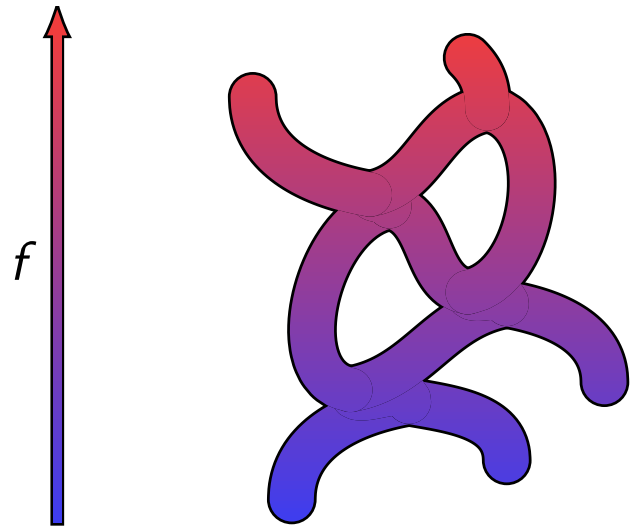
Sequential algorithm

Informal description

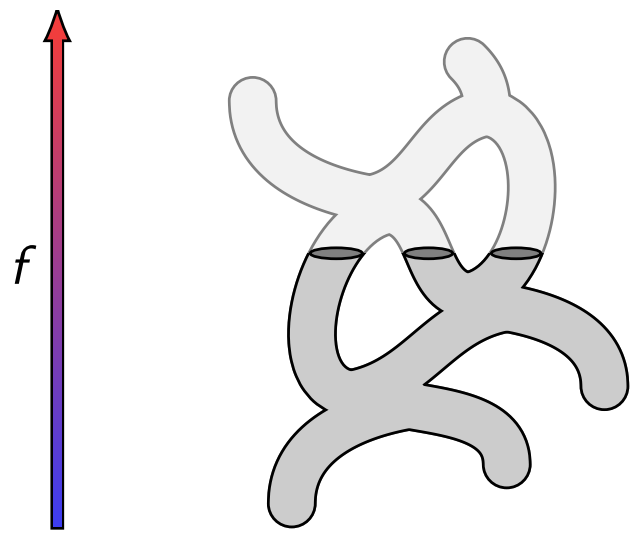
- Process the vertices of the mesh by **increasing** value of f.



Sequential algorithm

Informal description

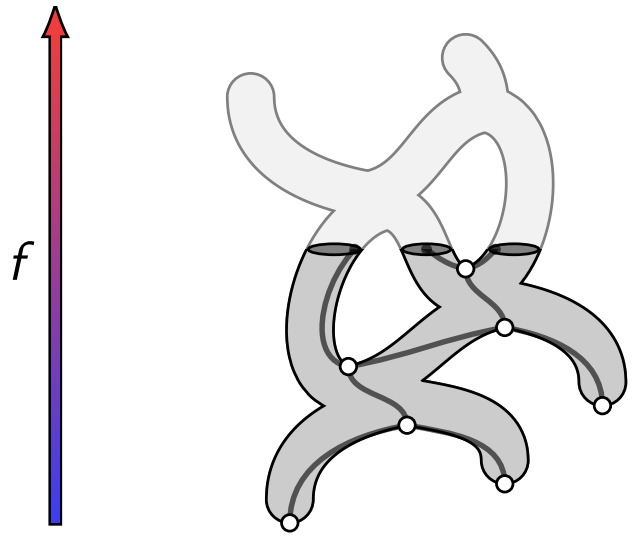
- Process the vertices of the mesh by **increasing** value of f.
- Construct the Reeb graph $\mathcal{R}(f)$ incrementally.



Sequential algorithm

Informal description

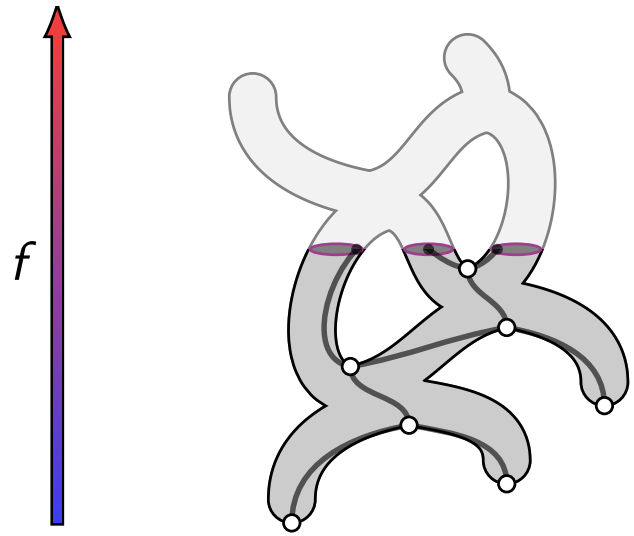
- Process the vertices of the mesh by **increasing** value of f.
- Construct the Reeb graph $\mathcal{R}(f)$ incrementally.
- While sweeping upwards, keep:
  - the **partial Reeb graph** constructed so far;



Sequential algorithm

Informal description

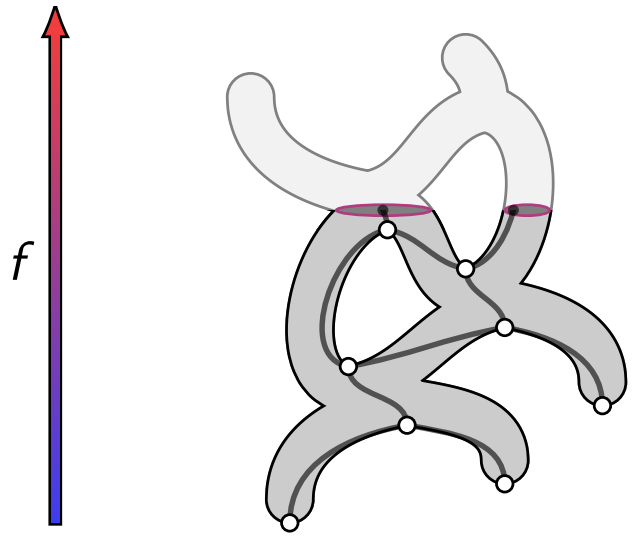
- Process the vertices of the mesh by **increasing** value of f.
- Construct the Reeb graph $\mathcal{R}(f)$ incrementally.
- While sweeping upwards, keep:
  - the **partial Reeb graph** constructed so far;
  - the current **level set** $f^{-1}(r)$.



Sequential algorithm

Informal description

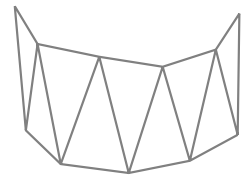
- Process the vertices of the mesh by **increasing** value of f.
- Construct the Reeb graph $\mathcal{R}(f)$ incrementally.
- While sweeping upwards, keep:
  - the **partial Reeb graph** constructed so far;
  - the current **level set** $f^{-1}(r)$.
- When processing a vertex, **update** the level set and the Reeb graph accordingly.



Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

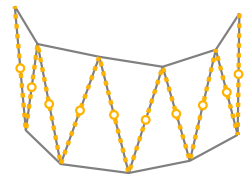


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;

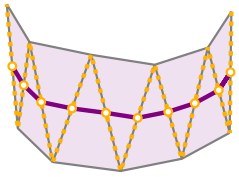


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



a triangle connects its two sides intersecting $f^{-1}(r)$

Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
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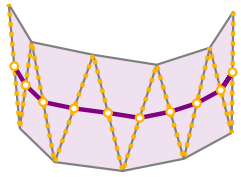


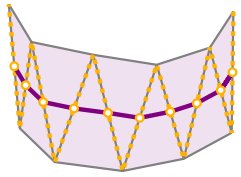
Updating G_r

Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

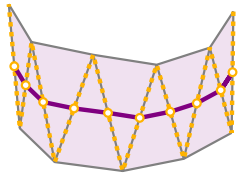
- **Trigger**: update when processing a vertex v.
  - from $r = f(v) - \epsilon$ to $r = f(v) + \epsilon$

Sequential algorithm

The preimage graph

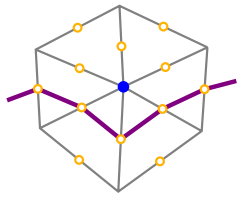
The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.

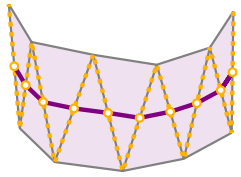


Sequential algorithm

The preimage graph

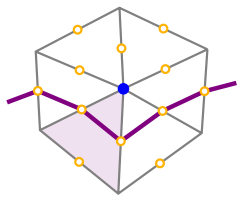
The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.

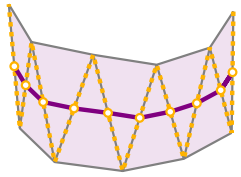


Sequential algorithm

The preimage graph

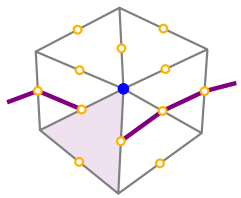
The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.

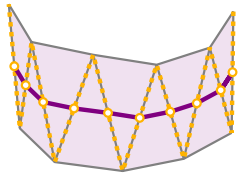


Sequential algorithm

The preimage graph

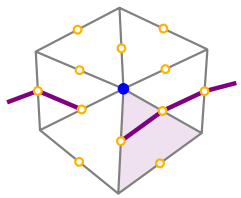
The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.

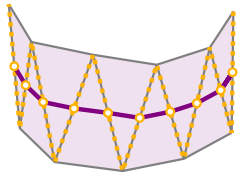


Sequential algorithm

The preimage graph

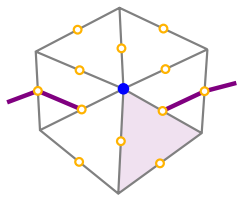
The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.

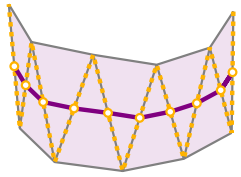


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.
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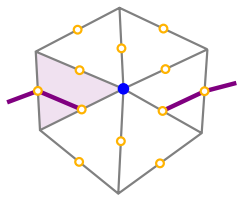
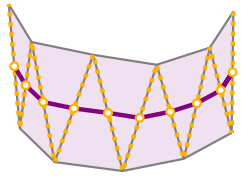


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.
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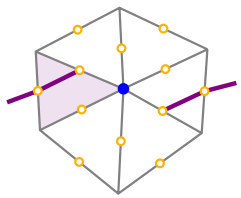
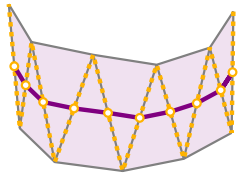


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.
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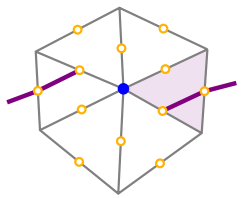
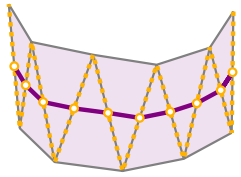


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.
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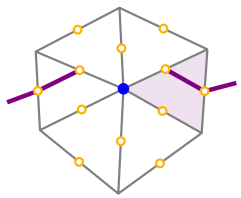
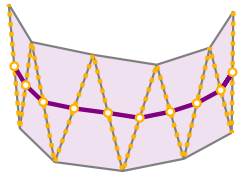


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.
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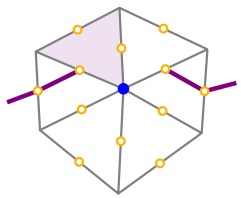
  3. v is the upper vertex of $\mathcal{T}$.

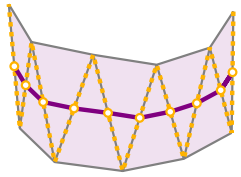


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
  1. v is the lower vertex of $\mathcal{T}$.
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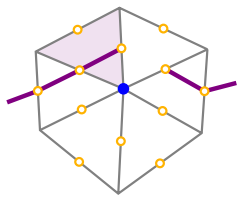
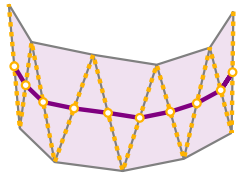


Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
- **edges** $\rightsquigarrow$ triangles of $\mathcal{M}$ intersecting $f^{-1}(r)$.



Updating G_r

- **Trigger**: update when processing a vertex v.
- **Action**: process each triangle $\mathcal{T}$ of Star(v) separately.
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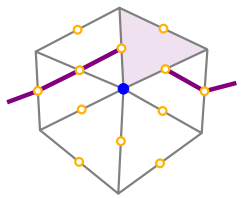

Sequential algorithm

The preimage graph

The level set $f^{-1}(r)$ can be represented by an abstract **graph** G_r:

- **nodes** $\rightsquigarrow$ edges of the mesh $\mathcal{M}$;
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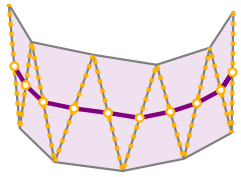

Updating G_r

- **Trigger**: update when processing a vertex v.
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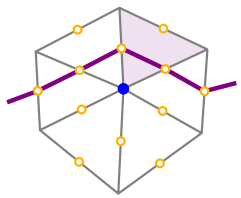

Sequential algorithm

The preimage graph

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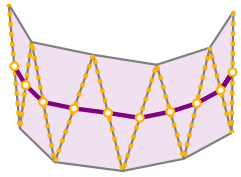

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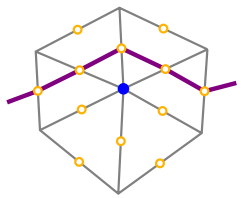

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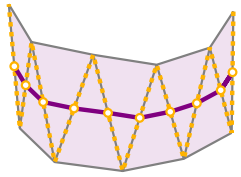

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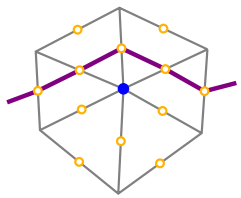

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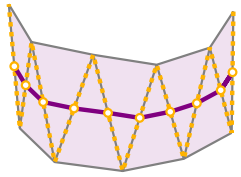

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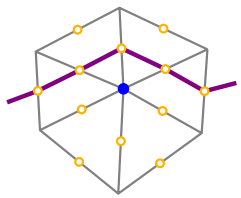

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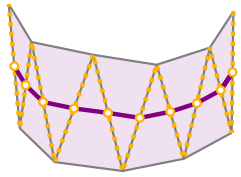

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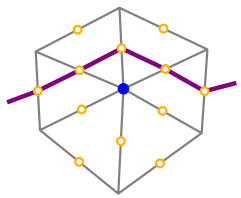

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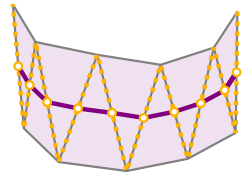

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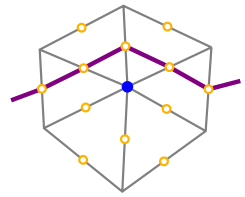

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$\rightsquigarrow$ offline dynamic connectivity problem

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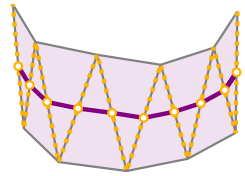

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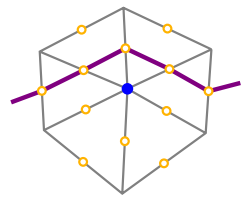

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$\rightsquigarrow$ offline dynamic connectivity problem $\rightsquigarrow$ **ST-trees**

support all the operations in $O(\log m)$