Black Hole Information Paradox Notes

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Notes for Suvrat Raju's Black Hole Information Paradox course at ICTP(well really online) during Spring 2021. Course website can be found here. This course closely follow a review posted here. Expect many typos and inaccuricies reflecting my understanding. If you have any comments let me know at hi@delonshen.com.

Contents

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Lecture 1: Introduction and Two-Point QFT Correlators

The main organization of this course

(a) Hawking's Original Paradox → Thermalization and exponentially small corrections.

(b) Paradoxes about interior of evaporating Black Holes → holography of information, islands and page curve.

(c) Paradoxes about large Black Holes in AdS/CFT→ Mirror operators, state-dependence, and firewalls/furrballs

Hawking Radiation

Lets start by talking about **Hawking Radiation**, it's the effect that underlies the information paradox. Take a black hole in asymptotically flat space. This black hole radiates with a temperature ∝ surface gravity(TODO ???). We should also recall that hawking radiation relies on short distance QFT physics and on global late-time properties of the black hole geometry. The interesting thing is that the derivation for Hawking's radiation also implies the existence of the entangled modes across horizons. So what are the common derivations of hawking radiation(TODO (a) is in appendix of review paper and (b) might be in wald)?

(a) Hawking's original derivation

(b) Rindler ↔ Minkowski Bogolivlov transformation

In this course we'll consider a different derivation from both of these

Quantum Fields near Null Surfaces

Lets take a second to step back from black hole and look at Quantum Fields near a null surface. We'll apply what we learn here to black holes later. What we want to show is that across any null surface in a smooth state (TODO smooth state who?) we can isolate a "local" QFT (which we'll define in a bit) with universal entanglement. This is useful because we'll find that in a black hole spacetime local degrees of freedom near the horizon gives global modes in blackhole geometry.

First lets define what we mean by a smooth metric around some point. Consider a point in some $D = d + 1$ space and let this point be the origin. We have $\mathcal{U}, \mathcal{V}$, two null coordinates, and $d - 1$ transverse coordinates. A metric is smooth around some point if around some point we can locally choose some coordinates so the metric takes the following form. (think light cone variant Kruskal coordinates in arbitrary dimensions?)

$$ds^2 = -d\mathcal{U}d\mathcal{V} + \delta_{\alpha\beta}dy^\alpha dy^\beta + \dots$$

Where $d\mathcal{U}d\mathcal{V}$ are two null coordinates and α, β is over $d - 1$ indices and where the $\dots$ terms vanish near origin.

$figure$

We also want to make an additional demand. Consider a scalar field ϕ and points near $U = 0$. If we're still thinking in terms of Kruskal coordinates this means we're thinking of things close to eachother on each side of the horizon? In the limit where x_1 approaches x_2 for any nonsingular state the two point correlation function (Wightman function?) becomes.

$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{N}{|x_1 - x_2|^{d-1}} + \ldots$$

We also impose the following scales

(a) $|x_1 - x_2| \ll \ell_{\text{curvature}}$

(b) $|x_1 - x_2| \ll \frac{1}{m}$

(c) $|x_1 - x_2| \gg \ell_{\text{Pl}}$ or any UV scale where EFT breaks down.

These length scales give us the normalization if we consider a free field (e.g. $\mathcal{L} = 1/2(\partial_\mu\phi)^2$). (TODO how???)

$$N = \frac{\Gamma(d-1)}{2^d \pi^{d/2}\Gamma(d/2)} \Rightarrow \langle \phi(x_1)\phi(x_2)\rangle = \frac{\Gamma(d-1)}{2^d \pi^{d/2}\Gamma(d/2)} \frac{1}{|x_1 - x_2|^{d-1}} + \ldots$$

Because of the length scales we assume we can say that the structure of the two point function is universal (TODO what in the world.) Before we continue lets look at a few things that will be useful

$$|x_1 - x_2|^2 = -\delta\mathcal{U}\delta\mathcal{V} + \delta_{\alpha\beta}\delta y^\alpha \delta y^\beta \qquad \delta O = O_1 - O_2$$

Also if we grind through some calculations we'll find that

$$\langle \partial_{U_1}\phi(x_1)\partial_{U_2}\phi(x_2)\rangle = -\frac{d^2-1}{4}\frac{N(\delta V)^2}{|x_1 - x_2|^{d+3}} + \ldots$$

Taking $\delta V \to 0$, e.g. we take δV to be the smallest separation, then we find that

$$\lim_{\delta V \to 0} \frac{(\delta V)^2}{(-\delta\mathcal{U}\delta\mathcal{V} + \delta y^\alpha \delta y^\beta \delta_{\alpha\beta})^{(d+3)/2}} \neq 0$$

It's not zero since it does receive a contribution when $y^\alpha = 0$. To see this we can do an integral over all the transverse separations.

$$\int \frac{(\delta V)^2}{(\delta\mathcal{U}\delta\mathcal{V} + \delta y^\alpha \delta y^\beta \delta_{\alpha\beta})^{(d+3)/2}} d^{d-1}\delta y^\alpha$$

We'll also take the $|x_1 - x_2|$ is positive. Now with the substitution $\delta\tilde{y}^\alpha = \delta y^\alpha / \sqrt{-\delta\mathcal{U}\delta\mathcal{V}}$ we get

$$\frac{1}{(\delta V)^2}\int \frac{d\delta\tilde{y}^\alpha}{[1 + \delta\tilde{y}^\alpha \delta\tilde{y}^\alpha]^{(d-3)/2}}$$

What happens in the end is that all factors of δV cancel. In the notes Suvrat says that for $\delta y^\alpha \neq 0$ the integral vanishes. I think this might be due to symmetry, there's a ring of δy^α with appropriate sign that cancels out when we do the integral. But since $\delta y^\alpha = 0$ doesn't have this cancellation it remains finite. In the end we get

$$\lim_{\delta V \to 0} \langle \partial_{U_1} \phi(x_1) \partial_{U_2} \phi(x_2) \rangle = -\frac{1}{4\pi} \frac{\delta^{d-1}(\delta y^\alpha)}{(\mathcal{U}_1 - \mathcal{U}_2 - i\epsilon)^2}$$

Something to note: the reason we did an integral was to pick up the coefficient of the delta function. How do we sniff out the presence of a delta function. We use the property $\int \delta(x) dx = 1$ with the fact that the integral vanishes for $x \neq 0$. Thus if the integral we considered above gave a finite answer then we know the two point correlation function of the derivatives of the states would be proportional to a delta function.

The next step which will happen in the next lecture would to define modes as approximately

$$\int \partial_\mu \phi (-U)^{i\omega}$$

What is this doing? It's picking up the right moving modes with constant $\mathcal{V}$. (TODO huh?)