Computer Vision - 16720
Homework 3

[redacted]
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Start date: Feb 24th
Due date: Mar 9th

Q1.1:

Our goal is to find the $\boldsymbol{\Delta p} = (u, v)^T$ that minimize the equation:

$$\sum_{(x,y)\in R^t} (I_{t+1}(x+u, y+v) - I_t(x,y))^2$$

$$\approx \sum_{(x,y)\in R^t} (I_{t+1}(x,y) + u(I_{t+1})_x(x,y) + v(I_{t+1})_y(x,y) - I_t(x,y))^2$$

$$= \sum_{(x,y)\in R^t} (\begin{bmatrix}(I_{t+1})_x(x,y) & (I_{t+1})_y(x,y)\end{bmatrix}\begin{bmatrix}u \\ v\end{bmatrix} - (I_t(x,y) - I_{t+1}(x,y)))^2$$

$$= \sum_{(x,y)\in R^t} (\boldsymbol{A\Delta p} - b)^2$$

We can take a derivative with respect to $\boldsymbol{\Delta p}$

$$\frac{\partial}{\partial \Delta p} \sum_{(x,y)\in R^t} (\boldsymbol{A\Delta p} - b)^2 = \sum_{(x,y)\in R^t} 2\boldsymbol{A^T}(\boldsymbol{A\Delta p} - b)$$

And set it to zero we could get:

$$= \sum_{(x,y)\in R^t} \boldsymbol{A^T A \Delta p} = \sum_{(x,y)\in R^t} \boldsymbol{A^T} b$$

From above we can see $\boldsymbol{A^T A} =$

$$\begin{bmatrix} (I_{t+1})_x^2(x,y) & I_{t+1})_x(x,y)(I_{t+1})_y(x,y) \\ (I_{t+1})_x(x,y)(I_{t+1})_y(x,y) & (I_{t+1})_y^2(x,y) \end{bmatrix}$$

Which is the Gauss-Newton approximation of Hessian matrix (No second derivative).

To solve $\boldsymbol{\Delta p}$, $\boldsymbol{A^T A}$should be invertible and the determinant of $\boldsymbol{A^T A}$ should not equal to zero, i.e, $det(\boldsymbol{A^T A}) \neq 0$

Q1.2: Lucas Kanade Inverse Compositional Implementation

Please see the code submitted on blackboard.

Q1.3: Test car sequence and Test ultrasound sequence

The output of Lucas-Kanade Tracking with One Single Template for the car sequence was as below.

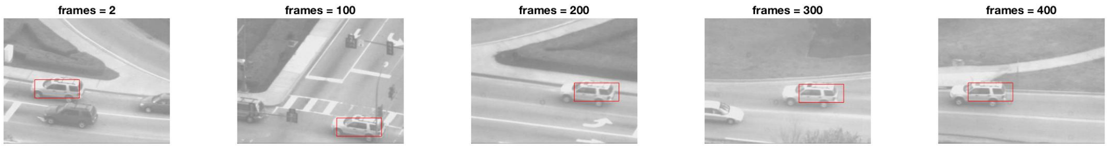

Figure 1: L-K Tracking with One Single Template for the car sequence

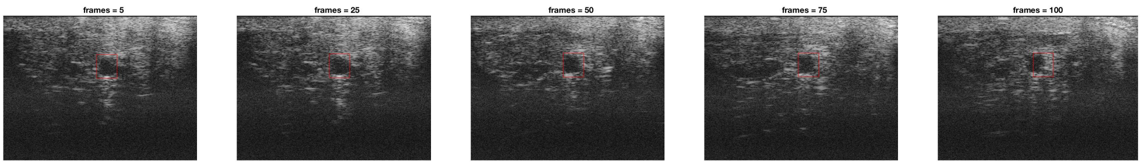

Figure 2: L-K Tracking with One Single Template for the ultrasound sequence

Q1.4 : Extra credits

Please see the code submitted on blackboard.

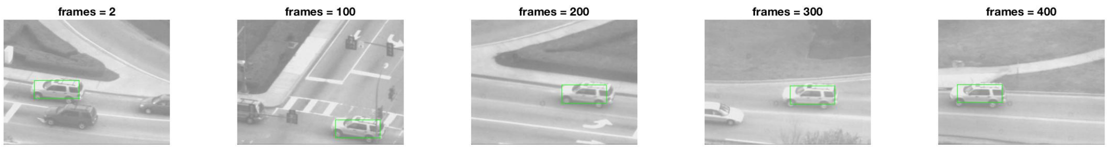

Figure 3: Lucas-Kanade Tracking with Template Correction for the car sequence

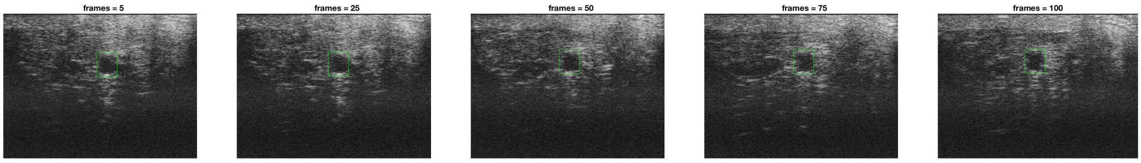

Figure 4: Lucas-Kanade Tracking with Template Correction for the ultrasound sequence

Q2.1: Appearance Basis

From the question we can know that $B_c's$ are bases and they're orthogonal to each other. It means that the mutually inner product (dot product) of B_cs are zero and the inner product with itself is a constant. Since we have :

$$I_{t+1} = I_t + \sum_{c=1}^{k} w_c B_c$$

It's equivalent to :

$$I_{t+1} - I_t = \sum_{c=1}^{k} w_c B_c$$

For a certain parameter w_i where $i \in [1,k]$, we could multiply the corresponding basis B_i in both side of the equation above then derive:

$$B_i \cdot (I_{t+1} - I_t) = B_i \cdot \sum_{c=1}^{k} w_c B_c$$

$$= B_i \cdot (w_1 B_1 + ... + w_i B_i + ... + w_k B_k)$$

$$= w_1 B_1 \cdot B_i + ... + w_i B_i \cdot B_i + ... + w_k B_k \cdot B_i$$

From above we could know that except $B_i \cdot B_i$, all other dot product $B_1 \cdot B_i ... B_k \cdot B_i$ are zero.So the equation above is equivalent to :

$$w_1 B_1 \cdot B_i + ... + w_i B_i \cdot B_i + ... + w_k B_k \cdot B_i$$

$$= 0 + ... + w_i B_i \cdot B_i + ... + 0$$

$$= w_i ||B_i||^2 = B_i \cdot (I_{t+1} - I_t)$$

So for a certain w_i,

$$w_i = \frac{B_i \cdot (I_{t+1} - I_t)}{||B_i||^2}$$

So for $c = 1, 2, ..., k$

$$w_c = \frac{B_c \cdot (I_{t+1} - I_t)}{||B_c||^2}$$

Q2.2: Tracking

Please see the code submitted on blackboard.

Q2.3: Lucas-Kanade Tracking with Appearance Basis

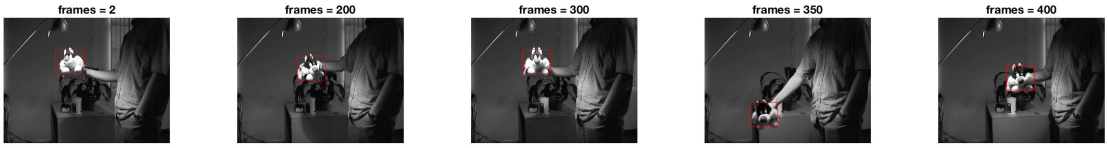

Figure 5: Lucas-Kanade Tracking with Appearance Basis

Q3.1: Dominant Motion Estimation

Q3.2: Moving Object Detection

Q3.3: Moving Object Detection

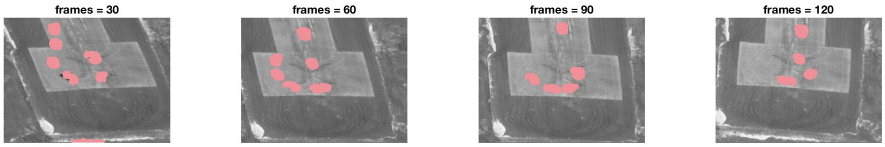

Figure 6: Lucas-Kanade Tracking of affine motion

From the images above we can see the moving objects has been detected.