

# Reeb graph


## Definition

Given:

- ▶ a manifold  $\mathcal{M}$ ;
- ▶ a Morse function  $f: \mathcal{M} \rightarrow \mathbb{R}$  with distinct critical values;

the **Reeb graph** of  $f$  is the 1-dimensional simplicial complex

$$\mathcal{R}(f) = \mathcal{M} / \sim.$$

The **segmentation map** is the quotient map  $\Phi: \mathcal{M} \rightarrow \mathcal{R}(f)$ .  
  $x \sim y$  if they belong to the same connected component of  $f^{-1}(f(x))$

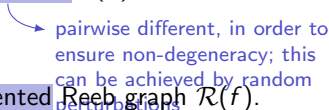

# Reeb graph

Desired algorithm


## Input:

- ▶ a 2-dimensional PL manifold  $\mathcal{M}$   
 $\rightsquigarrow$  a triangulated mesh  $\mathcal{M}$ ;
- ▶ a non-degenerate PL scalar field  $f$  on  $\mathcal{M}$   
 $\rightsquigarrow$  a scalar value  $f(v)$  for each vertex  $v$  of  $\mathcal{M}$ .

## Output:

- ▶ the augmented Reeb graph  $\mathcal{R}(f)$ .  
  


## Time complexity:

- ▶  $O(m \cdot \log m)$ , where  $m$  is the size of  $\mathcal{M}$ .  


**Parallelizable.**

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## Geometry of critical points

There are three kinds of critical points:

- ▶ (local) maximums  
 $\rightsquigarrow$  empty  $\text{Link}^+$ ;
- ▶ (local) minimums  
 $\rightsquigarrow$  empty  $\text{Link}^-$ ;
- ▶ saddles  
 $\rightsquigarrow$  disconnected  $\text{Link}^-$  and  $\text{Link}^+$ .

## How to detect them on a PL manifold?

Given a vertex  $v$ , the **star** of  $v$  is the union of all simplices containing  $v$ .

The **link** of  $v$  is the boundary of its star.

$$\text{Link}^+(v) = \{x \in \text{Link}(v) : f(x) > f(v)\}$$

$$\text{Link}^-(v) = \{x \in \text{Link}(v) : f(x) < f(v)\}$$