PHY 387M: Relativity Theory Notes

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Notes for Prof. Matzner's Relativity Theory(PHY 387M) course at UT Austin during Spring 2021. The course follows Misner, Thorne, and Wheeler's "Gravitation" as well as Prof. Matzner's own notes. If you have any comments let me know at hi@delonshen.com.

Last updated on January 17, 2021

Lecture 1a: Historical Background and Special Relativity

Historical background I'm leaving out goes here

Let $\mathcal{E}$ be an event in a $D = 4$ spacetime $\mathcal{M}$. $\mathcal{E}$ could be a camera flash going off at position $x^\mu = \{t, x, y, z\}$. Lets say we have two such events $\mathcal{E}_1$ and $\mathcal{E}_2$. The interval between these two events is

$$x^\mu x_\mu = x^\mu g_{\mu\nu} x^\nu = -c^2 t^2 + x^2 + y^2 + z^2$$

This interval is the same in any reference frame. Lets try to derive the Lorentz transform. Consider some fast guy walking towards you with speed v. You're standing still in your reference frame. Your position in the fast guy's reference frame x' if we assume Galilean relativity is $x = x' - vt$. However this clearly doesn't keep the speed of light c the same in every reference frame. Thus lets introduce an undetermined function $\gamma(|v|)$ where we use $|v|$ to impose isotropy

$$x' = \gamma(|v|)\left(x - \frac{v}{c}ct\right)$$

In Galilean relativity $ct' = ct$ but this also doesn't work. However, if we're somehow inspired to, we can also guess for special relativity

$$ct' = \gamma(|v|)\left(ct - \frac{v}{c}x\right)$$

Now we use the invariant interval to get γ

$$-c^2 t^2 + x^2 = -\gamma^2\left(ct - \frac{v}{c}x\right)^2 + \gamma^2\left(x - \frac{v}{c}ct\right)^2$$

Solving for γ with Mathematica gives us the following

$$\boxed{\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}}$$

We should also know that the invariant interval can become infinitesimal giving us an infinitesimal arc length in flat space time

$$-ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

This leads us to define the four velocity

$$\frac{dx^\mu}{ds} = \left\{c\frac{dt}{ds}, \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right\}$$

We define spacelike as $ds^2 > 0$, timelike as $ds^2 < 0$, and null (e.g. light ray) as $ds^2 = 0$. Note that if $ds^2 = 0$ we can't define the 4-velocity as above. We introduce some parameter (affine parameter?) λ and have

$$0 = -c^2\left(\frac{dt}{d\lambda}\right)^2 + \left(\frac{dx}{d\lambda}\right)^2 + \ldots$$

From now on we will let $c = 1$.