#ex1

$$eq\_1a := x(n+1) = \left(\frac{(n+1)}{(n+2)}\right)^2 \cdot x(n) + \frac{1}{(n+2)};$$

$$eq\_1a := x(n+1) = \frac{(n+1)^2\, x(n)}{(n+2)^2} + \frac{1}{n+2} \tag{1}$$

$$ans := rsolve(\{eq\_1a, x(0) = 1\}, x(n));$$

$$ans := \frac{n+2}{2\,(n+1)} \tag{2}$$

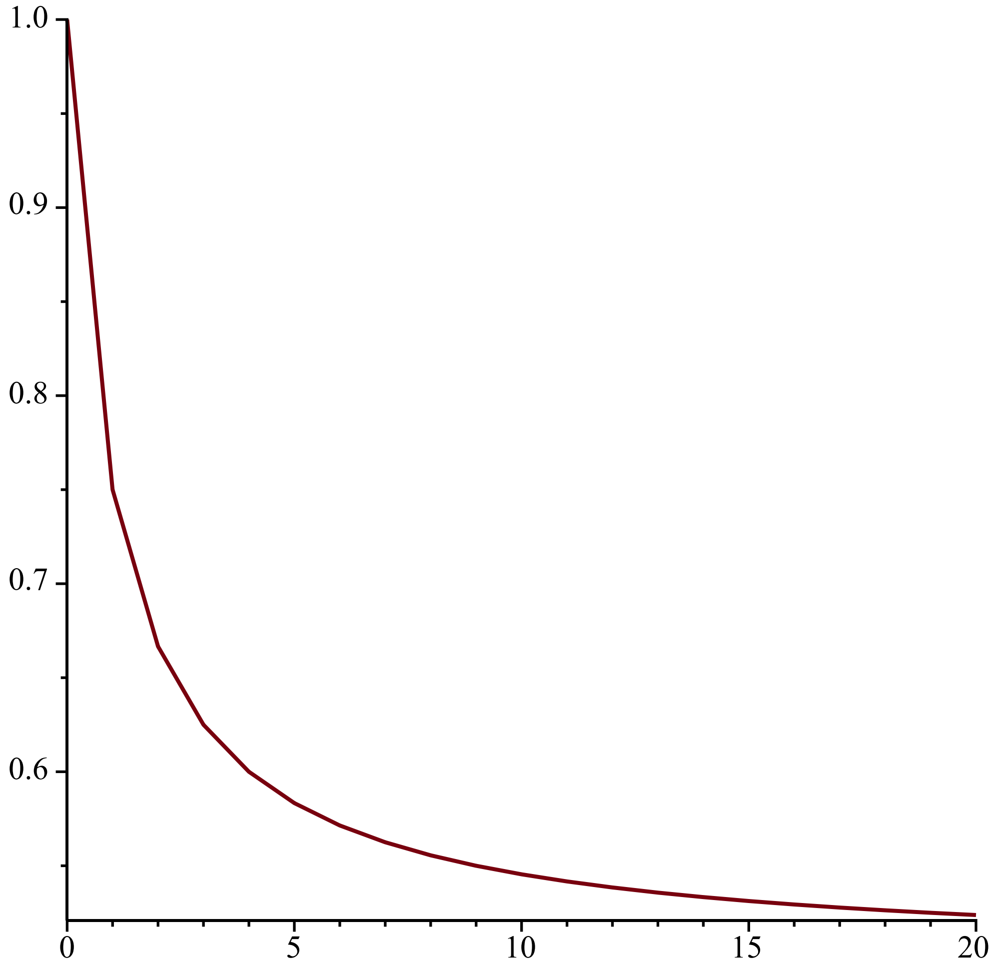
$$sol := unapply(ans, n);$$

$$sol := n \mapsto \frac{n+2}{2\cdot(n+1)} \tag{3}$$

$$[n, sol(n)]\$n = 0..10;$$

$$[0, 1], \left[1, \frac{3}{4}\right], \left[2, \frac{2}{3}\right], \left[3, \frac{5}{8}\right], \left[4, \frac{3}{5}\right], \left[5, \frac{7}{12}\right], \left[6, \frac{4}{7}\right], \left[7, \frac{9}{16}\right], \left[8, \frac{5}{9}\right], \left[9, \frac{11}{20}\right], \left[10, \frac{6}{11}\right] \tag{4}$$

$$plot([[n, sol(n)]\$n = 0..20]);$$



$eq\_1b := x(n+3) - 4\cdot x(n+2) + x(n+1) + 6\cdot x(n) = 60\cdot 4^n;$

$$eq\_1b := x(n+3) - 4\,x(n+2) + x(n+1) + 6\,x(n) = 60\,4^n \quad \textbf{(5)}$$

$ans := rsolve(\{eq\_1b, x(0) = 2, x(1) = 12, x(2) = 12\}, x(n));$

$$ans := -4\,(-1)^n - 16\,3^n + 16\,2^n + 6\,4^n \quad \textbf{(6)}$$

$sol := unapply(ans, n);$

$$sol := n \mapsto -4\cdot(-1)^n - 16\cdot 3^n + 16\cdot 2^n + 6\cdot 4^n \quad \textbf{(7)}$$

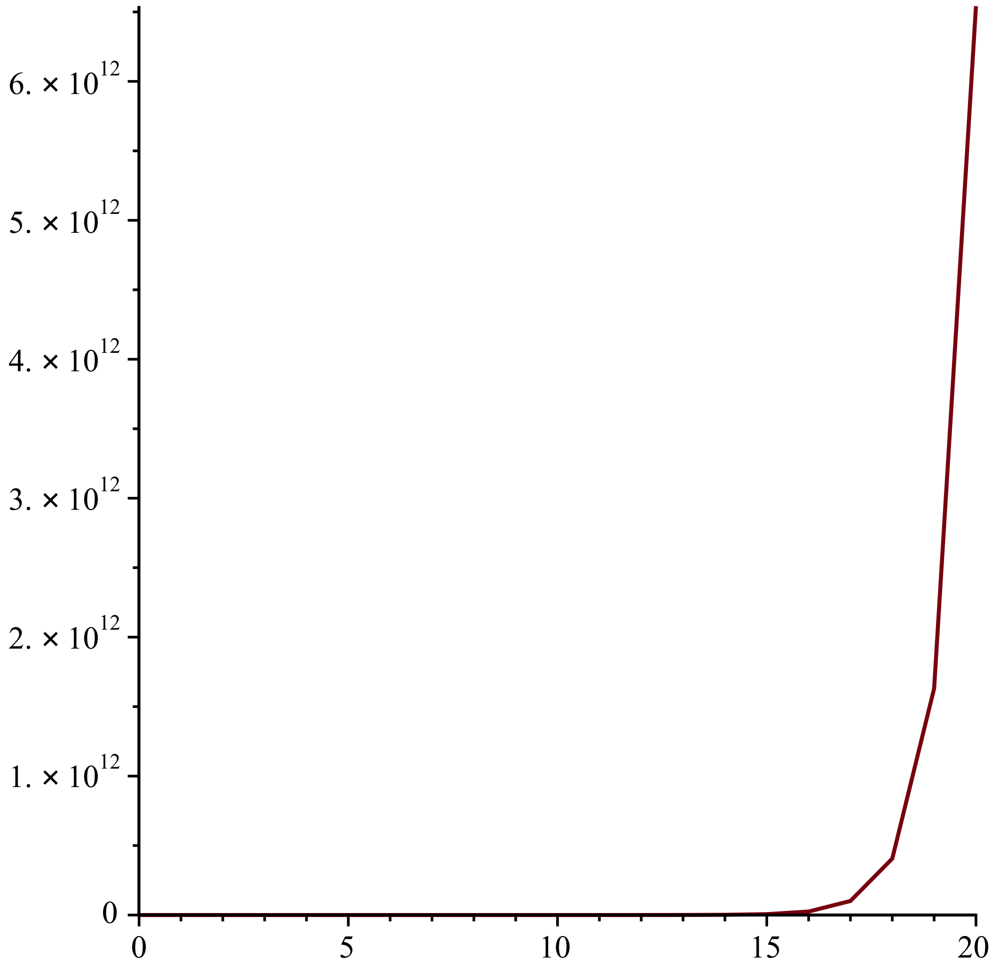
$sol := n \rightarrow evalf(ans);$

$$sol := n \mapsto evalf(ans) \quad \textbf{(8)}$$

$seq([n, sol(n)], n = 0..10);$

$[0, 2.], [1, 12.], [2, 12.], [3, 84.], [4, 492.], [5, 2772.], [6, 13932.], [7, 65364.], [8, 292332.], [9, 1.266132 \times 10^6], [10, 5.363052 \times 10^6]$ **(9)**

$plot([[n, sol(n)]\$n = 0..20]);$



$eq\_1c := x(n+1) = \frac{(2 \cdot x(n))}{(1 + 4 \cdot x(n))};$

$$eq\_1c := x(n+1) = \frac{2\,x(n)}{1 + 4\,x(n)} \quad \textbf{(10)}$$

$eq\_1c\_y := y(n+1) = \frac{1}{2} \cdot y(n) + 2;$

$$eq\_1c\_y := y(n+1) = \frac{y(n)}{2} + 2 \quad \textbf{(11)}$$

$ans\_y := rsolve(\{eq\_1c\_y, y(0) = 1\}, y(n));$

$$ans\_y := -3\left(\frac{1}{2}\right)^n + 4 \quad \textbf{(12)}$$

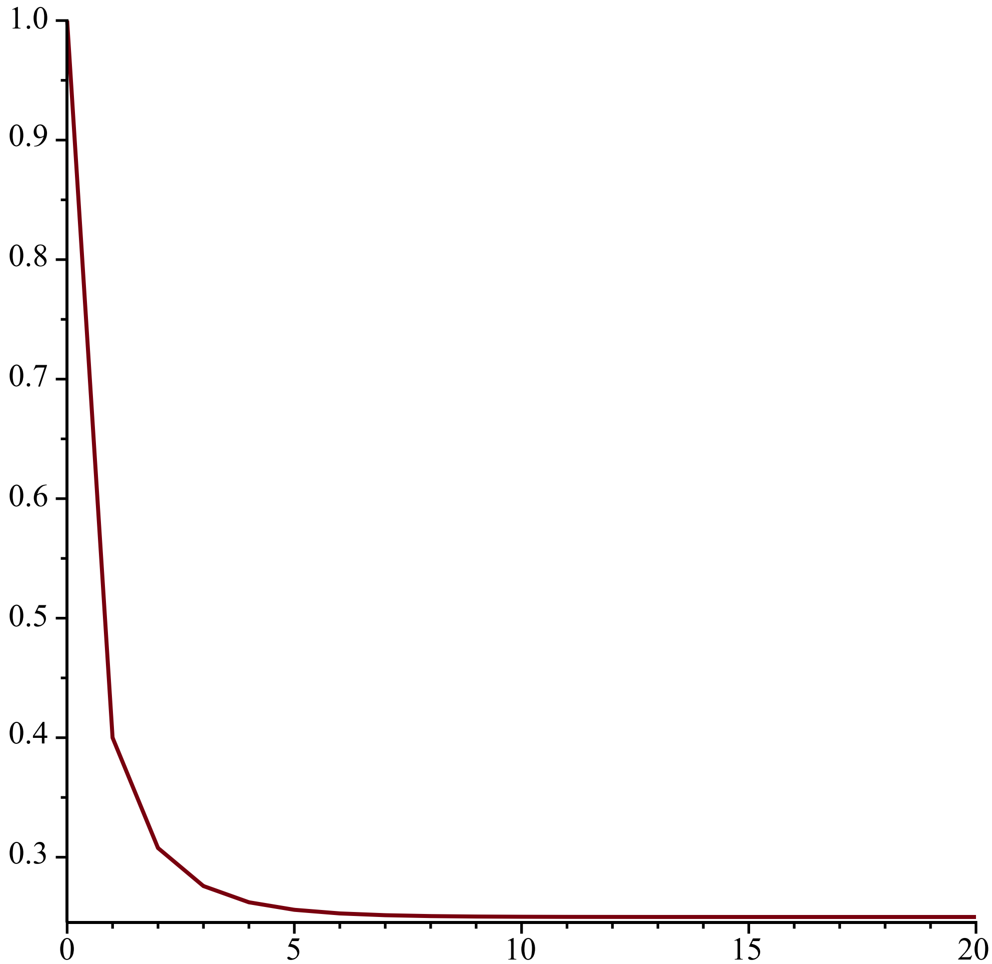
$sol := n \rightarrow \frac{1}{\left(4 - 3 \cdot \left(\frac{1}{2}\right)^n\right)};$

$$sol := n \mapsto \frac{1}{4 - 3 \cdot \left(\frac{1}{2}\right)^n} \tag{13}$$

$seq([n, sol(n)]\$n = 0..10);$

$$[0, 1], \left[1, \frac{2}{5}\right], \left[2, \frac{4}{13}\right], \left[3, \frac{8}{29}\right], \left[4, \frac{16}{61}\right], \left[5, \frac{32}{125}\right], \left[6, \frac{64}{253}\right], \left[7, \frac{128}{509}\right], \left[8, \frac{256}{1021}\right], \left[9, \frac{512}{2045}\right], \left[10, \frac{1024}{4093}\right] \tag{14}$$

$plot([[n, sol(n)]\$n = 0..20]);$



#ex 2

$f := x \rightarrow \frac{1}{2} \cdot \left(x + \frac{7}{x}\right);$

$$f := x \mapsto \frac{x}{2} + \frac{7}{2 \cdot x} \tag{15}$$

$fixedpoint := solve(x = f(x), x);$

$$fixedpoint := \sqrt{7}, -\sqrt{7} \tag{16}$$

#convergent to sqrt(7)

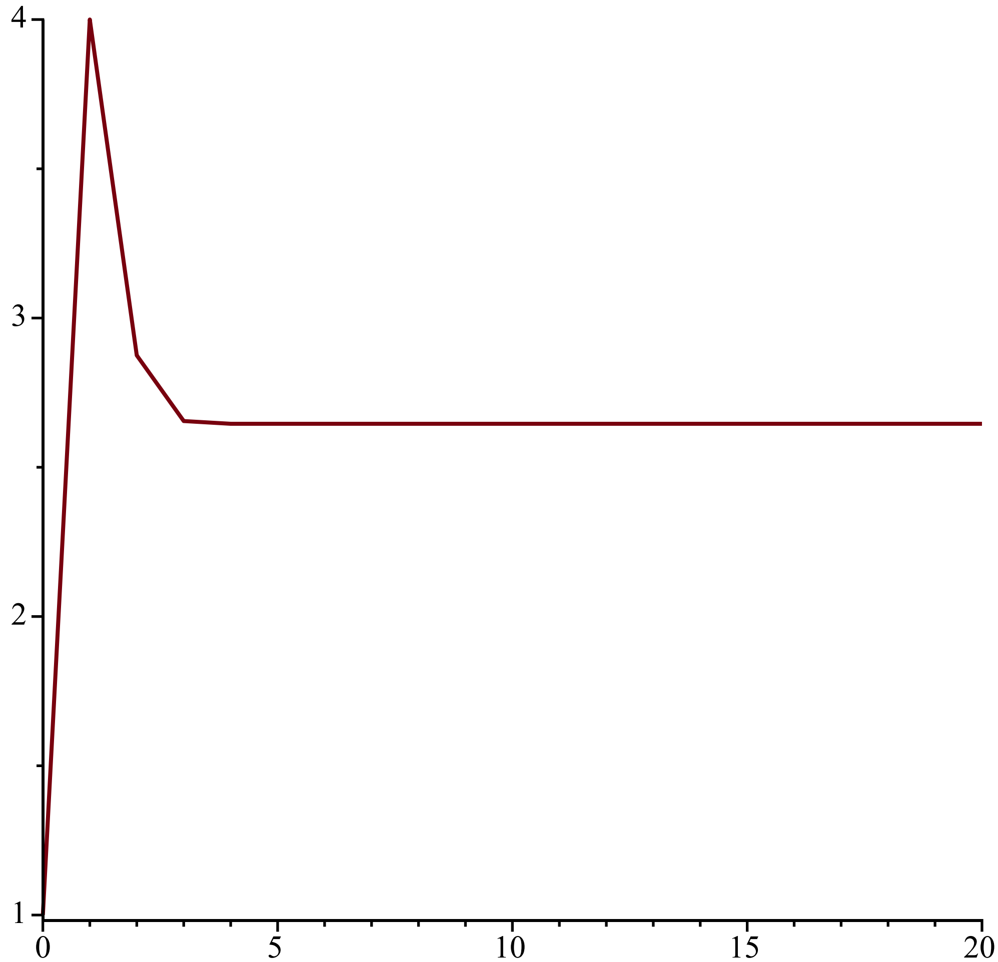
D$(f)(x)$;

$$\frac{1}{2} - \frac{7}{2\,x^2} \tag{17}$$

```
FixedPointIter := proc(f, x0, N)
   local A, i;
   A := array(0..N);
   A[0] := x0;
   for i from 0 to N-1 do
      A[i+1] := f(A[i]);
   end do;
   return A;
end proc:
```

```
N := 20 :
A := FixedPointIter(f, 1, N) :
```

```
plot([seq([i, A[i]], i = 0..N)]);
```



D(f) (10);

$$\frac{93}{200} \tag{18}$$

D(f) (1.23456);

$$-1.796379394 \tag{19}$$

D(f) ($-$9999);

$$\frac{49989997}{99980001} \tag{20}$$

D(f) (80085);

$$\frac{3206803609}{6413607225} \tag{21}$$

D(f) ($-$2);

$$-\frac{3}{8} \tag{22}$$

D(f) (2);

$$-\frac{3}{8} \tag{23}$$

D(f) (sqrt(7));

$$0 \tag{24}$$

D(f) (−sqrt(7));

$$0 \tag{25}$$

#ex 3

$with(linalg):$

$f1 := (x, y) \to x - x^2 - x \cdot y;$

$$f1 := (x, y) \mapsto x - x^2 - y \cdot x \tag{26}$$

$f2 := (x, y) \to 2 \cdot y - y^2 - 3 \cdot x \cdot y;$

$$f2 := (x, y) \mapsto 2 \cdot y - y^2 - 3 \cdot y \cdot x \tag{27}$$

$eqpoints := solve(\{f1(x, y) = x, f2(x, y) = y\}, \{x, y\});$

$$eqpoints := \left\{x = \frac{1}{2}, y = -\frac{1}{2}\right\}, \{x = 0, y = 0\}, \{x = 0, y = 1\} \tag{28}$$

$J := jacobian([f1(x, y), f2(x, y)], [x, y]);$

$$J := \begin{bmatrix} -2x - y + 1 & -x \\ -3y & -3x - 2y + 2 \end{bmatrix} \tag{29}$$

$M := eval(J, \{x = 0, y = 0\});$

$$M := \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \tag{30}$$

$M := matrix(M);$

$$M := \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \tag{31}$$

$e := eigenvals(M);$

$$e := 1, 2 \tag{32}$$

$M2 := eval(J, \{x = 0, y = 1\});$

$$M2 := \begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix} \tag{33}$$

$M2 := matrix(M2);$

$$M2 := \begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix} \quad (34)$$

$e2 := eigenvals(M2);$

$$e2 := 0, 0 \quad (35)$$

$M3 := eval\left(J, \left\{x = \frac{1}{2}, y = -\frac{1}{2}\right\}\right);$

$$M3 := \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix} \quad (36)$$

$M3 := matrix(M3);$

$$M3 := \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix} \quad (37)$$

$e3 := eigenvals(M3);$

$$e3 := 1 + \frac{I\sqrt{2}}{2}, 1 - \frac{I\sqrt{2}}{2} \quad (38)$$

$N := 10;$

$$N := 10 \quad (39)$$

$x[0] := -0.01; y[0] := 7$

$$x_0 := -0.01$$

$$y_0 := 7 \quad (40)$$

for i **from** 0 **to** $N - 1$ **do**
$x[i+1] := f1(x[i], y[i]);$
$y[i+1] := f2(x[i], y[i]);$
end do;

$$x_1 := 0.0599$$

$$y_1 := -34.79$$

$$x_2 := 2.14023299$$

$$y_2 := -1273.672337$$

$$x_3 := 2723.515190$$

$$y_3 := -1.616610700 \times 10^{6}$$

$$x_4 := 4.395448987 \times 10^{9}$$

$$y_4 := -2.600224797 \times 10^{12}$$

$$x_5 := 1.140983548 \times 10^{22}$$

$$y_5 := -6.726881529 \times 10^{24}$$

$$x_6 := 7.662242719 \times 10^{46}$$

$$y_6 := -4.502067728 \times 10^{49}$$

$$x_7 := 3.443722571 \times 10^{96}$$

$$y_7 := -2.016512602 \times 10^{99}$$

$$x_8 := 6.932450737 \times 10^{195}$$

$$y_8 := -4.045490144 \times 10^{198}$$

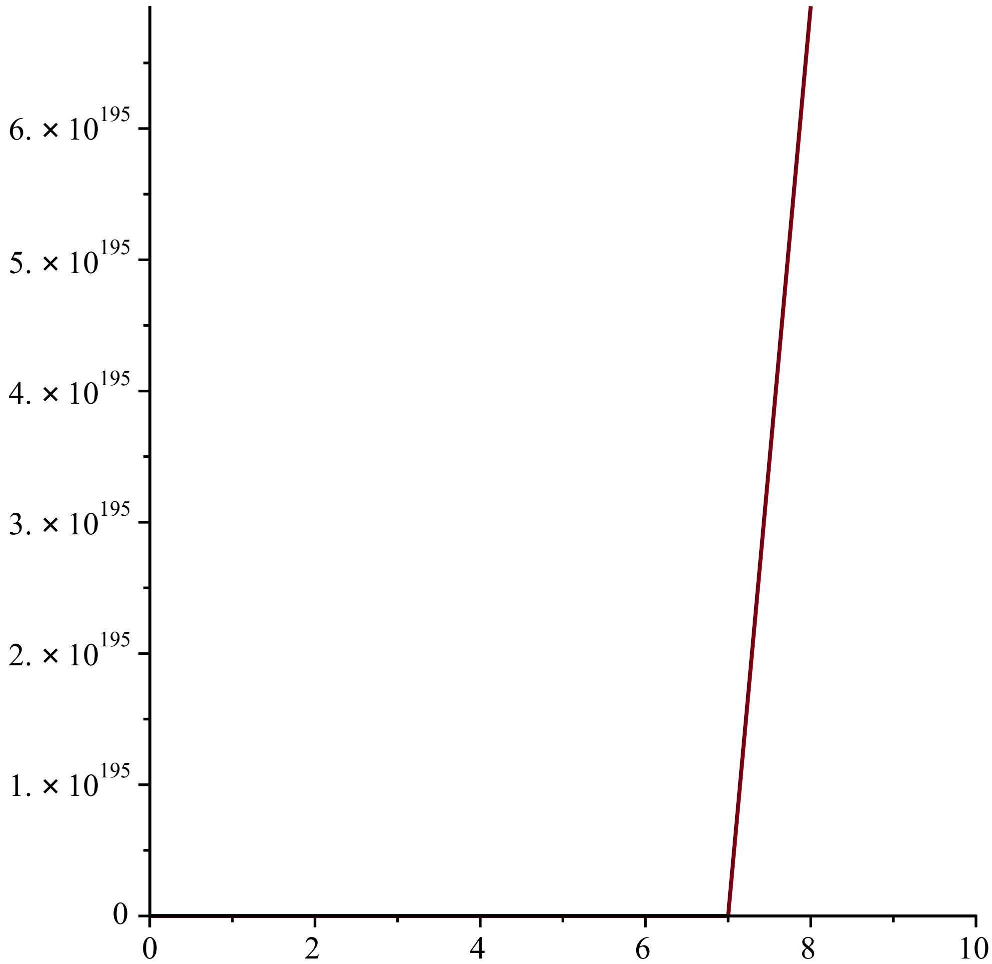
$$x_9 := 2.799710226 \times 10^{394}$$

$$y_9 := -1.628185503 \times 10^{397}$$

$$x_{10} := 4.550609226 \times 10^{791}$$

$$y_{10} := -2.637312689 \times 10^{794} \tag{41}$$

$p1 := plot([[n, x[n]] \$ n = 0 ..N]);$



#ex 4

$years := [5, 10, 15, 20]$

$$years := [5, 10, 15, 20] \quad (42)$$

$pA := 0.04;$

$$pA := 0.04 \quad (43)$$

$pB := 0.03;$

$$pB := 0.03 \quad (44)$$

$eq\_1 := S(n+1) = S(n) + p \cdot S0$

$$eq\_1 := S(n+1) = S(n) + p\, S0 \quad (45)$$

$soll := rsolve(\{eq\_1, S(0) = S0\}, S(n));$

$$soll := S0 + p\, S0\, (n+1) - p\, S0 \quad (46)$$

$s1 := simplify(\%);$

$$s1 := p\, S0\, n + S0 \quad (47)$$

$s1 := unapply(s1, n, p, S0);$

$$s1 := (n, p, S0) \mapsto S0 \cdot n \cdot p + S0 \quad (48)$$

$eq\_2 := S(n+1) = S(n) + \frac{p}{r} \cdot S(n);$

$$eq\_2 := S(n+1) = S(n) + \frac{p\,S(n)}{r} \tag{49}$$

$sol2 := rsolve(\{eq\_2, S(0) = S0\}, S(n));$

$$sol2 := S0 \left(\frac{p+r}{r}\right)^n \tag{50}$$

$s2 := unapply(sol2, n, p, r, S0);$

$$s2 := (n, p, r, S0) \mapsto S0 \cdot \left(\frac{p+r}{r}\right)^n \tag{51}$$

$s2f := (n, S0) \rightarrow s2(n, pB, 12, S0);$

$$s2f := (n, S0) \mapsto s2(n, pB, 12, S0) \tag{52}$$

$S0 := 1000$

$$S0 := 1000 \tag{53}$$

```
for i in years do
   printf("Year: %d\n", i);
   s1_val := s1(i, pA, S0);
   s2_val := s2f(12 * i, S0);
   printf("A: %.4f, B: %.4f\n", s1_val, s2_val);
end do:
```

```
Year: 5
A: 1200.0000, B: 1161.6168
Year: 10
A: 1400.0000, B: 1349.3535
Year: 15
A: 1600.0000, B: 1567.4317
Year: 20
A: 1800.0000, B: 1820.7550
```

```
for i in years do
   printf("Year: %d\n", i);
   s1_val := s1(i, pA, S0);
   s2_val := s2f(12 * i, S0);
   if s2_val < s1_val then
      printf("Company A maximizes with %.4f\n", s1_val);
   else
      printf("Company B maximizes with %.4f\n", s2_val);
   end if;
```

end do:

```
Year: 5
Company A maximizes with 1200.0000
Year: 10
Company A maximizes with 1400.0000
Year: 15
Company A maximizes with 1600.0000
Year: 20
Company B maximizes with 1820.7550
```