第 16 周作业

注：本次作业 3 4 5 题答案不唯一。

1

(2)

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \end{pmatrix}$$

(4)

$$\begin{pmatrix} 1 & 0 & -1 & & \\ 0 & 1 & \ddots & \ddots & \\ -1 & \ddots & 2 & \ddots & -1 \\ & & \ddots & \ddots & 0 \\ & & -1 & 0 & 1 \end{pmatrix}$$

3

(2)

$$\begin{pmatrix} 1 & 4 & -8 \\ 4 & 7 & 4 \\ -8 & 4 & 1 \end{pmatrix}$$

求特征值可得 $\begin{vmatrix} \lambda-1 & -4 & 8 \\ -4 & \lambda-7 & -4 \\ 8 & -4 & \lambda-1 \end{vmatrix} = (\lambda-1)^2(\lambda-7)+256-64(\lambda-7)-32(\lambda-1) = \lambda^3-9\lambda^2-81\lambda+729=(\lambda-9)^2(\lambda+9)$

求特征向量得 $x_1=(1,4,1)^T, x_2=(1,0,-1)^T, x_3=(2,-1,2)^T$

$$P=\begin{pmatrix} \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{2}{3} \\ \frac{4}{3\sqrt{2}} & 0 & -\frac{1}{3} \\ \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{2}{3} \end{pmatrix}$$

(4)

$$\begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & -1 \\ & & -1 & \end{pmatrix}$$

求特征值得$(\lambda-1)^2(\lambda+1)^2=0$

$$x_1=(1,1,0,0)^T,x_2=(0,0,-1,1)^T,x_3=(1,-1,0,0)^T,x_4=(0,0,1,1)^T$$

$$P=\begin{pmatrix}\frac{1}{\sqrt{2}} & & \frac{1}{\sqrt{2}} & \\ \frac{1}{\sqrt{2}} & & -\frac{1}{\sqrt{2}} & \\ & -\frac{1}{\sqrt{2}} & & \frac{1}{\sqrt{2}} \\ & \frac{1}{\sqrt{2}} & & \frac{1}{\sqrt{2}}\end{pmatrix}$$

4

(2)

$$y_2=\frac{1}{2}(x_2+x_3),y_3=\frac{1}{2}(x_2-x_3),y_1=x_1$$

$$Q=y_1^2+y_2^2-y_3^2$$

$$P=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 1 & -1\end{pmatrix}$$

(3)

令$x_1=y_1+y_2,x_2=y_1-y_2,x_3=y_3+y_4,x_4=y_3-y_4$

$$Q=(y_1+y_3)^2-(y_2+y_4)^2$$

令$y_1=z_1-z_3,y_2=z_2-z_4,y_3=z_3,y_4=z_4$

$$Q=z_1^2-z_2^2$$

$$P=\begin{pmatrix}1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{pmatrix}$$

5

(2)

$$\begin{pmatrix}0 & -1 & 0\\ -1 & 1 & 1\\ 0 & 1 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\to\begin{pmatrix}-1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\to\begin{pmatrix}-1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 1 & 2\\ 1 & 0 & 1\\ 1 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}\to\begin{pmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$P=\begin{pmatrix}1&0&1\\1&1&0\\0&0&1\end{pmatrix}$$

$$Q=-y_1^2+y_2^2+y_3^2$$

（4）

$$\begin{pmatrix}&\frac{1}{2}&&\frac{1}{2}\\\frac{1}{2}&&\frac{1}{2}&\\&\frac{1}{2}&&\frac{1}{2}\\\frac{1}{2}&&\frac{1}{2}&\\1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}\to\begin{pmatrix}&\frac{1}{2}&&\\\frac{1}{2}&&\frac{1}{2}&\\&\frac{1}{2}&&0\\&&0&\\1&&&\\&1&&-1\\&&1&\\&&&1\end{pmatrix}\to\begin{pmatrix}&\frac{1}{2}&&\\\frac{1}{2}&&0&\\&0&&0\\&&0&\\1&&-1&\\&1&&-1\\&&1&\\&&&1\end{pmatrix}$$

$$\to\begin{pmatrix}1&&&\\&-\frac{1}{4}&&\\&&0&\\&&&0\\1&-\frac{1}{2}&-1&\\1&\frac{1}{2}&&-1\\&&1&\\&&&1\end{pmatrix}$$

$$P=\begin{pmatrix}1&-\frac{1}{2}&-1&\\1&\frac{1}{2}&&-1\\&&1&\\&&&1\end{pmatrix}$$

$$Q=y_1^2-\frac{1}{4}y_2^2$$

7

$$A^2=A\to r(A)+r(I-A)=n$$

设$r(A)=r$，则$r(I-A)=n-r$

设A的特征值为λ，相应的特征向量为 x，则$A^2x=\lambda^2x,Ax=\lambda x,\lambda=0,1$

$Ax=0$解空间维数为$n-r$，$(I-A)x=0$解空间维数为r

相合标准型为$diag(I_r,O)$