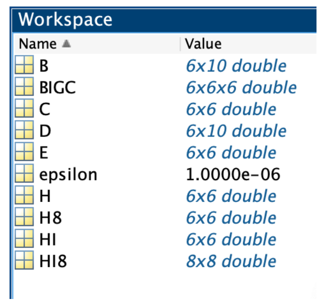
faculty of science
department of mathematics

MACM 316 COMPUTING ASSIGNMENT #3

Submit on Crowdmark by Tuesday, June 30, 2020, 11pm

Upload one .pdf file with 2 pages: Page 1 is your typed report (your discussions, data and figures on a single page); Page 2 is a listing of your code(s). The assignment is due at 11:00pm. You will receive a Crowdmark link for the upload.

Matlab scripts and functions for this assignment can be downloaded from the Canvas Homework page. Make sure to either set your "Matlab path", or run Matlab in the directory where your scripts and functions are located.



This computing assignment is an exploration of condition numbers, perturbations, and the numerical behaviour of random and not-so-random matrices.
You will need to download `gendata.p` from Canvas to get all the data for the assignment, including the matrices E, H, HI, $H8$, and $HI8$ referred to below. (Note: `gendata` is encoded, displaying it will show a sequence of strange characters.) Run the script, and look at the work space or type `whos` to get a list of the data.
For all your computations use $\epsilon = 10^{-6}$ - a variable `epsilon` with the proper value is included in your data.

C1. For $A = E$, $A = H$, compare the 1-condition number $\kappa_1(A)$ (in Matlab simply `cond(A,1)`) to the observed amplification in perturbations as well as to the Matlab estimate `rcond(A)`. Note, that `rcond(A)` estimates the reciprocal $1/\kappa_1(A)$.

1. Perturbations in the right-hand side.

   For each of these two matrices you will solve a total of 100 systems. You pair each right side `b = B(:,j)` with each perturbation direction `d = D(:,k)`; note, that all column vectors in your data have length 1 in the $||\cdot||_1$ norm. Compute (simply use the Matlab "\" backslash command) the solution of

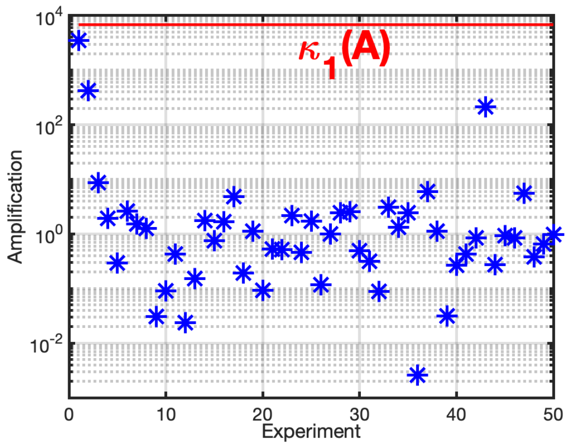
   $$Ax = b, \qquad \text{and} \qquad Ay = b + \epsilon d,$$

   and compare the amplification of the relative errors

   $$e = \frac{\frac{||y-x||_1}{||x||_1}}{\frac{||\epsilon d||_1}{||b||_1}} = \frac{||y - x||_1}{\epsilon ||x||_1}$$

   to the upper bound $\kappa_1(A)$.

   Look at the average, median, and maximum of the amplification factors. Describe your observations (supported by a plot), and comment on your results. See the sample below for a possible visualization.



2. Perturbations of the matrix.

   For each of the two matrices E and H, solve a total of 60 linear systems to compute amplification factors. You use the same 10 right hand sides b from the first part; to get your perturbation matrices, type `C=BIGC(:,:,k)`, for $k = 1, \ldots, 6$. All the data matrices have $||C||_1 = 1$.

   Compute (simply use the Matlab "\" backslash command) the solution of

$$Ax = b, \qquad \text{and} \qquad (A + \epsilon C)z = b,$$

   and compare the amplification of the relative errors

$$e = \frac{\frac{||z-x||_1}{||x||_1}}{\frac{||\epsilon C||_1}{||A||_1}} = ||A||_1 \frac{||z-x||_1}{\epsilon ||x||_1}$$

   to the upper bound $\kappa_1(A)$ and the Matlab estimate 1/`rcond(A)`.

   Look at averages, median, and maxima of amplification factors. Plot your results, and comment on your observations.

C2. Short (and sweet - depending on your taste). Use the Matlab command `AINV=inv(A)` to find the inverse of a matrix A, and compute the inverse of this inverse, `AC=inv(AINV)`, which mathematically equals $A \; (= \left(A^{-1}\right)^{-1}$. The matrix I is the identity matrix.

1. For $A = E$, compute $||A * AINV - I||_1$, and $||AC - A||_1$.
2. For $A = H$, compute $||A * AINV - I||_1$, $||AC - A||_1$. For this matrix, also compare the computed inverse to the exact inverse HI provided, i.e., compute $||AINV - HI||_1$.
3. Repeat item 2 for the matrix $A = H8$ with exact inverse $HI8$. Compute $\kappa_1(H8)$.

Summarize your observations and highlight anything that might seem surprising.