

IT-Security Cryptography and Secure Communications

Exercise: Public Key Cryptography

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1. Execute the Square-and-Multiply algorithm for $3^{17} \bmod 23$.

Solution

```
k = 0001 0001b

i = 4; f = 3 =>
i = 3; f = 9 =>
i = 2; f = 81 mod 23 = 12 =>
i = 1; f = 144 mod 23 = 6 =>
i = 0; f = ((6 * 6) mod 23) * 3 mod 23 = 16
```

2. Perform an encryption of a message using RSA.

I.e., choose 2 small prime numbers, compute e, d, n . Then encrypt the message (i.e., a (rather) small value) using the public key of a fellow student and send him the encrypted message. Let her/him decrypt your message. Afterwards validate that the encryption is successful.

Solution

Let's assume that $p = 7$ and $q = 11$.

$$n = p \cdot q = 77$$

$$\phi(n) = (p - 1)(q - 1) = 6 \cdot 10 = 60;$$

Hence the message has to be "less than" 60.

Compute e such that $\gcd(\phi(n), e) = 1$.

In this case, 2 to 6 are not possible because they all divide 60. We will select $e = 7$

Compute d ; i.e., $ed \bmod \phi(n) = 1$. $d = 43$; $(43 \times 7) \bmod \phi(60)$

Now: $PU = \{7, 77\}$, $PR = \{43, 77\}$.

Let the message M be "13": $C = 13^7 \bmod 77 = 62$.

To get the plaintext compute $P = 62^{43} \bmod 77$.

3. Can you think of a scenario in which fault-based attacks may be practical?

Solution

It is always practical when you have physical access to a device for a reasonable time to execute the attack. E.g., in IT-forensics.