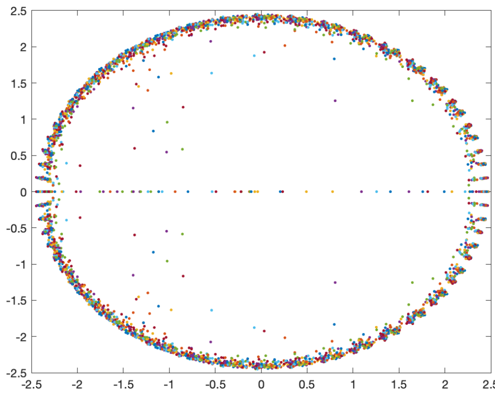
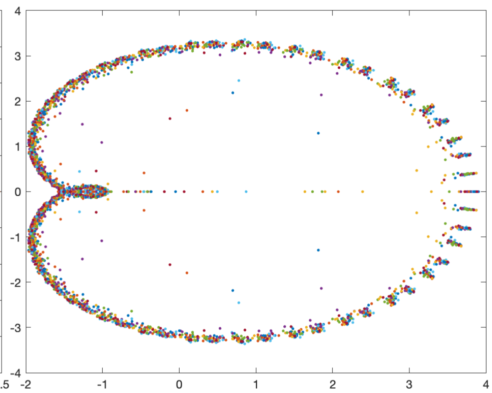
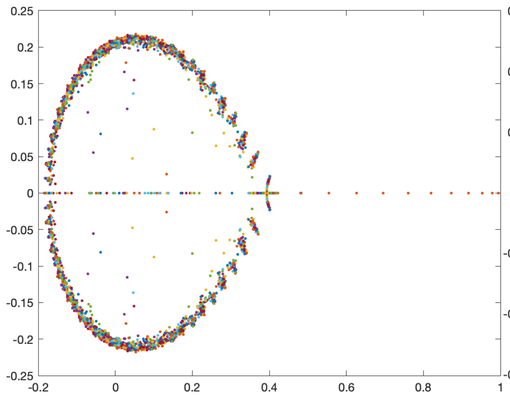


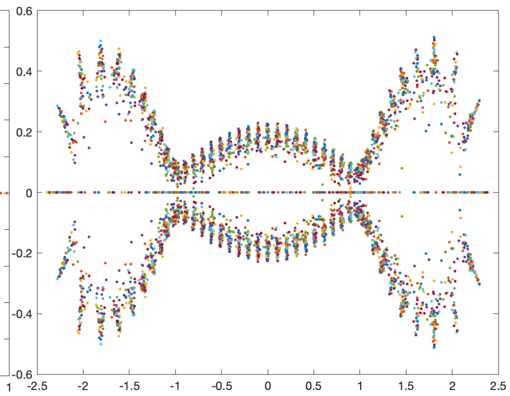
Ex1



Ex2



Ex3



Ex4

In Ex1, for A = 4J, the exact eigenvalues are all 0, and exact eigenvectors are $[1,0,0,0,\ldots]^T$, $[-1, 1.05219e\text{-}291,0,0,\ldots]^T$, and $[1, 1.05219e\text{-}291,0,0,\ldots]^T$. The condition numbers of the eigenvector matrix computing for ten random perturbations Q are 2496058182.53716, 17160427517.5256, 5248922285.14607, 6452564901.08892, 13478254296.4287, 3134551169.45479, 4643707360.30116, 4683312663.2115, 10396537251.0819, 2736524056.4801.
We can find that most of these condition numbers are from 1e^10 to 5e^10. If we perturb the equation from $z^n = 0$ to $z^n = \delta$, the solution will decrease. As δ gets larger, the solution gets larger as well.

In Ex2, for $A = 4J+4J^2$, the exact eigenvalues are all 0, and eigenvectors are $[1,0,0,0,\ldots]^T$, $[-1, 1.05219e\text{-}291,0,0,\ldots]^T$, and $[1, 1.05219e\text{-}291,0,0,\ldots]^T$, which are same as those in Ex1. The condition numbers of the eigenvector matrix computing for ten random perturbations Q are 5580248633.36534 20903069599.3746, 5338247720.44868, 9824117434.18279, 5093397451.75752, 5000170078.04864, 3930065831.10834, 6218579270.22663, 19855559624.4365 4467255975.31548. We can find that most of these condition numbers fluctuate around 5e^10.

In Ex3, for $A = -(LD)^{-1}U$, the condition numbers of the eigenvector matrix computing for ten random perturbations Q are 1417000362.73712 811981548.77583 682980042.700847 2611074408.13903 961170593.436261 6072009038.50794 1707229882.49536 1830077014.67361 3525739776.3822 2528830179.18396.
We can find that most of these condition numbers fluctuate around 1e^10. $\lambda = 0$ is an eigenvalue of A.

In Ex4, the condition numbers of the eigenvector matrix computing for ten random perturbations Q are 7836450637.83039 11721371764.4364 51003839774.0684 11328785306.746 11666131798.1925. 8527912145.40173 24779904063.0207 16512399094.497 9185753874.48598 21317165299.5545, which mostly are from 1e^10 to 2e^10.

In Ex5, as n grows from 1 to 10, the eigenvalues increase from -2 to 2. The reason for obtaining quite different results with perturbations from Ex4 is that in Ex5, B is generate randomly, so A is randomly generated. Then, results in Ex5 are quite different from those in Ex4.

Ex1

```
clc
clear
n = 42;
delta = 10^-8;

J = diag(ones(n-1,1),1);
A = 4*J;
[V,D] = eig(A);
zeig = [];
con = [];

for i = 1:100
    B = 2*rand(n)-eye(n);
    [Q,R] = qr(B);
    zeig = [zeig eig(A+delta*Q)];
    if mod(i,10)==0
        [V1,D1] = eig(A+delta*Q);
        con = [con cond(V1,1)];
    end
end
plot(zeig,'.');
disp(con);
```

Ex2

```
clc
clear
n = 42;
delta = 10^-8;

J = diag(ones(n-1,1),1);
A = 4*J+4*J^2;
[V,D] = eig(A);
zeig = [];
con = [];

for i = 1:100
    B = 2*rand(n)-eye(n);
    [Q,R] = qr(B);
    zeig = [zeig eig(A+delta*Q)];
    if mod(i,10)==0
        [V1,D1] = eig(A+delta*Q);
        con = [con cond(V1,1)];
    end
end
plot(zeig,'.');
disp(con);
```

Ex3

```
clc
clear
n = 42;
delta = 10^-8;

S = -2*diag(ones(n,1)) + diag(ones(n-1,1),-1) + diag(ones(n-1,1),1);
LD = tril(S);
U = triu(S,1);
A = -inv(LD)*U;
[V,D] = eig(A);
zeig = [];
con = [];
for i = 1:100
    B = 2*rand(n)-eye(n);
    [Q,R] = qr(B);
    zeig = [zeig eig(A+delta*Q)];
    if mod(i,10)==0
        [V1,D1] = eig(A+delta*Q);
        con = [con cond(V1,1)];
    end
end
plot(zeig,'.');
disp(con)
```

Ex4

```
clc
clear

n = 42;
delta = 10^-8;
t=4*[0:n-1]/(n-1) - 2;
p=poly(t);
A=compan(p);
[V,D] = eig(A);
zeig = [];
con = [];
for i = 1:100
    B = 2*rand(n)-eye(n);
    [Q,R] = qr(B);
    zeig = [zeig eig(A+delta*Q)];
    if mod(i,10)==0
        [V1,D1] = eig(A+delta*Q);
        con = [con cond(V1,1)];
    end
end
plot(zeig,'.');
disp(con);
```

Ex5

```
clc
clear

delta = 10^-8;
n = 10;
for n = 1:10
    t = 4*(n-1)/9-2;
    B = 2*rand(n) - eye(n);
    [Q, R] = qr(B);
    A = Q*diag(t)*Q';
    disp(eig(A));
    disp(eig(A+delta*Q));
    disp(eig(A+delta*(Q+transpose(Q))));
end
```