18 (1) (3) 21 23 26 27 28

18 (1) $|\lambda I-A| = \begin{vmatrix} \lambda-1 & 1 \\ 1 & \lambda-2 \end{vmatrix} = (\lambda-1)(\lambda-3)$

$\lambda=1$ 时解得特征向量 $\vec{e_1} = \frac{1}{\sqrt{2}}(1,1)^T$

$\lambda=3$ $\quad \vec{e_2} = \frac{1}{\sqrt{2}}(1,-1)^T$

故 $T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(3) $|\lambda I-A| = \begin{vmatrix} \lambda-1 & -2 & -4 \\ -2 & \lambda+2 & -2 \\ -4 & -2 & \lambda-1 \end{vmatrix} = (\lambda+3)^2(\lambda-6)$

$\lambda=-3$ 时得特征向量为 $c_1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_2\begin{bmatrix} -1 \\ 4 \\ -1 \end{bmatrix}$

标准正交化得 $\vec{e_1} = \frac{1}{\sqrt{2}}(1,0,-1)^T \quad \vec{e_2} = \frac{1}{3\sqrt{2}}(-1,4,-1)^T$

$\lambda=6$ 时特征向量为 $c_3\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \vec{e_3} = \frac{1}{3}(2,1,2)^T$

故 $T = \frac{1}{3\sqrt{2}}\begin{bmatrix} 3 & -1 & 2\sqrt{2} \\ 0 & 4 & \sqrt{2} \\ -3 & -1 & 2\sqrt{2} \end{bmatrix}$

21. (1) $Ux = \lambda_1 x \quad Uy = \lambda_2 y \Rightarrow \bar{y}^T U^H = \bar{\lambda}_2 y^H$

$y^H U^H U x = \lambda_1\bar{\lambda}_2 y^H x$ ，即有 $y^H x = \lambda_1\bar{\lambda}_2 y^H x$

由于 $\lambda_1 \neq \lambda_2$，故 $\lambda_1\bar{\lambda}_2 \neq 1$，故 $y^H x = 0$ $\quad x,y$ 正交

(2) $Ax = \lambda_1 x \quad Ay = \lambda_2 y \Rightarrow y^H A^H = \bar{\lambda}_2 y^H = \lambda_2 y^H$

$y^H A x = \lambda_1 y^H x \quad y^H A^H x = \lambda_2 y^H x$ 故 $\lambda_1 y^H x = \lambda_2 y^H x$

由 $\lambda_1 \neq \lambda_2$ 知 $y^H x = 0$ $\quad x,y$ 正交

23. (1) A的特征值均为实数，设为 $\lambda_1,\cdots,\lambda_n$

则 $I\pm iA$ 的特征值为 $1\pm i\lambda_1,\cdots,1\pm i\lambda_n$ 非零

故 $\det(I\pm iA) = \prod_{k=1}^{n}(1\pm i\lambda_k) \neq 0 \Rightarrow I+iA$ 可逆

(2) $(I+iA)(I-iA) = I+iA-iA+A^2 = I+A^2$

$(I-iA)(I+iA) = I-iA+iA+A^2 = I+A^2$

(3) $UU^H = (I+iA)(I-iA)^{-1}\left[(I+iA)(I-iA)^{-1}\right]^H$

$= (I+iA)(I-iA)^{-1}\left[(I-iA)^H\right]^{-1}(I+iA)^H$

$= (I+iA)(I-iA)^{-1}(I+iA)^{-1}(I-iA)$

由(2)知 $= (I+iA)(I+iA)^{-1}(I-iA)^{-1}(I-iA) = I$

25. 证：设 $Ax_k = \lambda_k x_k$ ， $A^H x_k = \bar{\lambda}_k x_k$

故 $A^H A x_k = \bar{\lambda}_k \lambda_k x_k \Rightarrow \bar{\lambda}_k \lambda_k = |\lambda_k|^2$ 为 $A^H A$ 特征值

故 $\sum_{i=1}^{n} \mu_i = \sum_{i=1}^{n} |\lambda_i|^2$

27. 证： $M^H M = \begin{bmatrix} A^H & O \\ C^H & B^H \end{bmatrix}\begin{bmatrix} A & C \\ O & B \end{bmatrix} = \begin{bmatrix} A^H A & A^H C \\ C^H A & C^H C + B^H B \end{bmatrix}$

$M M^H = \begin{bmatrix} A & C \\ O & B \end{bmatrix}\begin{bmatrix} A^H & O \\ C^H & B^H \end{bmatrix} = \begin{bmatrix} AA^H + CC^H & CB^H \\ BC^H & BB^H \end{bmatrix}$

$tr(A^H A) = tr(AA^H + CC^H) = tr(A^H A) + tr(CC^H)$ 　　故 $tr(CC^H) = 0 \Rightarrow C = O$

故 $A^H A = AA^H$ ， $B^H B = BB^H$

28. $\det(\lambda I - H) = \begin{vmatrix} \lambda & i & -1 \\ -i & \lambda & -i \\ -1 & i & \lambda \end{vmatrix} = (\lambda-2)(\lambda+1)^2$

$\lambda=2$ 时得 　$c_1\begin{bmatrix} 1 \\ i \\ 1 \end{bmatrix} \Rightarrow \vec{e}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ i \\ 1 \end{bmatrix}$

$\lambda=-1$ 时得特征向量为 　$c_2\begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \Rightarrow \vec{e}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$ 　$\vec{e}_3 = \sqrt{\frac{2}{3}}\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2}i \\ -1 \end{bmatrix}$

令 $U = [\vec{e}_1 \ \vec{e}_2 \ \vec{e}_3]$ 即可