REPORT

Zajęcia: Analog and digital electronic circuits

Teacher: prof. dr hab. Vasyl Martsenyuk

Lab 6

Date: 27.03.2024

Topic: "Quantization and Signal-to-Noise Ratio"

Variant: 13

Agnieszka Białecka

Informatyka II stopień,

stacjonarne,

1 semestr,

Gr.1a

1. Abstract

The objective is to investigate Signal-to-Noise Ratio for different signals.

2. Theoretical introduction

From theory, the 6dB / Bit rule of thumb is well known for uniform quantization. It states that the signal-to-noise ratio increases by 6 dB for every additional bit that is spent to quantize the input data. Hence,

$$\text{SNR in dB} = 6 \cdot B + \gamma,$$

where γ is a offset value in dB that depends on the PDF of the signal to be quantized. Note that this rule of thumb assumes that the quantization error exhibits uniform PDF and is not correlated with the quantized signal.

We can see that this rule of thumb is inaccurate when quantizing a sine signal with small number of bits or an amplitude in the range of the quantization step. Then, the mentioned assumptions are not fulfilled.

3. Input data (Variant)

This report was created with base of variant 13:

$\Omega_c = t^2 + t^3$

GitHub repository:

https://github.com/Delisolara/AaDEC

4. Course of actions

```python
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
from scipy import signal
import soundfile as sf
```

Picture 1. Uploaded libraries

```python
def my_quant(x, Q):
    r"""Saturated uniform midtread quantizer

    input:
    x  input signal
    Q  number of quantization steps
    output:
    xq quantized signal

    Note: for even Q in order to retain midtread characteristics,
    we must omit one quantization step, either that for lowest or the highest
    amplitudes. Typically the highest signal amplitudes are saturated to
    the 'last' quantization step. Then, in the special case of log2(N)
    being an integer the quantization can be represented with bits.
    """
    tmp = Q//2  # integer div
    quant_steps = (np.arange(Q) - tmp) / tmp  # we don't use this

    # forward quantization, round() and inverse quantization
    xq = np.round(x*tmp) / tmp
    # always saturate to -1
    xq[xq < -1.] = -1.
    # saturate to ((Q-1) - (Q\2)) / (Q\2), note that \ is integer div
    tmp2 = ((Q-1) - tmp) / tmp  # for odd N this always yields 1
    xq[xq > tmp2] = tmp2
    return xq
```

Picture 2. Implemented code

```python
def check_quant_SNR(x, dBoffset, title):
    print('std: {0:f}, var: {1:f}, mean: {2:f} of x'.format(np.std(x), np.var(x), np.mean(x)))
    Bmax = 24
    SNR = np.zeros(Bmax+1)
    SNR_ideal = np.zeros(Bmax+1)

    for B in range(1, Bmax+1):  # start at 1, since zero Q is not meaningful
        xq = my_quant(x, 2**B)
        SNR[B] = 10*np.log10(np.var(x) / np.var(xq-x))
        SNR_ideal[B] = B*20*np.log10(2) + dBoffset  # 6dB/bit + offset rule

    plt.figure(figsize=(5, 5))
    plt.plot(SNR_ideal, 'o-', label='theoretical', lw=3)
    plt.plot(SNR, 'x-', label='simulation')
    plt.xticks(np.arange(0, 26, 2))
    plt.yticks(np.arange(0, 156, 12))
    plt.xlim(2, 24)
    plt.ylim(6, 148)
    plt.xlabel('number of bits')
    plt.ylabel('SNR / dB')
    plt.title(title)
    plt.legend()
    plt.grid(True)
    print('maximum achievable SNR = {0:4.1f} dB at 24 Bit (i.e. HD audio)'.format(SNR[-1]))
```

Picture 3. Implemented code

```python
N = 10000
k = np.arange(N)
```

Picture 4. Implemented code

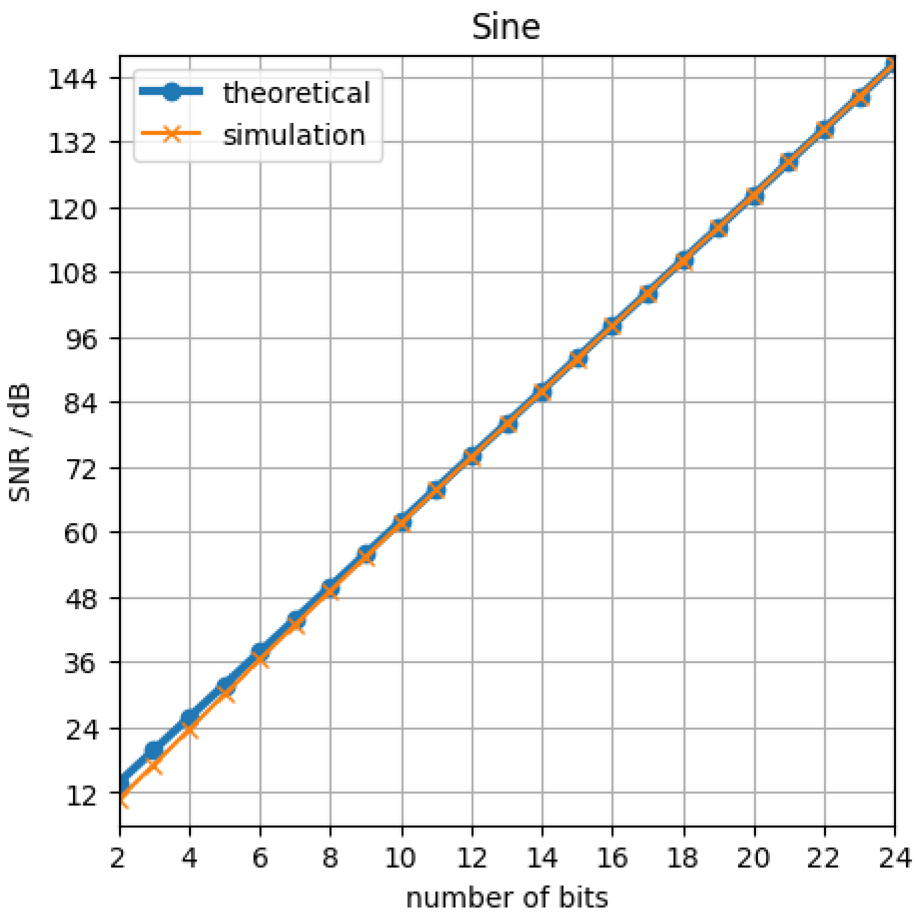
```
np.random.seed(4)
x = np.random.rand(N)
x -= np.mean(x)
x *= np.sqrt(1/3) / np.std(x)
dBoffset = 0
check_quant_SNR(x, dBoffset, 'Uniform PDF')
```

Picture 5. Implemented code

```
Omega = 2*np.pi * 997/44100
sigma2 = 1/2
dBoffset = -10*np.log10(2 / 3)
x = np.sqrt(2*sigma2) * np.sin(Omega*k)
check_quant_SNR(x, dBoffset, 'Sine')
```

Picture 6. Implemented code

```
std: 0.706997, var: 0.499845, mean: 0.000058 of x
maximum achievable SNR = 146.3 dB at 24 Bit (i.e. HD audio)
```

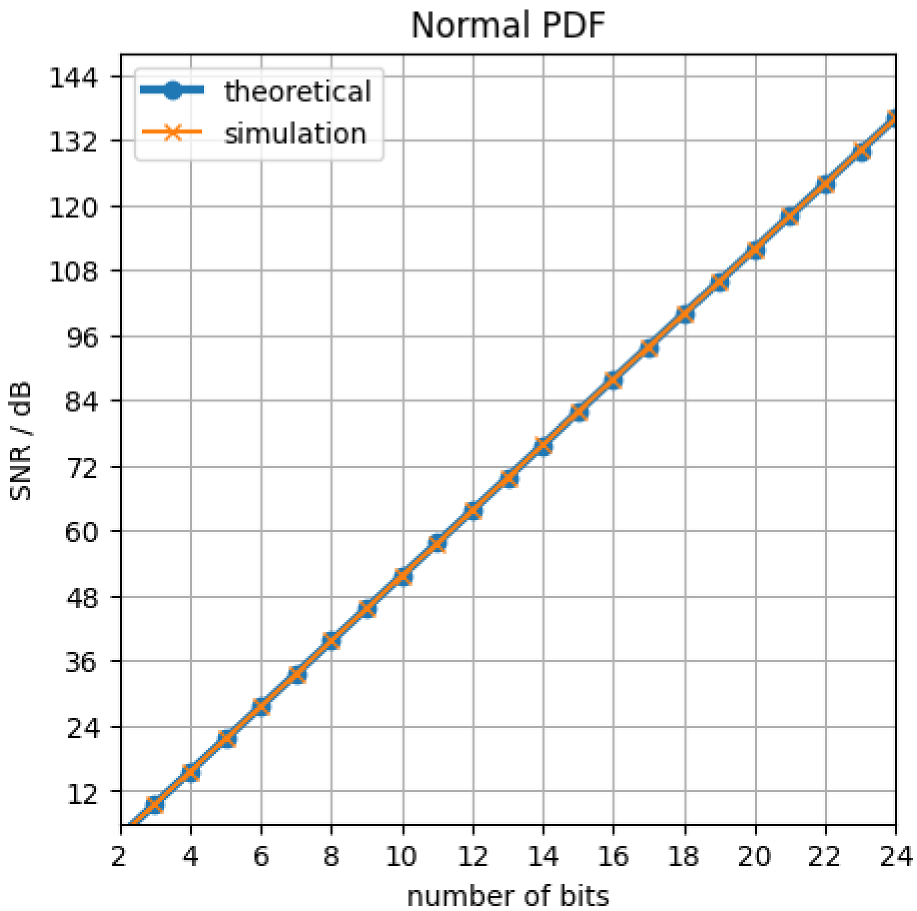


Picture 7. The result

```python
np.random.seed(4)
x = np.random.randn(N)
x -= np.mean(x)
x *= np.sqrt(0.0471) / np.std(x)
dBoffset = -8.5  # from clipping propability 1e-5
check_quant_SNR(x, dBoffset, 'Normal PDF')
```

Picture 8. Implemented code

```
std: 0.217025, var: 0.047100, mean: -0.000000 of x
maximum achievable SNR = 135.9 dB at 24 Bit (i.e. HD audio)
```

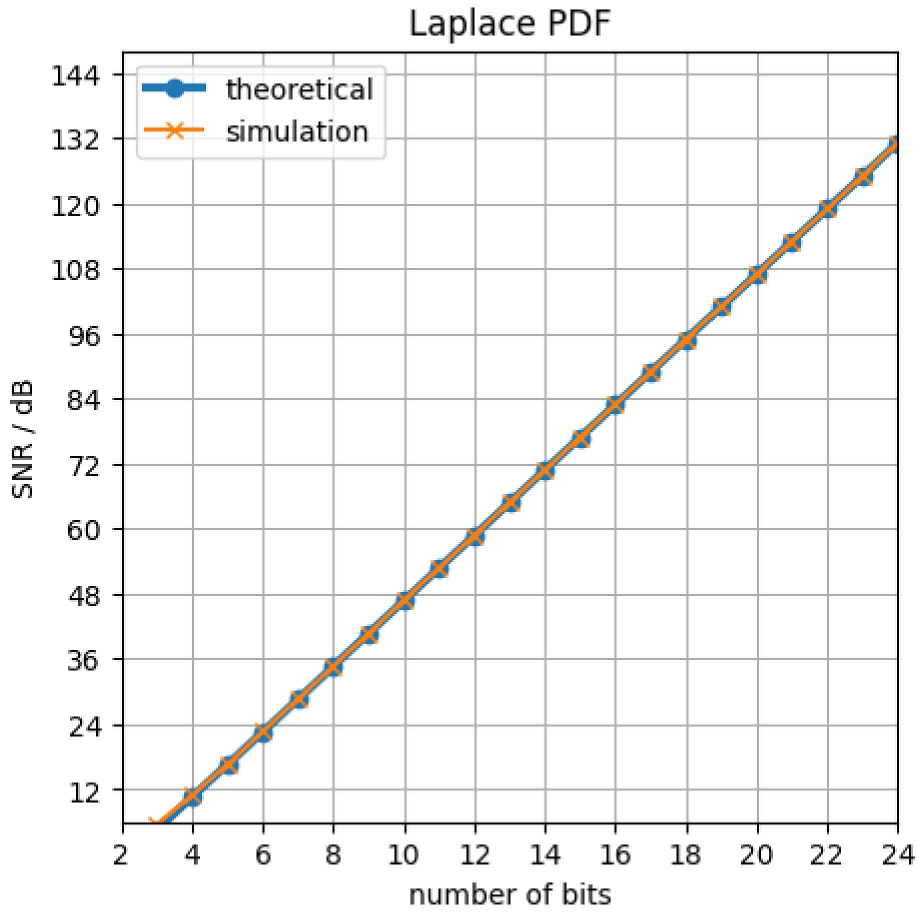


Picture 9. The result

```python
np.random.seed(4)
x = np.random.laplace(size=N)
pClip = 1e-5  # clipping propability
sigma = -np.sqrt(2) / np.log(pClip)
x -= np.mean(x)
x *= sigma / np.std(x)
dBoffset = -13.5  # empircially found for pClip = 1e-5
check_quant_SNR(x, dBoffset, 'Laplace PDF')
```

Picture 10. Implemented code

```
std: 0.122837, var: 0.015089, mean: 0.000000 of x
maximum achievable SNR = 131.1 dB at 24 Bit (i.e. HD audio)
```



Picture 11. The result

5. Conclusions

In this lab, we delved into the concept of Signal-to-Noise Ratio (SNR) and its application across various types of signals. Our exploration involved examining how SNR varies across different signal types, considering factors such as noise levels, signal strength, and the clarity of signal transmission. By analyzing SNR for diverse signal scenarios, we aimed to gain insights into the effectiveness of signal processing techniques in mitigating noise and enhancing signal quality.