模式识别课程作业 —— CRF

问题描述

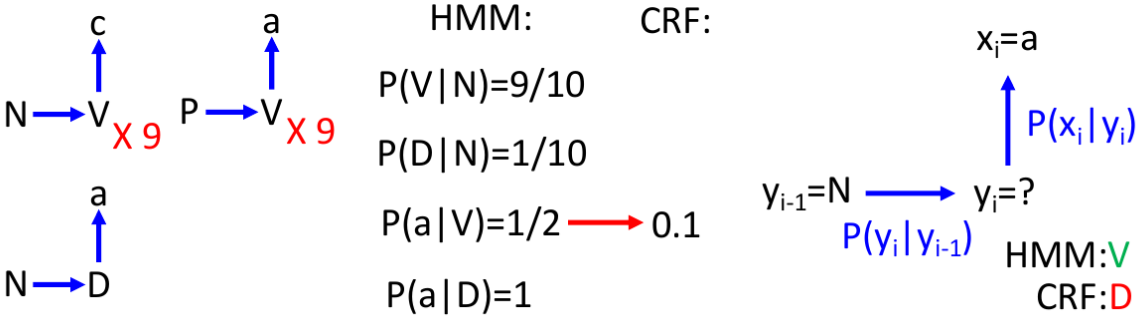
> 给定左侧19条序列和右侧任务（已知yi-1=N, xi=1, 求解yi），证明CRF的推断结果为D

- CRF: increase $P(x, \hat{y})$, decrease $P(x, y')$
  - To obtain correct results ...

$$(x, \hat{y}): \quad P(x, \hat{y}) > P(x, y)$$

HMM does not do that

CRF more likely to achieve that than HMM



Solution

1. 简化问题 && 约定

- 约定Tag集合 S = {N,V,P,V}， 单词集合 T = {a,c}

```
# S,T集合
setS = ('D','N','P','V')
setT = ('a','c')
```

- 约定语句向量长度为3（即Tag1 -> Tag2 -> word1），并且我们只需要推断长度为3的语句
- ydot 集合由训练集合中没有出现的语句向量所构成

2. Feature Vector

- 由于我们要推断的序列长度为3, 因此要推断的目标序列只有如下四个

```
# 构造目标特征序列
objSeqList = ["NVa","NDa","NPa","NNa"]
```

- 构造特征feature的时候，参考课件中的`Feature Vector` 的两个部分 `part1` = relations between tags and words, `part2` = relations between tags，这样会得到相当高维度的向量，很多分量的值为，因此可以将其剔除，最终只需要构造`1x4`的特征向量

```
def getFai(objSeq,seq):
    # 加上 start and end，分别标定为0,1
    objSeq = '0' + objSeq + '1'
    seq = '0' + seq + '1'
    fai = np.zeros((len(objSeq)-1),dtype=np.int16)
    for i in range(len(objSeq)-1):
        for j in range(len(seq)-1):
            if objSeq[i] == seq[i] and objSeq[i+1] == seq[i+1]:
                fai[i] = fai[i] + 1
    return fai
```

3. Training

- Gradient Ascent

$$w \rightarrow w + \eta \left(\phi(x^n, \hat{y}^n) - \sum_{y'} P(y'|x^n)\, \phi(x^n, y') \right)$$

```
def Train(objSeq,eta,e,times):
    # generate W randomly
    W = np.random.rand(len(objSeq)+1)
    for i in range(times):
        for seq in trainingData:
            dOW = getFai(objSeq,seq) - getSum(objSeq[-1],W,objSeq)
            if abs(dOW).all() < e:
                return W
            W = W + eta*(dOW)
    return W
```

4. Inference

$$y = \arg\max_{y \in Y} P(y|x) = \arg\max_{y \in Y} P(x,y)$$

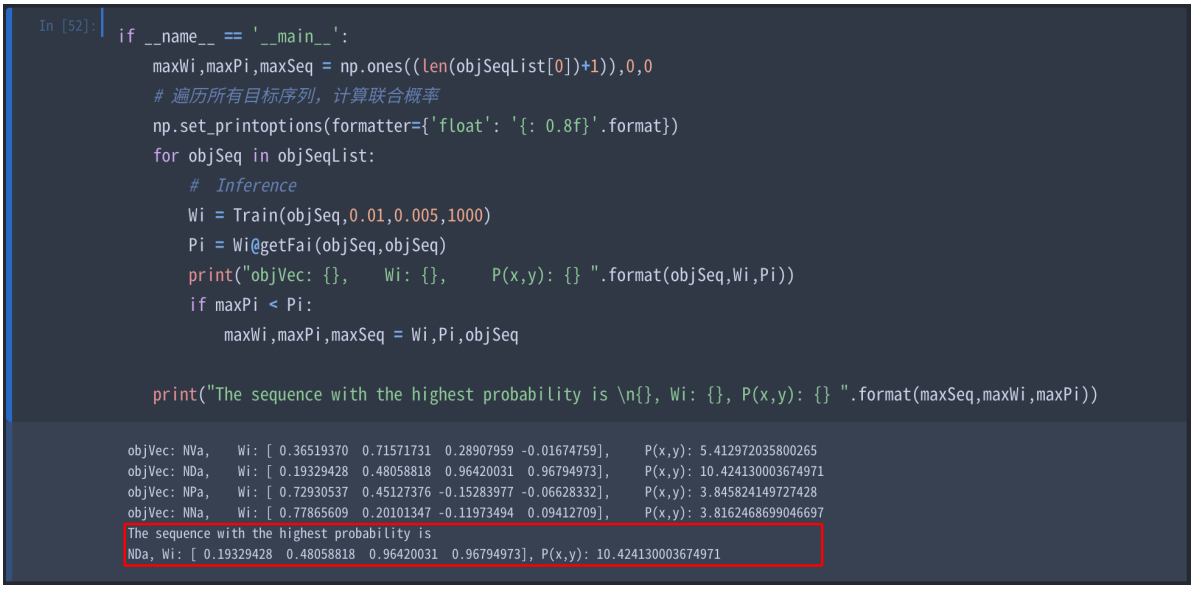
$$= \arg\max_{y \in Y} w \cdot \phi(x,y)$$ Done by Viterbi as well

```
if __name__ == '__main__':
    maxWi,maxPi,maxSeq = np.ones((len(objSeqList[0])+1)),0,0
    # 遍历所有目标序列，计算联合概率
    np.set_printoptions(formatter={'float': '{: 0.8f}'.format})
    for objSeq in objSeqList:
        #  Inference
        Wi = Train(objSeq,0.01,0.005,1000)
        Pi = Wi@getFai(objSeq,objSeq)
        print("objVec: {},     Wi: {},      P(x,y): {} ".format(objSeq,Wi,Pi))
        if maxPi < Pi:
            maxWi,maxPi,maxSeq = Wi,Pi,objSeq

    print("The sequence with the highest probability is \n{}, Wi: {}, P(x,y):
{} ".format(maxSeq,maxWi,maxPi))
```

5. result

> 学习率eta = 0.01, 误差下界e = 0.005，训练 1000 次数得到的结果如下图，四个目标序列中，`NDa`序列推断概率最大，因此推断结果为`NDa`



问题 & 改进

- 使用 Viterbi 算法优化。
- 对于这个小case，特殊化了特征集合，应该将构造的特征向量扩充到整个S集合和T集合。