

2)

$$\text{a. } \lim_{n \rightarrow \infty} \frac{(n+1)^5}{n^5} = \lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^5 = (\text{by L'Hopitals Rule}) \lim_{n \rightarrow \infty} 5 * \left(\frac{1}{1} \right) = 5$$

Therefore $(n+1)^5$ is $O(n^5)$

$$\text{b. } \lim_{n \rightarrow \infty} \frac{2^{n+1}}{2^n} = \lim_{n \rightarrow \infty} \frac{2^1}{2^0} = 2$$

Therefore 2^{n+1} is $O(2^n)$

$$\text{c. } \lim_{n \rightarrow \infty} \frac{n}{n \log n} = \lim_{n \rightarrow \infty} \frac{1}{\log n} = \frac{1}{\infty}$$

Therefore n is $O(n \log n)$

$$\text{d. } \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{(n^2+n)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$\lim_{n \rightarrow \infty} \frac{2n^3 + 3n^2 + n}{6n^3} = \frac{1}{3}$$

Therefore $\sum_{i=1}^n i^2$ is $O(n^3)$

$$\text{e. } \lim_{n \rightarrow \infty} \frac{7n^2+5n+3}{n} = (\text{by L'Hopitals Rule}) \lim_{n \rightarrow \infty} \frac{14n+5}{1} = \infty$$

Therefore $7n^2 + 5n + 3$ is not $O(n)$