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## C1 (part 1)

For A = E, the average of amplification factor is 258.2577, and the median is 1.9622. The maximum is 3.4880e+03, which is close to the value of the exact k1(A) = 6.8296e+03. From the graph, we can find that first 30 values are closer to the k1(A), while most values are around  $10^{\circ}0$ . There are also seven values are around  $10^{\circ}-2$ . All values are quite far from the reciprocal 1/k1(A) = 1.4642e-04.

For A = H, the average of amplification factor is 7.4294e+05, and the median is 2.5611. The maximum is 1.4951e+07, which is close to the value of the exact k1(A) = 2.9070e+07. From the graph, we can find that  $11^{th}$  to  $20^{th}$  values and  $31^{th}$  to  $40^{th}$  values are closer to the k1(A), while most values are around 10. All values are quite far from the reciprocal 1/k1(A) = 3.4399e-08.

## C1 (part 2)

For A = E, the average of amplification factor is 950.4024, and the median is 1.0359e+03. The maximum is 2.1835e+03, which is close to the value of the exact k1(A) = 6.8296e+03. From the graph, we can find that most values are around  $10^3$ . There are also seven values are around  $10^2$ . All values are quite far from the reciprocal 1/k1(A) = 1.4642e-04.

For A = E, the average of amplification factor is 2.1144e+06, and the median is 1.2282e+06. The maximum is 9.4726e+06, which is close to the value of the exact k1(A) = 2.9070e+07. From the graph, we can find that most values are around  $10^6$ , which are all quite close to the exact k1(A). All values are quite far from the reciprocal 1/k1(A) = 3.4399e-08.

## C2

- 1. For A = E,  $||A * AINV-||_1 = 1.5632e-13$ , and  $||AC-A||_1 = 1.8803e-12$ . It is shown that error is becoming larger as we use more inverse, for the value of using one inverse is smaller than that of using two inverses.
- 2. For A = H, || A \* AINV-||<sub>1</sub> = 6.1118e-10, and || AC-A||<sub>1</sub> = 5.8995e-11. Surprisingly, it is shown that error is getting smaller as we use more inverses. || AINV-HI||<sub>1</sub> = 0.00047445, which is the largest error.
- 3. For A = H8, || A \* AINV-||<sub>1</sub> = 3.6515e-06, and || AC-A||<sub>1</sub> = 8.7919e-09, which verifies the conclusion in question 1. However, || AINV-HI||<sub>1</sub> = 730.3153, which is much larger than the two errors above. K1(A) = 3.3873e+10, which is greater than 1.0. Hence, it is ill-conditioned.

```
C1 (part 1)
1 -
        clear
2 -
        clc
3 -
        gendata
4
5 -
       arr = [];
6 -
       A = H;
7
8 -
     \neg for j = 1:10
9 -
            for k =1:10
10 -
                b = B(:,j);
11 -
                d = D(:,k);
12 -
                x = A \setminus b;
13 -
                y = A (b+epsilon*d);
14 -
                e = norm((y-x),1)/(epsilon*norm(x,1));
15 -
                arr = [arr, e];
16 -
            end
17 -
       end
18
19
        % get the average, median, and maximum
20 -
        average = mean(arr);
21 -
        median = median(arr);
22 -
       maximum = max(arr);
23
24 -
        scatter(linspace(1,100), arr, 100, 'b*');
25 -
        set(gca, 'YScale', 'log');
26 -
        xlabel('experiment');
27 -
       ylabel('Amplification');
                                                                      C2
28
       aline = yline(cond(A,1), 'color', 'r', 'LineWidth', 3);
bline = yline(rcond(A), 'color', 'b', 'LineWidth', 3);
legend([aline, bline], {'K1(A)','1/K1(A)'});
29 -
                                                                     1 -
                                                                             clear
30 -
                                                                     2 -
                                                                             clc
31 -
                                                                     3 -
                                                                             gendata
                                                                     4 -
                                                                             A = E;
      C1 (part 2)
                                                                     5 -
                                                                             AINV = inv(A);
   1 -
          clear
                                                                     6 -
                                                                             AC = inv(AINV);
   2 -
          clc
                                                                     7 -
                                                                             norma = norm(A*AINV-eye(6),1);
   3 -
          gendata
                                                                     8 -
                                                                             normb = norm(AC-A, 1);
   4
   5 -
                                                                     9 -
                                                                             disp("The first norm is "+norma);
          arr = [];
   6 -
          A = H:
                                                                    10 -
                                                                             disp("The second norm is "+normb);
   7 -
          cond = cond(A,1);
                                                                    11
   8 -
        \Box for k = 1:6
   9 -
                                                                    12 -
                                                                             A = H;
              for j = 1:10
  10 -
                  c = BIGC(:,:,k);
                                                                    13 -
                                                                             AINV = inv(A);
  11 -
                  b = B(:,j);
                                                                    14 -
                                                                             AC = inv(AINV);
  12 -
                  x = A b:
                                                                    15 -
                                                                             norma = norm(A*AINV-eye(6),1);
  13 -
                  z = (A+epsilon*c)\b;
                                                                             normb = norm(AC-A, 1);
                  e = norm(A,1)*norm(z-x,1)/(epsilon*norm(x,1)); 16 -
  14 -
  15 -
                  arr = [arr, e];
                                                                    17 -
                                                                             normc = norm(AINV-HI,1);
  16 -
17 -
              end
                                                                             disp("The first norm is "+norma);
                                                                    18 -
         ∟ end
                                                                             disp("The second norm is "+normb);
                                                                    19 -
  18
                                                                    20 -
                                                                             disp("The third norm is "+normc);
  19
          % get the average, median, and maximum
  20 -
          average = mean(arr);
                                                                    21
  21 -
          median = median(arr);
                                                                    22 -
                                                                             A = H8;
  22 -
          maximum = max(arr);
                                                                    23 -
                                                                             AINV = inv(A);
  23
                                                                    24 -
                                                                             AC = inv(AINV);
          scatter(1:60, arr, 100, '*');
  24 -
  25 -
          set(gca, 'YScale', 'log');
                                                                    25 -
                                                                             norma = norm(A*AINV-eye(8),1);
          xlabel('experiment');
  26 -
                                                                    26 -
                                                                             normb = norm(AC-A, 1);
  27 -
          ylabel('Amplification');
                                                                    27 -
                                                                             normc = norm(AINV-HI8,1);
  28
          aline = yline(cond, 'color', 'r', 'LineWidth', 3);
bline = yline(rcond(A), 'color', 'b', 'LineWidth', 3); 29 -
  29 -
                                                                             disp("The first norm is "+norma);
  30 -
                                                                             disp("The second norm is "+normb);
          legend([aline, bline], \{'K1(A)', '1/K1(A)'\});
  31 -
                                                                             disp("The third norm is "+normc);
                                                                    30 -
                                                                             disp("k1(A) = "+cond(H8,1));
                                                                    31 -
```