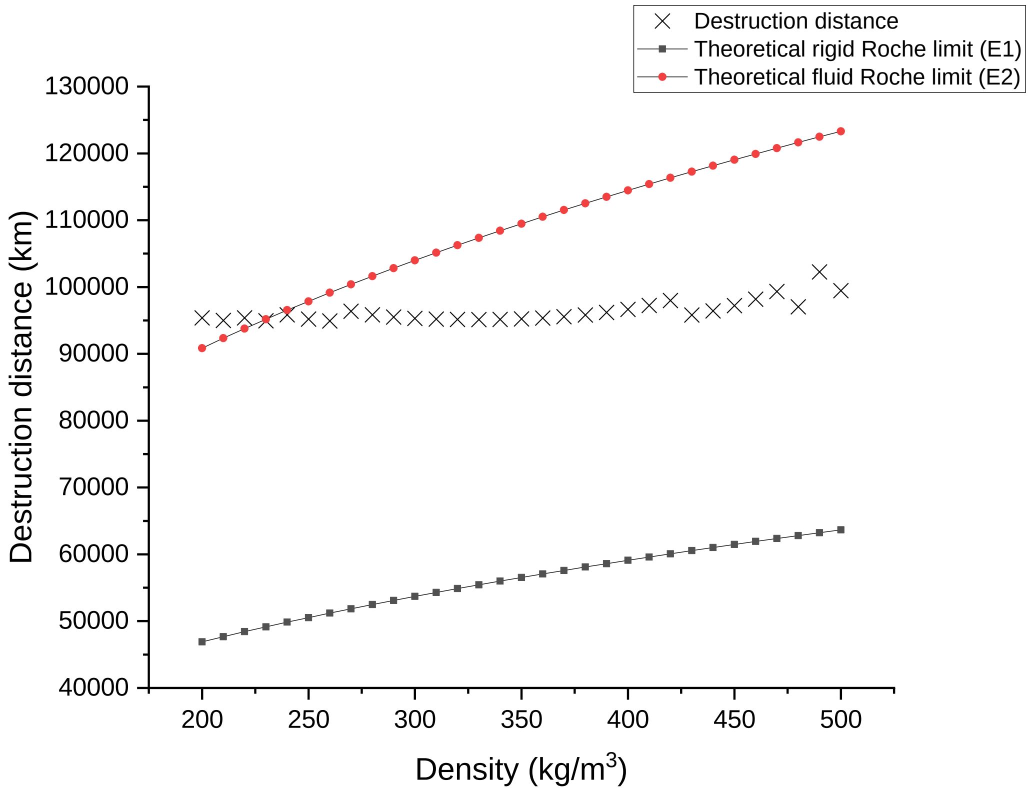

Fig.10
Destruction distance vs. density graph with a destruction standard of 0.00001 km

Most of the data points are within the range of fluid and rigid Roche limit, which fits the theory.

Discussion

There are still some aspects that can be improved for the methods in this experiment.

First of all, although the amount of data is very sufficient by using various types of programming tools, the utilization of data is still insufficient. In the follow-up study, it will be better to draw histograms of particle distances to show the particle distance distribution more clearly and to show the evolution of the distribution of particle distances.

On the other hand, the determination of the standard of destruction still requires a certain degree of in-depth discussion. Although some reasonable attempts have been made in this article, its quantitative and rigorous level is still insufficient.

In fig.10, the first few data points are off the prediction range, which might also be caused by the potentially improperly-set standard of destruction distance that requires further investigation.

Also, the method for this experiment to find the destruction distance is running the simulation with the same initial condition every time, and determine the destruction distance by determining the destruction time. However, it is not the only choice. For experiments in the future, it is also possible to change initial condition i.e. the orbit characteristic to tune the perigee and use the outcome after passing Jupiter as the guideline of whether the perigee distance is a destruction distance. One advantage of using such an approach is that there is no need to choose any quantitative standard to determine the destruction, as the determination is relatively complicated.

Finally, density is not the only property that can impact on the tidal reaction. Further research can also focus on how the radius or the particle number affects the tidal force reaction outcome, which can be achieved easily by modifying setup.par for RocheeeLimit program.

Conclusion

For comets with different densities, there are roughly three types of behaviour. For low-density comets, they will tend to be torn apart and form a pearl-string pattern; For medium-density comets and high-density comets, major structures will remain, but medium-density ones usually have a small number of escaped particles, while high-density ones are expected to constraint all particles on its surface.

The analyse also shows that orbit characteristic has a huge impact on the outcome of the tidal force reaction process. Medium-density comets and high-density comets are all torn apart as passing the perigee, which is identical to the outcome for low-density comets. But the orbit, as well as the velocity, help their gravity to gain back control later in time.

By setting a reasonable value as the standard of destruction, the theory of Roche limit was verified.

References

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Appendix

Link to Exp_T Programs and Scripts:
https://github.com/Deleeete/2nd-Year-Exp-T-Programs-and-Scripts

Usage has been stated in README.md.