HW12.

5. (1) $EX=\int_0^{+\infty}\frac{4x^3}{\theta^3\sqrt{\pi}}e^{-\frac{x^2}{\theta^2}}dx. =\int_0^{+\infty}\frac{2\theta}{\sqrt{\pi}}\left(\frac{x}{\theta}\right)^2e^{-\frac{x^2}{\theta^2}}\cdot\frac{2x}{\theta^2}dx$

$\overset{y=\frac{x^2}{\theta^2}}{=}\int_0^{+\infty}\frac{2\theta}{\sqrt{\pi}}ye^{-y}dy=-\frac{2\theta}{\sqrt{\pi}}(y+1)e^{-y}\Big|_0^{+\infty}=\frac{2\theta}{\sqrt{\pi}}.$

$\therefore\ \hat{\theta}=\frac{\sqrt{\pi}}{2}\bar{X}.$

(2) $\mathrm{Var}\,\hat{\theta}=\frac{\pi}{4}\mathrm{Var}\,\bar{X}=\frac{\pi}{4}\mathrm{Var}\frac{\sum_{i=1}^n X_i}{n}=\frac{\pi}{4n^2}\mathrm{Var}\sum_{i=1}^n X_i\overset{X_1\cdots X_n相互独立.}{=}\frac{\pi n}{4n^2}\mathrm{Var}X$

$EX^2=\int_0^{+\infty}\frac{4x^4}{\theta^3\sqrt{\pi}}e^{-\frac{x^2}{\theta^2}}dx=\int_0^{+\infty}\frac{2\theta^2}{\sqrt{\pi}}\left(\frac{x^2}{\theta^2}\right)^{\frac{3}{2}}e^{-\frac{x^2}{\theta^2}}\cdot\frac{2x}{\theta^2}dx$

$=\int_0^{+\infty}\frac{2\theta^2}{\sqrt{\pi}}y^{\frac{3}{2}}e^{-y}dy=\frac{2\theta^2}{\sqrt{\pi}}\Gamma\left(\frac{5}{2}\right)=\frac{2\theta^2}{\sqrt{\pi}}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}=\frac{3\theta^2}{2}$

$\therefore\ \mathrm{Var}X=EX^2-(EX)^2=\frac{3\theta^2}{2}-\frac{4\theta^2}{\pi}$

$\therefore\ \mathrm{Var}\hat{\theta}=\frac{\pi}{4n}\left(\frac{3\theta^2}{2}-\frac{4\theta^2}{\pi}\right)=\left(\frac{3\pi}{8}-1\right)\frac{\theta^2}{n}$

注. 第二问求$\hat{\theta}$的方差, 与样本无关、写样本方差S^2是不对的.

8. (1) $EX=\int_0^{\theta}x\cdot\frac{1}{2\theta}dx+\int_\theta^1\frac{x}{2(1-\theta)}dx=\frac{1}{2\theta}\frac{\theta^2}{2}+\frac{1}{2(1-\theta)}\frac{1-\theta^2}{2}$

$=\frac{\theta}{4}+\frac{1+\theta}{4}=\frac{1+2\theta}{4}$

$\therefore\ \hat{\theta}=\frac{4\bar{X}-1}{2}$

(2) $E\bar{X}^2=E\frac{1}{n^2}\left(\sum_{i=1}^n X_i\right)^2=\frac{1}{n^2}E\left[\sum_{i=1}^n X_i^2+\sum_{i\neq j}X_i\cdot X_j\right]$

$=\frac{1}{n^2}\left[n\cdot EX_i^2+(n^2-n)(EX_i)^2\right]=\frac{1}{n}EX_i^2+\frac{n-1}{n}(EX_i)^2.$

$EX^2=\int_0^{\theta}x^2\cdot\frac{1}{2\theta}dx+\int_\theta^1\frac{x^2}{2(1-\theta)}dx=\frac{1}{2\theta}\frac{\theta^3}{3}+\frac{1}{2(1-\theta)}\frac{1-\theta^3}{3}$

$=\frac{\theta^2}{6}+\frac{\theta^2+\theta+1}{6}=\frac{2\theta^2+\theta+1}{6}$

$\therefore\ E\bar{X}^2=\frac{2\theta^2+\theta+1}{6n}+\frac{n-1}{n}\frac{4\theta^2+4\theta+1}{16}=\left(\frac{1}{3n}+\frac{n-1}{4n}\right)\theta^2+\left(\frac{1}{6n}+\frac{n-1}{4n}\right)\theta+\frac{1}{6n}+\frac{n-1}{16n}$

$\therefore\ E4\bar{X}^2=\left(\frac{4}{3n}+\frac{n-1}{n}\right)\theta^2+\left(\frac{2}{3n}+\frac{n-1}{n}\right)\theta+\frac{2}{3n}+\frac{n-1}{4n}$

$=\frac{3n+1}{3n}\theta^2+\frac{3n-1}{3n}\theta+\frac{3n-5}{12n}\cdot\neq\theta^2.$

$\therefore\ 4\bar{X}^2$不是θ^2的无偏估计.

扫描全能王 创建

22. (1) $E(X_{i+1}-X_i)^2 = EX_{i+1}^2 + EX_i^2 - 2EX_iX_{i+1}$

$= 2EX^2 - 2(EX)^2 = 2\mathrm{Var}X = 2\sigma^2$

$\therefore E\, c\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2 = 2c(n-1)\sigma^2 = \sigma^2$

$\therefore c = \frac{1}{2(n-1)}$

(2) $E\bar{X}^2 = \mathrm{Var}\,\bar{X} + (E\bar{X})^2 = \frac{1}{n^2}\mathrm{Var}\sum_{i=1}^{n} X_i + (E\bar{X})^2$

$X_1 \dots X_n$ iid

$= \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}X_i + \left(\frac{1}{n}\sum_{i=1}^{n}EX_i\right)^2 = \frac{\sigma^2}{n} + u^2$.

$ES^2 = \frac{1}{n-1}E\sum_{i=1}^{n}(X_i-\bar{X})^2 = \frac{1}{n-1}\sum_{i=1}^{n}E(X_i-\bar{X})^2$

$E(X_i-\bar{X})^2 = EX_i^2 + E\bar{X}^2 - 2EX_i\bar{X} = \mathrm{Var}X_i + (EX_i)^2 + E\bar{X}^2 - 2EX_i\bar{X}$

$EX_i\cdot\bar{X} = \frac{1}{n}\sum_{j=1}^{n}EX_iX_j = \frac{1}{n}[EX_i^2 + (n-1)EX_iX_j] = \frac{1}{n}[EX_i^2 + (n-1)(EX_i)^2]$

$\therefore ES^2 = \frac{n}{n-1}\left\{\mathrm{Var}X_i + (EX_i)^2 + E\bar{X}^2 - \frac{2}{n}[EX_i^2 + (n-1)(EX_i)^2]\right\}$

$= \frac{n}{n-1}\left\{\sigma^2 + u^2 + \frac{\sigma^2}{n} + u^2 - \frac{2}{n}[\sigma^2 + u^2 + (n-1)u^2]\right\}$

$= \frac{n}{n-1}\left\{\sigma^2 + \frac{\sigma^2}{n} + 2u^2 - \frac{2}{n}[\sigma^2 + nu^2]\right\} = \frac{n}{n-1}\left\{\sigma^2 - \frac{\sigma^2}{n}\right\} = \sigma^2$. 记住这个结论 ($X_1 \dots X_n$ iid时)

$\therefore E(\bar{X}^2 - cS^2) = \frac{\sigma^2}{n} + u^2 - c\sigma^2 = u^2$

$\therefore c = \frac{1}{n}$

47. (1) 矩估计: $EX = \int_0^{+\infty} x\cdot\lambda\alpha\cdot x^{\alpha-1}e^{-\lambda x^\alpha}dx \overset{y=\lambda x^\alpha}{=} \int_0^{+\infty} x\cdot e^{-y}dy = \lambda^{-\frac{1}{\alpha}}\int_0^{+\infty} y^{\frac{1}{\alpha}}e^{-y}dy$

$= \lambda^{-\frac{1}{\alpha}}\Gamma\left(\frac{1}{\alpha}+1\right) = \frac{1}{\alpha\lambda^{\frac{1}{\alpha}}}\Gamma\left(\frac{1}{\alpha}\right)$

$\therefore \hat{\lambda} = \left[\frac{\Gamma(\frac{1}{\alpha})}{\alpha\bar{X}}\right]^\alpha$

(2) MLE: $f(x;\lambda) = \prod_{i=1}^{n}\lambda\alpha x_i^{\alpha-1}e^{-\lambda x_i^\alpha}$

$l(\lambda, x) = \sum_{i=1}^{n}\ln\lambda - \lambda x_i^\alpha + \ln\alpha\cdot x_i^{\alpha-1}$

$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n}x_i^\alpha = 0$. $\therefore \hat{\lambda} = \frac{n}{\sum_{i=1}^{n}x_i^\alpha}$

49. $f(x;\theta) = \prod_{i=1}^{n}\frac{1}{\theta}I_{\{\theta\le x_i\le 2\theta\}}$

$L(\theta) = \frac{1}{\theta^n}I_{\{\theta\le x_1\dots x_n\le 2\theta\}} = \frac{1}{\theta^n}I_{\{\theta\le x_{(1)}\le x_{(n)}\le 2\theta\}}$

$\therefore \frac{x_{(n)}}{2}\le\theta\le x_{(1)}$ 时, $L(\theta)$ 随 θ 单减.

$\therefore \theta = \frac{x_{(n)}}{2}$ 时 $L(\theta)$ 取最大值.

$\therefore \hat{\theta} = \frac{X_{(n)}}{2}$

扫描全能王 创建

$$P(X_{(n)}\le x)=\prod_{i=1}^{n}P(X_i\le x)=\prod_{i=1}^{n}\frac{(x-\theta)}{\theta}=\frac{(x-\theta)^n}{\theta^n}$$

$\therefore X_{(n)}$ pdf: $f(x)=\frac{n(x-\theta)^{n-1}}{\theta^n}$.

$$\therefore EX_{(n)}=\int_\theta^{2\theta}x\cdot\frac{n(x-\theta)^{n-1}}{\theta^n}dx=\frac{n}{\theta^n}\int_\theta^{2\theta}(x-\theta)(x-\theta)^{n-1}+\theta(x-\theta)^{n-1}dx$$

$$=\frac{n}{\theta^n}\int_\theta^{2\theta}(x-\theta)^n+\theta(x-\theta)^{n-1}dx=\frac{n}{\theta^n}\left[\frac{(x-\theta)^{n+1}}{n+1}+\frac{\theta(x-\theta)^n}{n}\right]\Big|_{x=\theta}^{2\theta}$$

$$=\frac{n}{\theta^n}\left[\frac{\theta^{n+1}}{n+1}+\frac{\theta^{n+1}}{n}\right]=\frac{2n+1}{n+1}\theta.$$

$\therefore$ 不是无偏估计

修正为 $\hat{\theta}'=\frac{n+1}{2n+1}X_{(n)}$

注. 求均匀分布次序统计量期望.（不能作为严格证明）

设 $X_1\ldots X_n\sim U(a,b)$

将 (a,b) 均匀分为 $n+1$ 份. 有 n 个分点. "理论上" $X_1\ldots X_n$ 会落在这 n 个分点上.

$\therefore EX_{(i)}=a+\frac{(b-a)}{n+1}i$

50. $X_1\ldots X_m, Y_1\ldots Y_n$ 均相互独立

（$\therefore$ 作为一个在体一起对参数进行估计）.

联合分布 $f(x,y)=f(x_1,\ldots x_m,y_1\ldots y_n)=\prod_{i=1}^{m}(2\pi\sigma^2)^{-\frac{1}{2}}\exp\{-\frac{(x_i-u_1)^2}{2\sigma^2}\}\prod_{j=1}^{n}(2\pi\sigma^2)^{-\frac{1}{2}}\exp\{-\frac{(y_j-u_2)^2}{2\sigma^2}\}$

$$l(\theta)=\sum_{i=1}^{m}-\frac{1}{2}\ln\sigma^2-\frac{(x_i-u_1)^2}{2\sigma^2}+\sum_{j=1}^{n}-\frac{1}{2}\ln\sigma^2-\frac{(y_j-u_2)^2}{2\sigma^2}+C$$ C 与 u_1, u_2, σ^2 均无关的常数.

$$=-\frac{m+n}{2}\ln\sigma^2-\frac{\sum_{i=1}^{m}(x_i-u_1)^2+\sum_{j=1}^{n}(y_j-u_2)^2}{2\sigma^2}+C.$$

$$\frac{\partial l}{\partial u_1}=0\Rightarrow\sum_{i=1}^{m}(x_i-u_1)=0.\Rightarrow\hat{u}_1=\frac{\sum_{i=1}^{m}x_i}{n}=\bar{X}$$

$$\frac{\partial l}{\partial u_2}=0\Rightarrow\sum_{j=1}^{n}(y_j-u_2)=0\Rightarrow\hat{u}_2=\frac{\sum_{j=1}^{n}Y_j}{n}=\bar{Y}$$

$$\frac{\partial l}{\partial\sigma^2}=0\Rightarrow-\frac{m+n}{2}\frac{1}{\sigma^2}+\frac{\sum_{i=1}^{m}(x_i-u_1)^2+\sum_{j=1}^{n}(y_j-u_2)^2}{2(\sigma^2)^2}=0.$$

$$\therefore\sigma^2=\frac{\sum_{i=1}^{m}(x_i-u_1)^2+\sum_{j=1}^{n}(y_j-u_2)^2}{m+n}$$

代入 $\hat{u}_1, \hat{u}_2$. $$\hat{\sigma}^2=\frac{\sum_{i=1}^{m}(x_i-\bar{x})^2+\sum_{j=1}^{n}(y_j-\bar{Y})^2}{m+n}.$$

1° 题目中 X_i. Y_j 共享同一个 σ^2, 所以对 σ^2 估计时可以同时使用X、Y的信息、

2°. 似然函数形式较复杂时, 应关注待估参数. 其他常数求导结果为0.

扫描全能王 创建

HW13

11. $E\bar{X} = EX = \theta$. $\therefore$ $\bar{X}$是θ的无偏估计.

$X \sim U(a,b)$ $\quad VarX = \frac{(b-a)^2}{12}$

$Var\bar{X} = \frac{1}{n} VarX = \frac{1}{12n}$

$P(X_{(n)} \le x) = \prod_{i=1}^{n} P(X_i \le x) = (\theta - \frac{1}{2} + x)^n$.

$\therefore EX_{(n)} = \int_{\theta-\frac{1}{2}}^{\theta+\frac{1}{2}} x \cdot n(\theta - \frac{1}{2} + x)^{n-1} dx = \theta - \frac{1}{2} + \frac{n}{n+1} = \theta + \frac{n-1}{2(n+1)}$ $\quad \therefore X_{(n)}$不是无偏估计.

修正: $X_{(n)} - \frac{n-1}{2(n+1)} \triangleq \hat{\theta}$

$Var\,\hat{\theta} = Var\,X_{(n)}$

$EX_{(n)}^2 = \int_{\theta-\frac{1}{2}}^{\theta+\frac{1}{2}} x^2 \cdot n(x-\theta+\frac{1}{2})^{n-1}dx = x^2(x-\theta+\frac{1}{2})^n\Big|_{x=\theta-\frac{1}{2}}^{\theta+\frac{1}{2}} - \int_{\theta-\frac{1}{2}}^{\theta+\frac{1}{2}} 2x(x-\theta+\frac{1}{2})^n dx$

$= (\theta+\frac{1}{2})^2 - \frac{2x}{n+1}(x-\theta+\frac{1}{2})^{n+1}\Big|_{x=\theta-\frac{1}{2}}^{\theta+\frac{1}{2}} + \int_{\theta-\frac{1}{2}}^{\theta+\frac{1}{2}} \frac{2}{n+1}(x-\theta+\frac{1}{2})^{n+1}dx$

$= (\theta+\frac{1}{2})^2 - \frac{2(\theta+\frac{1}{2})}{n+1} + \frac{2}{(n+1)(n+2)}$

$Var\,X_{(n)} = EX_{(n)}^2 - E^2X_{(n)} = \frac{n}{(n+1)^2(n+2)}$

令 $\frac{1}{12n} = \frac{n}{(n+1)^2(n+2)}$

$12n^2 = (n^2+2n+1)(n+2) = n^3+4n^2+5n+2$.

$n^3-8n^2+5n+2=0$. $\quad (n-1)(n^2-7n-2)=0$.

$n=1$. $\quad n=\frac{7\pm\sqrt{49+8}}{2}$.

$\therefore$ $n=1$时 $\quad Var\bar{X} = Var\hat{\theta}$ $\quad$ 一样

$2\le n\le 7$时 $\quad Var\bar{X} < Var\hat{\theta}$ $\quad$ $\bar{X}$更有效

$n\ge 8$ 时 $\quad Var\bar{X} > Var\hat{\theta}$ $\quad$ $\hat{\theta}$更有效.

13. (1) $P(X_{(n)} \le x) = \prod_{i=1}^{n} \frac{x}{\theta} = \frac{x^n}{\theta^n}$.

$\therefore f_1(x) = \frac{nx^{n-1}}{\theta^n} I_{(0,\theta)}$

$EX_{(n)} = \frac{n\theta}{n+1}$

$\therefore E\hat{\theta}_1 = \theta$ 是θ的无偏估计

$P(X_{(1)} \le x) = 1 - P(X_{(1)} > x) = 1 - (1-\frac{x}{\theta})^n$.

$f_2(x) = \frac{n}{\theta}(1-\frac{x}{\theta})^{n-1} I_{(0,\theta)}$

$EX_{(1)} = \frac{\theta}{n+1}$.

(2) $E\hat{\theta}_2 = C_n \cdot \frac{\theta}{n+1} = \theta$.

$\therefore C_n = n+1$

(3) $Var\,\hat{\theta}_2 = (n+1)^2 Var\,X_{(1)}$

$Var\,\hat{\theta}_3 = \frac{1}{n} Var\,X = \frac{\theta^2}{12n}$

$Var\,\hat{\theta}_4 = \frac{(n+1)^2}{n^2} Var\,X_{(n)}$.

方法同11. 可计算得 $Var\,X_{(1)} = \frac{n\theta^2}{(n+1)^2(n+2)}$

$Var\,X_{(n)} = \frac{n\theta^2}{(n+1)^2(n+2)}$

$\therefore Var\hat{\theta}_2 = \frac{n\theta^2}{n+2}$ $\quad Var\,\hat{\theta}_3 = \frac{\theta^2}{12n}$ $\quad Var\,\hat{\theta}_4 = \frac{\theta^2}{n(n+2)}$

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显然，$\hat{\theta}_2$的方差最大。

令 $\frac{\theta^2}{12n} = \frac{\theta^2}{n(n+2)}$　　$n(n+2)=12n$，　$n(n-10)=0$.

$n \le 9$ 时，　$Var\,\hat{\theta}_3 < Var\,\hat{\theta}_4 < Var\,\hat{\theta}_2$

$n=10$，　$Var\,\hat{\theta}_3 = Var\,\hat{\theta}_4 < Var\,\hat{\theta}_2$

$n \ge 11$，　$Var\,\hat{\theta}_4 < Var\,\hat{\theta}_3 < Var\,\hat{\theta}_2$

18. (1) $E\hat{\theta}_1 = a_n E\bar{X} = a_n \cdot EX = a_n \cdot \theta = \theta$.　$\Rightarrow a_n = 1$

$E\hat{\theta}_2 = b_n EX_{(1)}$

$P(X_{(1)} > x) = \prod_{i=1}^{n} P(X_i > x) = \prod_{i=1}^{n} \exp\{-\frac{x}{\theta}\} = \exp\{-\frac{nx}{\theta}\}$

$\therefore X_{(1)} \sim Exp(\frac{n}{\theta})$

$\therefore E\hat{\theta}_2 = b_n \cdot \frac{\theta}{n} = \theta$　$\Rightarrow b_n = n$.

指数分布：$X \sim Exp(\lambda)$

$\Rightarrow EX = \frac{1}{\lambda}$　$Var\,X = \frac{1}{\lambda^2}$

$f(x) = \lambda e^{-\lambda x} I_{(0,+\infty)}$

$F(x) = 1 - e^{-\lambda x}$　$(x \ge 0)$.

$P(X > x) = e^{-\lambda x}$　$(x \ge 0)$.

(2) $Var\,\hat{\theta}_1 = \frac{1}{n} Var\,X = \frac{\theta^2}{n}$

$Var\,\hat{\theta}_2 = n^2 \cdot Var(X_{(1)}) = n^2 \cdot (\frac{\theta}{n})^2 = \theta^2$

$\therefore Var\,\hat{\theta}_1 \le Var\,\hat{\theta}_2$　　$\therefore \hat{\theta}_1$ 更有效

19. (1) pdf：$f(x) = \frac{2x}{\theta} e^{-\frac{x^2}{\theta}} I_{(0,+\infty)}$

$\therefore EX = \int_0^{+\infty} \frac{2x^2}{\theta} e^{-\frac{x^2}{\theta}} dx = -x \cdot e^{-\frac{x^2}{\theta}} \Big|_0^{+\infty} + \int_0^{+\infty} e^{-\frac{x^2}{\theta}} dx$

$= \sqrt{2\pi \cdot \frac{\theta}{2}} \cdot \int_0^{+\infty} \frac{1}{\sqrt{2\pi \cdot \frac{\theta}{2}}} \exp\{-\frac{x^2}{2 \cdot \frac{\theta}{2}}\} dx$.

$= \frac{\sqrt{\pi\theta}}{2}$.

$EX^2 = \int_0^{+\infty} x^2 \cdot \frac{2x}{\theta} e^{-\frac{x^2}{\theta}} dx = -x^2 e^{-\frac{x^2}{\theta}} \Big|_0^{+\infty} + \int_0^{+\infty} 2x\, e^{-\frac{x^2}{\theta}} dx$

$\overset{y=x^2}{=} \int_0^{+\infty} y\, e^{-\frac{y}{\theta}} dy = -\theta e^{-\frac{y}{\theta}} \Big|_0^{+\infty} = \theta$

(2) $f(x) = \prod_{i=1}^{n} \frac{2x_i}{\theta} e^{-\frac{x_i^2}{\theta}}$

$l(\theta, x) = \sum_{i=1}^{n} -\ln\theta - \frac{x_i^2}{\theta} + \ln 2x_i$

$\frac{\partial l}{\partial \theta} = -\frac{n}{\theta} + \frac{\sum_{i=1}^{n} x_i^2}{\theta^2} = 0$

$\therefore \theta = \frac{\sum_{i=1}^{n} x_i^2}{n}$　　$\therefore \hat{\theta} = \frac{\sum_{i=1}^{n} X_i^2}{n}$

(3) 由(1)可知　$EX^2 = \theta$.

$\therefore \hat{\theta} = \frac{\sum_{i=1}^{n} X_i^2}{n} \xrightarrow{P} \theta$.

扫描全能王 创建

44.(1) $EX = \int_{\theta}^{+\infty} \frac{x}{\sigma} \exp\{-\frac{x-\theta}{\sigma}\} dx$

$= -xe^{-\frac{x-\theta}{\sigma}}\Big|_{\theta}^{+\infty} + \int_{\theta}^{+\infty} e^{-\frac{x-\theta}{\sigma}} dx = \theta - \sigma e^{-\frac{x-\theta}{\sigma}}\Big|_{\theta}^{+\infty} = \theta + \sigma$

$\therefore \hat{\theta}_1 = \bar{X} - \sigma$.

$f(x) = \prod_{i=1}^{n} \frac{1}{\sigma} \exp\{-\frac{(x_i-\theta)}{\sigma}\} I(x_i > \theta)$

$L(\theta) = \sigma^{-n} \exp\{-\frac{\sum_{i=1}^{n}(x_i-\theta)}{\sigma}\} I(x_{(1)} > \theta)$

$\therefore$ $\theta < X_{(1)}$ 时，$L(\theta)$ 关于 θ 单增.

$\theta = X_{(1)}$ 时 $L(\theta)$ 取最大值

$\therefore \hat{\theta}_2 = X_{(1)}$.

(2) $E\hat{\theta}_1 = E\bar{X} - \sigma = \theta$. $\hat{\theta}_1$ 无偏

$P(X > x) = \int_{x}^{+\infty} \frac{1}{\sigma} e^{-\frac{t-\theta}{\sigma}} dt = -e^{-\frac{t-\theta}{\sigma}}\Big|_{x}^{+\infty} = e^{-\frac{x-\theta}{\sigma}}$ ($x > \theta$ 时)

$P(X_{(1)} > x) = \prod_{i=1}^{n} P(X_i > x) = \exp\{-\frac{n(x-\theta)}{\sigma}\}$ $(x > \theta)$

$\therefore$ $X_{(1)}$ 的 pdf. $f_1(x) = \frac{n}{\sigma} \exp\{-\frac{n(x-\theta)}{\sigma}\} I(x > \theta)$

$EX_{(1)} = \int_{\theta}^{+\infty} f_1(x) dx = \frac{\sigma}{n} + \theta$.

$\therefore$ $\hat{\theta}_2$ 不是无偏估计. 修正为 $\tilde{\theta}_2 = X_{(1)} - \frac{\sigma}{n}$

(3) $Var\,\tilde{\theta}_1 = \frac{1}{n} Var X = \frac{\sigma^2}{n}$

$Var\,\tilde{\theta}_2 = Var\,X_{(1)} = (\frac{\sigma}{n})^2 = \frac{\sigma^2}{n^2}$

$\therefore Var\,\tilde{\theta}_1 \geqslant Var\,\tilde{\theta}_2$

$\therefore$ $\tilde{\theta}_2$ 更优

该题中的分布其实是平移后的指数分布.

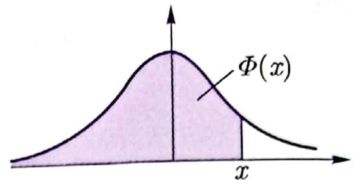
平移后期望改变，但方差是不变的.

实际和 6.18. 是一样的.

扫描全能王 创建

附　　表

附表 1　标准正态分布表



$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{u^2}{2}}\mathrm{d}u$$

x	0	1	2	3	4	5	6	7	8	9
0.0	0.500 0	0.504 0	0.508 0	0.512 0	0.516 0	0.519 9	0.523 9	0.527 9	0.531 9	0.535 9
0.1	0.539 8	0.543 8	0.547 8	0.551 7	0.555 7	0.559 6	0.563 6	0.567 5	0.571 4	0.575 3
0.2	0.579 3	0.583 2	0.587 1	0.591 0	0.594 8	0.598 7	0.602 6	0.606 4	0.610 3	0.614 1
0.3	0.617 9	0.621 7	0.625 5	0.629 3	0.633 1	0.636 8	0.640 6	0.644 3	0.648 0	0.651 7
0.4	0.655 4	0.659 1	0.662 8	0.666 4	0.670 0	0.673 6	0.677 2	0.680 8	0.684 4	0.687 9
0.5	0.691 5	0.695 0	0.698 5	0.701 9	0.705 4	0.708 8	0.712 3	0.715 7	0.719 0	0.722 4
0.6	0.725 7	0.729 1	0.732 4	0.735 7	0.738 9	0.742 2	0.745 4	0.748 6	0.751 7	0.754 9
0.7	0.758 0	0.761 1	0.764 2	0.767 3	0.770 3	0.773 4	0.776 4	0.779 4	0.782 3	0.785 2
0.8	0.788 1	0.791 0	0.793 9	0.796 7	0.799 5	0.802 3	0.805 1	0.807 8	0.810 6	0.813 3
0.9	0.815 9	0.818 6	0.821 2	0.823 8	0.826 4	0.828 9	0.831 5	0.834 0	0.836 5	0.838 9
1.0	0.841 3	0.843 8	0.846 1	0.848 5	0.850 8	0.853 1	0.855 4	0.857 7	0.859 9	0.862 1
1.1	0.864 3	0.866 5	0.868 6	0.870 8	0.872 9	0.874 9	0.877 0	0.879 0	0.881 0	0.883 0
1.2	0.884 9	0.886 9	0.888 8	0.890 7	0.892 5	0.894 4	0.896 2	0.898 0	0.899 7	0.901 5
1.3	0.903 2	0.904 9	0.906 6	0.908 2	0.909 9	0.911 5	0.913 1	0.914 7	0.916 2	0.917 7
1.4	0.919 2	0.920 7	0.922 2	0.923 6	0.925 1	0.926 5	0.927 8	0.929 2	0.930 6	0.931 9
1.5	0.933 2	0.934 5	0.935 7	0.937 0	0.938 2	0.939 4	0.940 6	0.941 8	0.943 0	0.944 1
1.6	0.945 2	0.946 3	0.947 4	0.948 4	0.949 5	0.950 5	0.951 5	0.952 5	0.953 5	0.954 5
1.7	0.955 4	0.956 4	0.957 3	0.958 2	0.959 1	0.959 9	0.960 8	0.961 6	0.962 5	0.963 3
1.8	0.964 1	0.964 8	0.965 6	0.966 4	0.967 1	0.967 8	0.968 6	0.969 3	0.970 0	0.970 6
1.9	0.971 3	0.971 9	0.972 6	0.973 2	0.973 8	0.974 4	0.975 0	0.975 6	0.976 2	0.976 7
2.0	0.977 2	0.977 8	0.978 3	0.978 8	0.979 3	0.979 8	0.980 3	0.980 8	0.981 2	0.981 7
2.1	0.982 1	0.982 6	0.983 0	0.983 4	0.983 8	0.984 2	0.984 6	0.985 0	0.985 4	0.985 7
2.2	0.986 1	0.986 4	0.986 8	0.987 1	0.987 4	0.987 8	0.988 1	0.988 4	0.988 7	0.989 0
2.3	0.989 3	0.989 6	0.989 8	0.990 1	0.990 4	0.990 6	0.990 9	0.991 1	0.991 3	0.991 6
2.4	0.991 8	0.992 0	0.992 2	0.992 5	0.992 7	0.992 9	0.993 1	0.993 2	0.993 4	0.993 6
2.5	0.993 8	0.994 0	0.994 1	0.994 3	0.994 5	0.994 6	0.994 8	0.994 9	0.995 1	0.995 2
2.6	0.995 3	0.995 5	0.995 6	0.995 7	0.995 9	0.996 0	0.996 1	0.996 2	0.996 3	0.996 4
2.7	0.996 5	0.996 6	0.996 7	0.996 8	0.996 9	0.997 0	0.997 1	0.997 2	0.997 3	0.997 4
2.8	0.997 4	0.997 5	0.997 6	0.997 7	0.997 7	0.997 8	0.997 9	0.997 9	0.998 0	0.998 1
2.9	0.998 1	0.998 2	0.998 2	0.998 3	0.998 4	0.998 4	0.998 5	0.998 5	0.998 6	0.998 6
3.	0.998 7	0.999 0	0.999 3	0.999 5	0.999 7	0.999 8	0.999 8	0.999 9	0.999 9	1.000 0

注：表中末行为函数值 $\Phi(3.0), \Phi(3.1), \cdots, \Phi(3.9)$.

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附表 5　泊松分布表

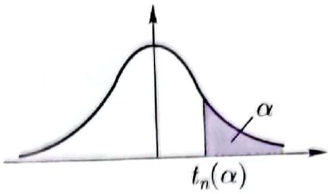
$$P(X \geqslant x) = \sum_{k=x}^{\infty} \frac{\lambda^k \mathrm{e}^{-\lambda}}{k!}$$

x	$\lambda=0.2$	$\lambda=0.3$	$\lambda=0.4$	$\lambda=0.5$	$\lambda=0.6$	$\lambda=0.7$
0	1.000 000 0	1.000 000 0	1.000 000 0	1.000 000	1.000 000	1.000 000
1	0.181 269 2	0.259 181 8	0.329 680 0	0.393 469	0.451 188	0.503 415
2	0.017 523 1	0.036 936 3	0.061 551 9	0.090 204	0.121 901	0.155 805
3	0.001 148 5	0.003 599 5	0.007 926 3	0.014 388	0.023 115	0.034 142
4	0.000 056 8	0.000 265 8	0.000 776 3	0.001 752	0.003 358	0.005 753
5	0.000 002 3	0.000 015 8	0.000 061 2	0.000 172	0.000 394	0.000 786
6	0.000 000 1	0.000 000 8	0.000 004 0	0.000 014	0.000 039	0.000 090
7			0.000 000 2	0.000 001	0.000 003	0.000 009
8						0.000 001

x	$\lambda=0.8$	$\lambda=0.9$	$\lambda=1.0$	$\lambda=1.2$	$\lambda=1.5$	$\lambda=2.0$
0	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
1	0.550 671	0.593 430	0.632 121	0.698 806	0.776 870	0.864 665
2	0.191 208	0.227 518	0.264 241	0.337 373	0.442 175	0.593 994
3	0.047 423	0.062 857	0.080 301	0.120 513	0.191 153	0.323 324
4	0.009 080	0.013 459	0.018 988	0.033 769	0.065 642	0.142 877
5	0.001 411	0.002 344	0.003 660	0.007 746	0.018 576	0.052 653
6	0.000 184	0.000 343	0.000 594	0.001 500	0.004 456	0.016 564
7	0.000 021	0.000 043	0.000 083	0.000 251	0.000 926	0.004 534
8	0.000 002	0.000 005	0.000 010	0.000 037	0.000 170	0.001 097
9			0.000 001	0.000 005	0.000 028	0.000 237
10				0.000 001	0.000 004	0.000 046
11					0.000 001	0.000 008
12						0.000 001

x	$\lambda=2.5$	$\lambda=3.0$	$\lambda=3.5$	$\lambda=4.0$	$\lambda=4.5$	$\lambda=5.0$
0	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
1	0.917 915	0.950 213	0.969 803	0.981 684	0.988 891	0.993 262
2	0.712 703	0.800 852	0.864 112	0.908 422	0.938 901	0.959 572
3	0.456 187	0.576 810	0.679 153	0.761 897	0.826 422	0.875 348
4	0.242 424	0.352 768	0.463 367	0.566 530	0.657 704	0.734 974
5	0.108 822	0.184 737	0.274 555	0.371 163	0.467 896	0.559 507
6	0.042 021	0.083 918	0.142 386	0.214 870	0.297 070	0.384 039
7	0.014 187	0.033 509	0.065 288	0.110 674	0.168 949	0.237 817
8	0.004 247	0.011 905	0.026 739	0.051 134	0.086 586	0.133 372
9	0.001 140	0.003 803	0.009 874	0.021 363	0.040 257	0.068 094
10	0.000 277	0.001 102	0.003 315	0.008 132	0.017 093	0.031 828
11	0.000 062	0.000 292	0.001 019	0.002 840	0.006 669	0.013 695
12	0.000 013	0.000 071	0.000 289	0.000 915	0.002 404	0.005 453
13	0.000 002	0.000 016	0.000 076	0.000 274	0.000 805	0.002 019
14		0.000 003	0.000 019	0.000 076	0.000 252	0.000 698
15		0.000 001	0.000 004	0.000 020	0.000 074	0.000 226
16			0.000 001	0.000 005	0.000 020	0.000 069
17				0.000 001	0.000 005	0.000 020
18					0.000 001	0.000 005
19						0.000 001

扫描全能王 创建



$P(t_n > t_n(\alpha)) = \alpha$

n	α = 0.25	0.10	0.05	0.025	0.01	0.005
1	1.000 0	3.077 7	6.313 8	12.706 2	31.820 7	63.657 4
2	0.816 5	1.885 6	2.920 0	4.302 7	6.964 6	9.924 8
3	0.764 9	1.637 7	2.353 4	3.182 4	4.540 7	5.840 9
4	0.740 7	1.533 2	2.131 8	2.776 4	3.746 9	4.604 1
5	0.726 7	1.475 9	2.015 0	2.570 6	3.364 9	4.032 2
6	0.717 6	1.439 8	1.943 2	2.446 9	3.142 7	3.707 4
7	0.711 1	1.414 9	1.894 6	2.364 6	2.998 0	3.499 5
8	0.706 4	1.396 8	1.859 5	2.306 0	2.896 5	3.355 4
9	0.702 7	1.383 0	1.833 1	2.262 2	2.821 4	3.249 8
10	0.699 8	1.372 2	1.812 5	2.228 1	2.763 8	3.169 3
11	0.697 4	1.363 4	1.795 9	2.201 0	2.718 1	3.105 8
12	0.695 5	1.356 2	1.782 3	2.178 8	2.681 0	3.054 5
13	0.693 8	1.350 2	1.770 9	2.160 4	2.650 3	3.012 3
14	0.692 4	1.345 0	1.761 3	2.144 8	2.624 5	2.976 8
15	0.691 2	1.340 6	1.753 1	2.131 5	2.602 5	2.946 7
16	0.690 1	1.336 8	1.745 9	2.119 9	2.583 5	2.920 8
17	0.689 2	1.333 4	1.739 6	2.109 8	2.566 9	2.898 2
18	0.688 4	1.330 4	1.734 1	2.100 9	2.552 4	2.878 4
19	0.687 6	1.327 7	1.729 1	2.093 0	2.539 5	2.860 9
20	0.687 0	1.325 3	1.724 7	2.086 0	2.528 0	2.845 3
21	0.686 4	1.323 2	1.720 7	2.079 6	2.517 7	2.831 4
22	0.685 8	1.321 2	1.717 1	2.073 9	2.508 3	2.818 8
23	0.685 3	1.319 5	1.713 9	2.068 7	2.499 9	2.807 3
24	0.684 8	1.317 8	1.710 9	2.063 9	2.492 2	2.796 9
25	0.684 4	1.316 3	1.708 1	2.059 5	2.485 1	2.787 4
26	0.684 0	1.315 0	1.705 6	2.055 5	2.478 6	2.778 7
27	0.683 7	1.313 7	1.703 3	2.051 8	2.472 7	2.770 7
28	0.683 4	1.312 5	1.701 1	2.048 4	2.467 1	2.763 3
29	0.683 0	1.311 4	1.699 1	2.045 2	2.462 0	2.756 4
30	0.682 8	1.310 4	1.697 3	2.042 3	2.457 3	2.750 0
40	0.681	1.303	1.684	2.021	2.423	2.704
60	0.679	1.296	1.671	2.000	2.390	2.660
120	0.677	1.289	1.658	1.980	2.358	2.617
∞	0.674	1.282	1.654	1.960	2.326	2.576

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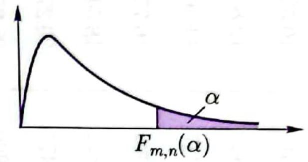
附表 4　*F* 分布表



$$P(F_{m,n} > F_{m,n}(\alpha)) = \alpha$$

$\alpha = 0.10$

n \ m	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	4.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61

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