Mean Local Time of the Ascending Node

This memo describes the relationship between the mean local time of the ascending node (MLTAN) and right ascension of the ascending node (RAAN) of Earth-orbiting spacecraft. This geometric relationship defines the orientation of the spacecraft's orbital plane with respect to the mean Sun. The MLTAN/RAAN relationship is important for mission design of sun-synchronous, repeating ground track, and other types of mission orbits that must satisfy Sun-orbit plane and other lighting constraints. A numerical example is provided that uses the Multiyear Interactive Computer Almanac (MICA) to compute the characteristics of a typical spacecraft mission.

The relationship between the mean local time of the ascending node in hours and the right ascension of the ascending node (Ω) in degrees of a spacecraft is given by

$$MLTAN = \left(\Omega - \alpha_{\odot}^{m}\right)/15 + 12$$

where $\alpha_{\odot}^{m}$ is the right ascension of the mean Sun, also in degrees, computed at the time of the ascending node crossing. In this equation, the constant 15 represents the number of degrees of Earth angular motion in one solar hour. Both the spacecraft's RAAN and the right ascension of the mean Sun should be defined in the same coordinate system, usually true equator and equinox of date (TOD) or perhaps Earth mean equator and equinox of J2000 (EME2000).

We can re-arrange this equation and solve for the spacecraft RAAN required for a given MLTAN with

$$\Omega = \alpha_{\odot}^{m} + 15\left(MLTAN - 12\right)$$

The right ascension of the mean Sun is determined from the following expression

$$\alpha_{\odot}^{m} = \alpha_{\odot}^{a} + EqT$$

where $\alpha_{\odot}^{a}$ is the right ascension of the *apparent* Sun and EqT is called the *equation of time*.

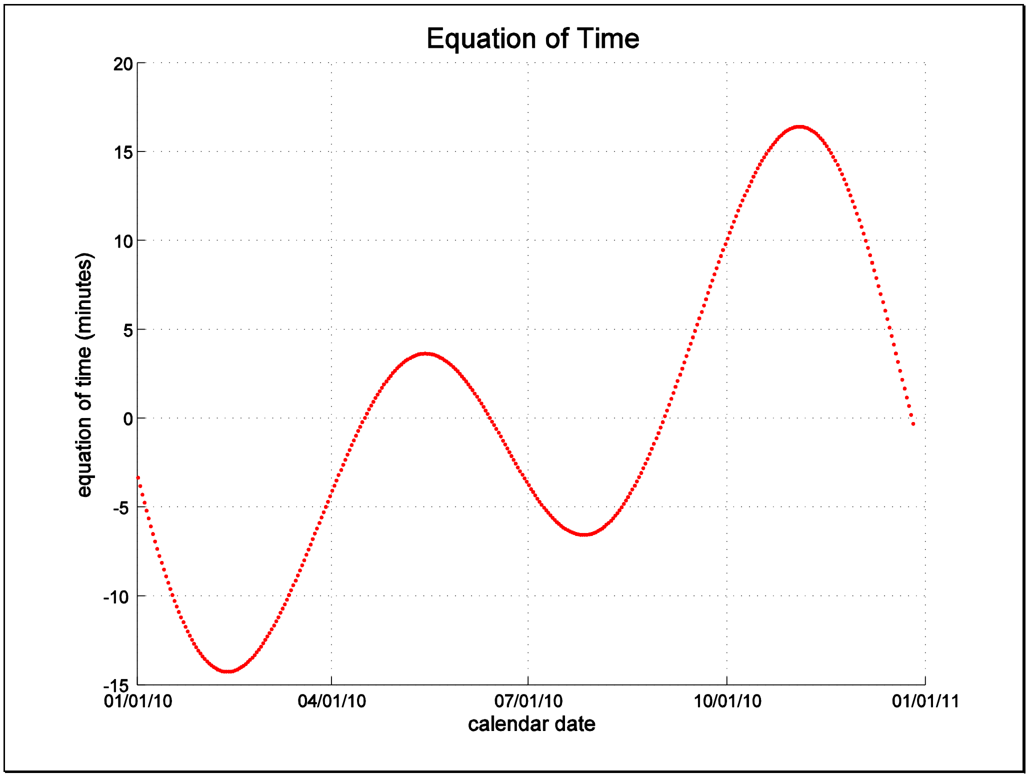
As defined by the American astronomer Simon Newcomb in 1895, the fictitious mean Sun travels along the celestial equator at a uniform rate, completing one revolution in the same time period as the apparent Sun. The time required for one revolution is called the tropical year.

According to the *Astronomical Almanac*,

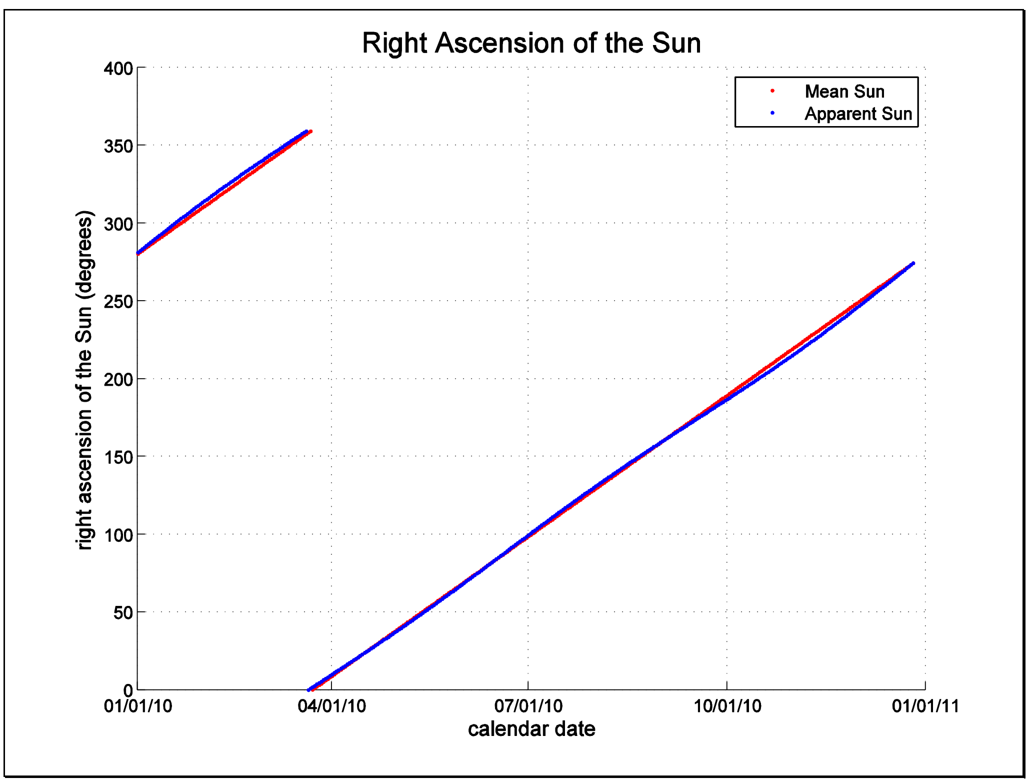
> Apparent position implies the coordinates of the Sun at a specific date, obtained by removing from the directly observed position the effects that depend on the topocentric location of the observer; i.e., refraction, diurnal aberration, and geocentric (diurnal) parallax. Thus, the position at which the Sun would actually be seen from the center of the Earth, if the Earth were transparent, non-refracting, and massless, referred to the true equator and equinox.

It is important not to confuse the mean sun with mean coordinate systems or mean orbital elements. Additional information and clarifications extracted from MICA and the *Explanatory Supplement to the Astronomical Almanac* can be found in the Useful Time and Astronomical Definitions section later in this document. The appendix describes several algorithms for computing the equation of time.

The equation of time indicates how the apparent Sun leads or trails the mean Sun. The following is a plot of the behavior of the equation of time for the 2010 calendar year.



The following is a plot of the behavior of the right ascension of the mean and apparent Sun for the 2010 calendar year.



Alternative Formulation

The equation of time, in degrees, can also be expressed as

$$\begin{aligned} EqT &= GHA_{\odot}^{a} - GHA_{\odot}^{m} \\ &= \left(GAST - \alpha_{\odot}^{a}\right) - \left(15UT - 180^{\circ}\right) \end{aligned}$$

where GHA is the Greenwich hour angle of the apparent or mean Sun in degrees. In the second form of this equation, GAST represents the Greenwich apparent sidereal time, in degrees, and UT is the "universal time", in hours.

If we substitute this form of the EqT equation into the equation for $\alpha_{\odot}^{m}$ given previously,

$$\begin{aligned} \alpha_{\odot}^{m} &= \alpha_{\odot}^{a} + \left\{\left(GAST - \alpha_{\odot}^{a}\right) - \left(15UT - 180^{\circ}\right)\right\} \\ &= \alpha_{\odot}^{a} + GAST - \alpha_{\odot}^{a} - 15UT + 180^{\circ} \\ &= GAST - 15UT + 180^{\circ} \end{aligned}$$

Now substitute this expression into the original MLTAN equation as follows,

$$\begin{aligned} MLTAN &= \left\{\Omega - \left(GAST - 15UT + 180^{\circ}\right)\right\}/15 + 12 \\ &= \Omega/15 - GAST^{h} + UT - 12 + 12 \\ &= \Omega^{h} - GAST^{h} + UT \end{aligned}$$

The part of this last equation given by $\Omega - GAST$ is the east longitude of the spacecraft. In this form of the MLTAN equation, UT, Ω and $GAST$ are expressed in hours.

Finally, since we are interested in conditions at the ascending node, the MLTAN in hours is

$$MLTAN = \lambda_{AN}^{E} + UT_{AN}$$

where λ_{AN}^{E} is the east longitude of the ascending node evaluated at the time of the ascending node crossing, and UT_{AN} is the universal time of the ascending node crossing, both in the unit of hours. This form of the MLTAN equation is used by several spacecraft and launch vehicle organizations.

<u>Important Note</u>

Since the computation of Greenwich sidereal time is based on the UT1 time scale as is solar time-based Earth rotation, the "universal time" used in this alternate equation should be with respect to the UT1 time scale. However, the use of Coordinated Universal Time (UTC) in these calculations introduces a small error as explained later in this document.

Numerical Example

This section describes the computation of the RAAN required for a user-defined MLTAN. The numerical data required for the calculations is determined using version 2.2.1 of the MICA computer program created by the Astronomical Applications Department of the United States Naval Observatory (USNO) and published by Willmann-Bell, Inc., (www.willbell.com).

For this example, we will use a desired MLTAN value of 1800 hours (6 pm ascending node). Furthermore, we assume the UTC of the spacecraft's ascending node is 15:30:45 on June 20, 2010.

We can use the *apparent geocentric equator of date* position calculation feature of MICA to determine the right ascension of the apparent Sun along with the equation of time. Here's the MICA screen output for this example.

```
                                Sun

                      Apparent Geocentric Positions
                     True Equator and Equinox of Date

     Date        Time          Right      Declination    Distance    Equation
          (UT1)              Ascension                               of Time
                h  m   s      h                °             AU        m   s
  2010 Jun 20 15:30:45.0     5.9423375    + 23.435839   1.016176648   - 1 35.1
```

Notice that MICA uses UT1 as its fundamental time argument. We have specified the time of the ascending node crossing on the UTC time scale. However, UT1 and UTC never differ by more than 0.9 seconds which introduces a very small error in our calculations.

From this information, the right ascension of the mean Sun is

$$\alpha_{\odot}^{m} = \alpha_{\odot}^{a} + EqT = 5.9423375 \text{ hours} + (-0.0264) \text{ hours} = 5.9159375 \text{ hours} = 88.73906 \text{ degrees}$$

The required mission orbit RAAN is calculated according to

$$\Omega = \alpha_{\odot}^{m} + 15\left(MLTAN - 12\right) = 88.73906 \text{ degrees} + [15\ (18 - 12)] \text{ degrees} = 178.73906 \text{ degrees}$$

As a check, we can compute the MLTAN from the spacecraft RAAN according to

$$MLTAN = \left(\Omega - \alpha_{\odot}^{m}\right)/15 + 12 = [(178.73906 - 88.73906) / 15] \text{ hours} + 12 \text{ hours} = 18.0 \text{ hours}$$

Alternative Method

Recall the MLTAN alternative equation as

$$MLTAN = \lambda_{AN}^{E} + UT_{AN}$$

First we calculate the east longitude of the ascending node, in hours, using

$$\lambda_{AN}^{E} = \Omega^{h} - \alpha_{g}^{h}$$

where α_{g}^{h} is the right ascension of Greenwich in hours. Geographic longitude is measured positive east of the Greenwich or "prime" meridian.

For this example, MICA provides the following output for the Greenwich sidereal time.

```
                              SIDEREAL TIME


                Location:     0°00'00.0",   0°00'00.0",     0m
                  (Longitude referred to Greenwich meridian)
```

Date		Time	Greenwich Sidereal Time Mean	App.	Local Sidereal Time Mean	App.	Equation of the Equinoxes
(UT1)							
		h m s	h m s	s	h m s	s	s
2010 Jun 20		15:30:45.0	9 25 41.3409	42.3538	9 25 41.3409	42.3538	+1.0130

The Greenwich *apparent* sidereal time is

$$\text{GAST} = 9{:}25{:}42.3538 = 9.428316 \text{ hours}$$

Therefore, for this example the east longitude of the ascending node in hours is

$$\lambda_{AN}^{E} = \Omega - \alpha_g = (178.73906 / 15) \text{ hours} - 9.428316 \text{ hours} = 2.48762 \text{ hours}$$

Finally, the MLTAN in hours is

$$MLTAN = \lambda_{AN}^{E} + UT_{AN} = 2.48762 \text{ hours} + 15.5125 \text{ hours} = 18.0 \text{ hours}$$

Useful Time and Astronomical Definitions

Celestial sphere

The celestial sphere is an imaginary sphere of arbitrary radius upon which celestial bodies may be considered to be located. As circumstances require, the celestial sphere may be centered at the observer, at the Earth's center, or at any other location.

Equinox

An equinox is either of the two points on the celestial sphere at which the ecliptic intersects the celestial equator; also the time at which the Sun passes through either of these intersection points; i.e., when the apparent longitude of the Sun is 0° or 180°.

Mean equator and equinox

The celestial reference system defined by the orientation of the Earth's equatorial plane on some specified date together with the direction of the dynamical equinox on that date, neglecting nutation. Thus, the mean equator and equinox are affected only by *precession*.

Hour circle

The hour circle is a great circle on the celestial sphere that passes through the celestial pole and is therefore perpendicular to the celestial equator.

Hour angle

The angular distance on the celestial sphere measured westward along the celestial equator from the meridian to the hour circle that passes through a celestial object.

Universal Time, UT1

UT1 is a time scale that is based on the rotation of the Earth. UT1 is related to Terrestrial Time (TT) by the formula, UT1 = TT - ΔT where ΔT is determined from astronomical observations (its value is now about one minute). In current practice, UT1 is defined by its relationship to sidereal time, an observable quantity.

Coordinated Universal Time, UTC

The UTC time scale is the basis of international civil time keeping. For most places, civil time is simply an integral number of hours offset from UTC. UTC is a hybrid time scale: its rate is the same as that of International Atomic Time (TAI), but its epoch is occasionally adjusted in one-second steps (leap seconds) to keep it within 0.9 second of UT1.

Sidereal time

Sidereal time is a direct, observable measure of the rotation of the Earth with respect to the stars. The sidereal time at any location is equal to the apparent right ascensions of stars transiting the local meridian.

Greenwich Mean Sidereal Time, GMST

Greenwich mean sidereal time is the Greenwich hour angle of the mean equinox of date.

Local Hour Angle, LHA

It is the local sidereal time minus the right ascension, LHA = LST – RA, where LHA is the local hour angle, LST is the local sidereal time and RA is the right ascension. The local hour angle is measured in units of time: 1 hour for each 15 degrees from the local meridian.

Mean Solar Time

A measure of time based conceptually on the diurnal motion of a fiducial point, called the fictitious mean Sun, with uniform motion along the celestial equator.

Sidereal Time

Sidereal time is the hour angle of the equinox; hence a measure of the rotation of the Earth with respect to the stars rather than the Sun. If the mean equinox is used, the result is mean sidereal time; if the true equinox is used, the result is apparent sidereal time. The hour angle can be measured with respect to the local meridian or the Greenwich meridian, yielding, respectively, local or Greenwich (mean or apparent) sidereal times.

Tropical year

A tropical year is the period of time for the ecliptic longitude of the Sun to increase 360 degrees. Since the Sun's ecliptic longitude is measured with respect to the equinox, the tropical year comprises a complete cycle of seasons, and its length is approximated by the civil (Gregorian) calendar. The mean tropical year is approximately 365.24219 mean solar days.

Additional Equations

Local mean sidereal time

$$LMST = GMST + \lambda^E$$

where *GMST* is the Greenwich mean sidereal time and λ^E is the east longitude of the location of interest.

Local apparent sidereal time

$$LAST = GAST + \lambda^E$$

where *GAST* is the Greenwich apparent sidereal time and λ^E is the east longitude of the location of interest.

Mean solar time at Greenwich

$$MSTG = GMST - \alpha_\odot^m + 12$$

where *GMST* is the Greenwich mean sidereal time.

Local mean solar time

$$LMST = MSTG + \lambda^E$$

where *MSTG* is the Greenwich mean solar time and λ^E is the east longitude of the location of interest.

Algorithm Resources

Multiyear Interactive Computer Almanac, USNO and Willmann-Bell, Inc., 1998-2005.

Danby, J. M. A., *Fundamentals of Celestial Mechanics*, Willmann-Bell, Inc., 1988.

Meeus, J., *Astronomical Algorithms*, Willmann-Bell, Inc., 1991.

Moulton, F. R., *An Introduction to Celestial Mechanics*, Dover, 1970.

Seidelmann, P. K. (Editor), *Explanatory Supplement to the Astronomical Almanac*, University Science Books, 1992.

Hughes, David W., Yallop, B. D., and Hohenkerk, C. Y., “The Equation of Time”, *Monthly Notices of the Royal Astronomical Society* (1989) **238**, 1529-1535.

Appendix

Algorithms for Computing the Equation of Time

This appendix describes two numerical methods that can be used to compute the equation of time for a user-defined epoch. It also references two publications with algorithms suitable for computers.

Algorithm 1

This simple algorithm is described on page 484 of *Explanatory Supplement to the Astronomical Almanac*.

The fundamental time argument for this method is the number of Julian centuries given by

$$T = \frac{Jdate - 2451545.0}{36525}$$

where *Jdate* is the Julian date of the user-defined epoch.

The equation of time, in degrees, is determined from

$$EqT = -1.915 \sin G - 0.020 \sin 2G + 2.466 \sin 2\lambda - 0.053 \sin 4\lambda$$

where

$$G = 357.528 + 35999.050T$$

$$\lambda = L + 1.915 \sin G + 0.020 \sin 2G$$

$$L = 280.460 + 36000.770T$$

In these equations, G is the mean anomaly of the Sun, λ is the ecliptic longitude of the Sun, and L is the mean longitude of the Sun.

Algorithm 2

This more accurate algorithm is described in Chapter 27 of *Astronomical Algorithms* by Jean Meeus.

The fundamental time argument for this method is the number of Julian millennia given by

$$T = \frac{Jdate - 2451545.0}{365250}$$

where *Jdate* is the Julian date of the user-defined epoch.

The equation of time, in degrees, is determined from

$$EqT = L_0 - 0.0057183 - \alpha_{\odot} + \Delta\psi \cos\varepsilon$$

In this equation, $\alpha_{\odot}$ is the right ascension of the apparent Sun, $\Delta\psi$ is the nutation in longitude, ε is the true obliquity of the ecliptic, and L_0 is the mean longitude of the Sun given by

$$\begin{aligned} L_0 = {} & 280.4664567 + 360007.6982779T + 0.03032028T^2 \\ & + T^3/49931 - T^4/15299 - T^5/1988000 \end{aligned}$$

The equations for computing the apparent right ascension of the Sun can be found in Chapter 24 of *Astronomical Algorithms* and the algorithms for computing the nutation in longitude and obliquity of the ecliptic are described in Chapter 21 of the same reference.

Additional equation of time algorithms suitable for computer implementation can be found in the following publications;

Hughes, David W., Yallop, B. D., and Hohenkerk, C. Y., "The Equation of Time", *Monthly Notices of the Royal Astronomical Society* (1989) **238**, 1529-1535.

Vallado, David, *Fundamentals of Astrodynamics and Applications*, Third Edition, Microcosm Press, 2007.