

Table 1. Selected models for discrete random variables

Name Symbol	PMF $p(x)$	Parameter ranges	Mean, Standard deviation	Note
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, \dots, n$	$0 < p < 1,$ n a positive integer	$np,$ $\sqrt{np(1-p)}$	1
Geometric $Geo(p)$	$p(1-p)^{x-1}, \quad x = 1, 2, \dots$	$0 < p < 1$	$\frac{1}{p}, \frac{\sqrt{1-p}}{p}$	
Negative binomial $NB(k, p)$	$\binom{x-1}{k-1} p^k (1-p)^{x-k}, \quad x = k, k+1, \dots$	$0 < p < 1,$ k a positive integer	$\frac{k}{p}, \frac{\sqrt{k(1-p)}}{p}$	1
Hypergeometric $HG(N, n, k)$	$\frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}}, \quad \begin{cases} x \text{ integer} \\ x \leq k \\ n-x \leq N-k \end{cases}$	$0 < k, n \leq N$ k, n, N positive integers	$\frac{nk}{N},$ $\sqrt{\frac{nk(N-k)(N-n)}{N^2(N-1)}}$	1
Poisson $Psn(v)$	$\frac{(v)^x}{x!} \exp(-v), \quad x = 0, 1, \dots$	$0 < v$	v, \sqrt{v}	2

Notes:

1. $\binom{n}{x} = \frac{n!}{(n-x)!x!}$ is the *binomial factor*.
2. The *homogeneous Poisson process* is $Psn(\lambda t)$, where λ is the mean rate of occurrences and t is the time or space coordinate.

Table 2. Selected models for continuous random variables

Name symbol	PDF $f(x)$, CDF $F(x)$, bounds	Parameter ranges	Mean, Standard deviation	Note
Uniform U(a, b)	$\frac{1}{b-a}, \frac{x-a}{b-a}, a \leq x \leq b$	$a < b$	$\frac{a+b}{2}, \frac{b-a}{2\sqrt{3}}$	
Beta Bet(a, b, q, r)	$\frac{(x-a)^{q-1}(b-x)^{r-1}}{B(q,r)(b-a)^{q+r+1}}, a \leq x \leq b$ no closed form for CDF	$a < b, 0 < q, r$	$\frac{ar+bq}{q+r}, \frac{b-a}{q+r} \sqrt{\frac{qr}{q+r+1}}$	1,2
Exponential Exp(λ)	$\lambda \exp(-\lambda x), 1 - \exp(-\lambda x), 0 < x$	$0 < \lambda$	$\frac{1}{\lambda}, \frac{1}{\lambda}$	
Gamma Gam(k, λ)	$\frac{\lambda(\lambda x)^{k-1} \exp(-\lambda x)}{\Gamma(k)}, \frac{\Gamma(k, \lambda x)}{\Gamma(k)}, 0 < x$	$0 < \lambda, 0 < k$	$\frac{k}{\lambda}, \frac{\sqrt{k}}{\lambda}$	2,3,5
Chi-square $\chi^2(v)$	$\frac{(x/2)^{v/2-1} \exp(-x/2)}{2\Gamma(v/2)}, \frac{\Gamma(v/2, x/2)}{\Gamma(v/2)}, 0 < x$	$0 < v$	$2v, \sqrt{2v}$	5
Rayleigh Ray(u)	$\frac{2x}{u^2} \exp\left[-\left(\frac{x}{u}\right)^2\right], 1 - \exp\left[-\left(\frac{x}{u}\right)^2\right], 0 < x$	$0 < u$	$\frac{\sqrt{\pi}u}{2}, \frac{\sqrt{4-\pi}u}{2}$	6
Weibull Wbl(u, k)	$\frac{k}{u} \left(\frac{x}{u}\right)^{k-1} \exp\left[-\left(\frac{x}{u}\right)^k\right], 1 - \exp\left[-\left(\frac{x}{u}\right)^k\right], 0 < x$	$0 < u, 0 < k$	$u\Gamma(1-1/k), u\sqrt{\Gamma(1+2/k)-\Gamma^2(1+1/k)}$	6
Gumbel Gmb(u, α)	$\alpha \exp\{-\alpha(x-u) - \exp[-\alpha(x-u)]\}, \exp\{-\exp[-\alpha(x-u)]\}$	$0 < \alpha$	$u + \frac{\gamma}{\alpha}, \frac{\pi}{\sqrt{6}\alpha}$	8
Normal N(μ, σ)	$\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \Phi\left(\frac{x-\mu}{\sigma}\right)$	$0 < \sigma$	μ, σ	4
Lognormal LN(λ, ζ)	$\frac{1}{\sqrt{2\pi}\zeta x} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda}{\zeta}\right)^2\right], \Phi\left(\frac{\ln x - \lambda}{\zeta}\right), 0 < x$	$0 < \zeta$	$\exp(\lambda + 0.5\zeta^2), \exp(\lambda + 0.5\zeta^2)\sqrt{\exp(\zeta^2) - 1}$	4
Laplace Lap(α, β)	$\frac{\beta}{2} \exp(-\beta x-\alpha), \frac{1}{2} \exp(-\beta x-\alpha) \text{ for } x \leq \alpha, 1 - \frac{1}{2} \exp(-\beta x-\alpha) \text{ for } \alpha \leq x$	$0 < \beta$	$\alpha, \frac{\sqrt{2}}{\beta}$	
Pareto Par(k, u)	$\frac{k}{u} \left(\frac{u}{x}\right)^{k+1}, 1 - \left(\frac{u}{x}\right)^k, u \leq x$	$0 < u, 0 < k$	$\frac{ku}{k-1}, \sqrt{\frac{k}{k-2}} \frac{u}{k-1}$	7

Notes for Table 2:

1. $B(q, r) = \Gamma(q)\Gamma(r)/\Gamma(q+r)$ is the *beta* function. See Note 2.
2. $\Gamma(k) = \int_0^\infty u^{k-1} \exp(-u) du$ is the *gamma* function. See Table 3 for a tabulation of this function.
3. $\Gamma(k, x) = \int_0^x u^{k-1} \exp(-u) du$ is the *incomplete gamma function*. See Table 4 for a tabulation of the ratio $\Gamma(k, x)/\Gamma(k)$.
4. $\Phi(x) = (2\pi)^{-1/2} \int_{-\infty}^x \exp(-u^2/2) du$ is the *standard normal cumulative probability*. See Table 5 for a tabulation of this function.
5. $\chi^2(v)$ is identical to $\text{Gam}(v/2, 1/2)$.
6. $\text{Ray}(u)$ is identical to $\text{Wbl}(u, 2)$.
7. For the $\text{Par}(k, u)$, the mean exists only when $1 < k$ and the variance exists only when $2 < k$. Otherwise, these values are infinite.
8. $\gamma = 0.5772156649$ is Euler's constant.

Table 3. Gamma function $\Gamma(k) = \int_0^\infty u^{k-1} \exp(-u) du$

Note: $\Gamma(k+1) = k\Gamma(k)$, $\Gamma(k) = (k-1)!$ for integer k , and $\Gamma(0.5) = \sqrt{\pi}$

k	$\Gamma(k)$								
1.000	1.00000	1.200	0.91817	1.400	0.88726	1.600	0.89352	1.800	0.93138
1.005	0.99714	1.205	0.91686	1.405	0.88700	1.605	0.89409	1.805	0.93272
1.010	0.99433	1.210	0.91558	1.410	0.88676	1.610	0.89468	1.810	0.93408
1.015	0.99156	1.215	0.91433	1.415	0.88655	1.615	0.89529	1.815	0.93545
1.020	0.98884	1.220	0.91311	1.420	0.88636	1.620	0.89592	1.820	0.93685
1.025	0.98617	1.225	0.91192	1.425	0.88618	1.625	0.89657	1.825	0.93826
1.030	0.98355	1.230	0.91075	1.430	0.88604	1.630	0.89724	1.830	0.93969
1.035	0.98097	1.235	0.90962	1.435	0.88591	1.635	0.89793	1.835	0.94114
1.040	0.97844	1.240	0.90852	1.440	0.88581	1.640	0.89864	1.840	0.94261
1.045	0.97595	1.245	0.90745	1.445	0.88572	1.645	0.89937	1.845	0.94410
1.050	0.97350	1.250	0.90640	1.450	0.88566	1.650	0.90012	1.850	0.94561
1.055	0.97110	1.255	0.90539	1.455	0.88562	1.655	0.90088	1.855	0.94714
1.060	0.96874	1.260	0.90440	1.460	0.88560	1.660	0.90167	1.860	0.94869
1.065	0.96643	1.265	0.90344	1.465	0.88561	1.665	0.90247	1.865	0.95025
1.070	0.96415	1.270	0.90250	1.470	0.88563	1.670	0.90330	1.870	0.95184
1.075	0.96192	1.275	0.90160	1.475	0.88568	1.675	0.90414	1.875	0.95345
1.080	0.95973	1.280	0.90072	1.480	0.88575	1.680	0.90500	1.880	0.95507
1.085	0.95757	1.285	0.89987	1.485	0.88584	1.685	0.90588	1.885	0.95672
1.090	0.95546	1.290	0.89904	1.490	0.88595	1.690	0.90678	1.890	0.95838
1.095	0.95339	1.295	0.89824	1.495	0.88608	1.695	0.90770	1.895	0.96006
1.100	0.95135	1.300	0.89747	1.500	0.88623	1.700	0.90864	1.900	0.96177
1.105	0.94935	1.305	0.89672	1.505	0.88640	1.705	0.90960	1.905	0.96349
1.110	0.94740	1.310	0.89600	1.510	0.88659	1.710	0.91057	1.910	0.96523
1.115	0.94547	1.315	0.89531	1.515	0.88680	1.715	0.91157	1.915	0.96699
1.120	0.94359	1.320	0.89464	1.520	0.88704	1.720	0.91258	1.920	0.96877
1.125	0.94174	1.325	0.89400	1.525	0.88729	1.725	0.91361	1.925	0.97058
1.130	0.93993	1.330	0.89338	1.530	0.88757	1.730	0.91467	1.930	0.97240
1.135	0.93816	1.335	0.89278	1.535	0.88786	1.735	0.91574	1.935	0.97424
1.140	0.93642	1.340	0.89222	1.540	0.88818	1.740	0.91683	1.940	0.97610
1.145	0.93471	1.345	0.89167	1.545	0.88851	1.745	0.91793	1.945	0.97798
1.150	0.93304	1.350	0.89115	1.550	0.88887	1.750	0.91906	1.950	0.97988
1.155	0.93141	1.355	0.89066	1.555	0.88924	1.755	0.92021	1.955	0.98180
1.160	0.92980	1.360	0.89018	1.560	0.88964	1.760	0.92137	1.960	0.98374
1.165	0.92823	1.365	0.88974	1.565	0.89005	1.765	0.92256	1.965	0.98570
1.170	0.92670	1.370	0.88931	1.570	0.89049	1.770	0.92376	1.970	0.98768
1.175	0.92520	1.375	0.88891	1.575	0.89094	1.775	0.92499	1.975	0.98969
1.180	0.92373	1.380	0.88854	1.580	0.89142	1.780	0.92623	1.980	0.99171
1.185	0.92229	1.385	0.88818	1.585	0.89191	1.785	0.92749	1.985	0.99375
1.190	0.92089	1.390	0.88785	1.590	0.89243	1.790	0.92877	1.990	0.99581
1.195	0.91951	1.395	0.88755	1.595	0.89296	1.795	0.93007	1.995	0.99790

Table 4a. Gamma Cumulative Probability Function

Note: $\Gamma(k, \lambda x)/\Gamma(k)$ above double lines, $1 - \Gamma(k, \lambda x)/\Gamma(k)$ below double lines

λx	$k=1$	2	3	4	5	6	7	8	9	10
0.0001	1.00E-04	5.00E-09	1.67E-13	4.17E-18	8.33E-23	1.39E-27	1.98E-32	2.48E-37	2.76E-42	2.76E-47
0.0005	5.00E-04	1.25E-07	2.08E-11	2.60E-15	2.60E-19	2.17E-23	1.55E-27	9.68E-32	5.38E-36	2.69E-40
0.001	1.00E-03	5.00E-07	1.67E-10	4.16E-14	8.33E-18	1.39E-21	1.98E-25	2.48E-29	2.75E-33	2.75E-37
0.002	2.00E-03	2.00E-06	1.33E-09	6.66E-13	2.66E-16	8.87E-20	2.54E-23	6.34E-27	1.41E-30	2.82E-34
0.003	3.00E-03	4.49E-06	4.49E-09	3.37E-12	2.02E-15	1.01E-18	4.33E-22	1.62E-25	5.41E-29	1.62E-32
0.004	3.99E-03	7.98E-06	1.06E-08	1.06E-11	8.50E-15	5.67E-18	3.24E-21	1.62E-24	7.20E-28	2.88E-31
0.005	4.99E-03	1.25E-05	2.08E-08	2.59E-11	2.59E-14	2.16E-17	1.54E-20	9.65E-24	5.36E-27	2.68E-30
0.007	6.98E-03	2.44E-05	5.69E-08	9.95E-11	1.39E-13	1.62E-16	1.62E-19	1.42E-22	1.11E-25	7.73E-29
0.01	9.95E-03	4.97E-05	1.65E-07	4.13E-10	8.26E-13	1.38E-15	1.97E-18	2.46E-21	2.73E-24	2.73E-27
0.02	1.98E-02	1.97E-04	1.31E-06	6.56E-09	2.62E-11	8.74E-14	2.50E-16	6.24E-19	1.39E-21	2.77E-24
0.03	2.96E-02	4.41E-04	4.40E-06	3.30E-08	1.98E-10	9.87E-13	4.23E-15	1.58E-17	5.28E-20	1.58E-22
0.04	3.92E-02	7.79E-04	1.04E-05	1.03E-07	8.25E-10	5.50E-12	3.14E-14	1.57E-16	6.97E-19	2.79E-21
0.05	4.88E-02	1.21E-03	2.01E-05	2.50E-07	2.50E-09	2.08E-11	1.48E-13	9.27E-16	5.15E-18	2.57E-20
0.07	6.76E-02	2.34E-03	5.42E-05	9.46E-07	1.32E-08	1.54E-10	1.54E-12	1.34E-14	1.04E-16	7.30E-19
0.1	9.52E-02	4.68E-03	1.55E-04	3.85E-06	7.67E-08	1.27E-09	1.82E-11	2.27E-13	2.52E-15	2.52E-17
0.2	1.81E-01	1.75E-02	1.15E-03	5.68E-05	2.26E-06	7.49E-08	2.13E-09	5.32E-11	1.18E-12	2.35E-14
0.3	2.59E-01	3.69E-02	3.60E-03	2.66E-04	1.58E-05	7.83E-07	3.34E-08	1.25E-09	4.14E-11	1.24E-12
0.4	3.30E-01	6.16E-02	7.93E-03	7.76E-04	6.12E-05	4.04E-06	2.29E-07	1.14E-08	5.04E-10	2.01E-11
0.5	3.93E-01	9.02E-02	1.44E-02	1.75E-03	1.72E-04	1.42E-05	1.00E-06	6.22E-08	3.44E-09	1.71E-10
0.6	4.51E-01	1.22E-01	2.31E-02	3.36E-03	3.94E-04	3.89E-05	3.29E-06	2.45E-07	1.62E-08	9.67E-10
0.7	5.03E-01	1.56E-01	3.41E-02	5.75E-03	7.86E-04	9.00E-05	8.88E-06	7.69E-07	5.93E-08	4.13E-09
0.8	5.51E-01	1.91E-01	4.74E-02	9.08E-03	1.41E-03	1.84E-04	2.07E-05	2.05E-06	1.81E-07	1.43E-08
0.9	5.93E-01	2.28E-01	6.29E-02	1.35E-02	2.34E-03	3.43E-04	4.34E-05	4.82E-06	4.77E-07	4.25E-08
1	6.32E-01	2.64E-01	8.03E-02	1.90E-02	3.66E-03	5.94E-04	8.32E-05	1.02E-05	1.13E-06	1.11E-07
2	8.65E-01	5.94E-01	3.23E-01	1.43E-01	5.27E-02	1.66E-02	4.53E-03	1.10E-03	2.37E-04	4.65E-05
3	9.50E-01	8.01E-01	5.77E-01	3.53E-01	1.85E-01	8.39E-02	3.35E-02	1.19E-02	3.80E-03	1.10E-03
4	9.82E-01	9.08E-01	7.62E-01	5.67E-01	3.71E-01	2.15E-01	1.11E-01	5.11E-02	2.14E-02	8.13E-03
5	6.74E-03	9.60E-01	8.75E-01	7.35E-01	5.60E-01	3.84E-01	2.38E-01	1.33E-01	6.81E-02	3.18E-02
6	2.48E-03	9.83E-01	9.38E-01	8.49E-01	7.15E-01	5.54E-01	3.94E-01	2.56E-01	1.53E-01	8.39E-02
7	9.12E-04	7.30E-03	9.70E-01	9.18E-01	8.27E-01	6.99E-01	5.50E-01	4.01E-01	2.71E-01	1.70E-01
8	3.35E-04	3.02E-03	9.86E-01	9.58E-01	9.00E-01	8.09E-01	6.87E-01	5.47E-01	4.07E-01	2.83E-01
9	1.23E-04	1.23E-03	6.23E-03	9.79E-01	9.45E-01	8.84E-01	7.93E-01	6.76E-01	5.44E-01	4.13E-01
10	4.54E-05	4.99E-04	2.77E-03	9.90E-01	9.71E-01	9.33E-01	8.70E-01	7.80E-01	6.67E-01	5.42E-01
11	1.67E-05	2.00E-04	1.21E-03	4.92E-03	9.85E-01	9.62E-01	9.21E-01	8.57E-01	7.68E-01	6.59E-01
12	6.14E-06	7.99E-05	5.22E-04	2.29E-03	7.60E-03	9.80E-01	9.54E-01	9.10E-01	8.45E-01	7.58E-01
13	2.26E-06	3.16E-05	2.23E-04	1.05E-03	3.74E-03	9.89E-01	9.74E-01	9.46E-01	9.00E-01	8.34E-01
14	8.32E-07	1.25E-05	9.40E-05	4.74E-04	1.81E-03	5.53E-03	9.86E-01	9.68E-01	9.38E-01	8.91E-01
15	3.06E-07	4.89E-06	3.93E-05	2.11E-04	8.57E-04	2.79E-03	7.63E-03	9.82E-01	9.63E-01	9.30E-01
16	1.13E-07	1.91E-06	1.63E-05	9.31E-05	4.00E-04	1.38E-03	4.01E-03	1.00E-02	9.78E-01	9.57E-01
17	4.14E-08	7.45E-07	6.73E-06	4.06E-05	1.85E-04	6.75E-04	2.06E-03	5.43E-03	9.87E-01	9.74E-01
18	1.52E-08	2.89E-07	2.76E-06	1.76E-05	8.42E-05	3.24E-04	1.04E-03	2.89E-03	7.06E-03	9.85E-01
19	5.60E-09	1.12E-07	1.12E-06	7.53E-06	3.80E-05	1.54E-04	5.20E-04	1.51E-03	3.87E-03	8.86E-03
20	2.06E-09	4.33E-08	4.56E-07	3.20E-06	1.69E-05	7.19E-05	2.55E-04	7.79E-04	2.09E-03	5.00E-03

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Table 4b. Gamma Cumulative Probability Function

Note: $\Gamma(k, \lambda x)/\Gamma(k)$ above double lines, $1 - \Gamma(k, \lambda x)/\Gamma(k)$ below double lines

λx	$k = 11$	12	13	14	15	16	17	18	19	20
1	1.00E-08	8.32E-10	6.36E-11	4.52E-12	3.00E-13	1.87E-14	1.09E-15	6.06E-17	3.18E-18	1.59E-19
2	8.31E-06	1.36E-06	2.07E-07	2.93E-08	3.87E-09	4.80E-10	5.61E-11	6.19E-12	6.48E-13	6.44E-14
3	2.92E-04	7.14E-05	1.61E-05	3.40E-06	6.70E-07	1.24E-07	2.16E-08	3.57E-09	5.59E-10	8.31E-11
4	2.84E-03	9.15E-04	2.74E-04	7.63E-05	1.99E-05	4.89E-06	1.13E-06	2.48E-07	5.16E-08	1.02E-08
5	1.37E-02	5.45E-03	2.02E-03	6.98E-04	2.26E-04	6.90E-05	1.99E-05	5.42E-06	1.40E-06	3.45E-07
6	4.26E-02	2.01E-02	8.83E-03	3.63E-03	1.40E-03	5.09E-04	1.75E-04	5.69E-05	1.76E-05	5.18E-06
7	9.85E-02	5.33E-02	2.70E-02	1.28E-02	5.72E-03	2.41E-03	9.58E-04	3.62E-04	1.30E-04	4.44E-05
8	1.84E-01	1.12E-01	6.38E-02	3.42E-02	1.73E-02	8.23E-03	3.72E-03	1.59E-03	6.50E-04	2.53E-04
9	2.94E-01	1.97E-01	1.24E-01	7.39E-02	4.15E-02	2.20E-02	1.11E-02	5.32E-03	2.43E-03	1.06E-03
10	4.17E-01	3.03E-01	2.08E-01	1.36E-01	8.35E-02	4.87E-02	2.70E-02	1.43E-02	7.19E-03	3.45E-03
11	5.40E-01	4.21E-01	3.11E-01	2.19E-01	1.46E-01	9.26E-02	5.59E-02	3.22E-02	1.77E-02	9.29E-03
12	6.53E-01	5.38E-01	4.24E-01	3.18E-01	2.28E-01	1.56E-01	1.01E-01	6.30E-02	3.74E-02	2.13E-02
13	7.48E-01	6.47E-01	5.37E-01	4.27E-01	3.25E-01	2.36E-01	1.65E-01	1.10E-01	6.98E-02	4.27E-02
14	8.24E-01	7.40E-01	6.42E-01	5.36E-01	4.30E-01	3.31E-01	2.44E-01	1.73E-01	1.17E-01	7.65E-02
15	8.82E-01	8.15E-01	7.32E-01	6.37E-01	5.34E-01	4.32E-01	3.36E-01	2.51E-01	1.81E-01	1.25E-01
16	9.23E-01	8.73E-01	8.07E-01	7.25E-01	6.32E-01	5.33E-01	4.34E-01	3.41E-01	2.58E-01	1.88E-01
17	9.51E-01	9.15E-01	8.65E-01	7.99E-01	7.19E-01	6.29E-01	5.32E-01	4.36E-01	3.45E-01	2.64E-01
18	9.70E-01	9.45E-01	9.08E-01	8.57E-01	7.92E-01	7.13E-01	6.25E-01	5.31E-01	4.38E-01	3.49E-01
19	9.82E-01	9.65E-01	9.39E-01	9.02E-01	8.50E-01	7.85E-01	7.08E-01	6.22E-01	5.31E-01	4.39E-01
20	9.89E-01	9.79E-01	9.61E-01	9.34E-01	8.95E-01	8.43E-01	7.79E-01	7.03E-01	6.19E-01	5.30E-01
21	6.25E-03	9.87E-01	9.75E-01	9.57E-01	9.28E-01	8.89E-01	8.37E-01	7.73E-01	6.98E-01	6.16E-01
22	3.55E-03	7.63E-03	9.85E-01	9.72E-01	9.52E-01	9.23E-01	8.83E-01	8.31E-01	7.68E-01	6.94E-01
23	1.98E-03	4.43E-03	9.12E-03	9.83E-01	9.69E-01	9.48E-01	9.18E-01	8.77E-01	8.25E-01	7.62E-01
24	1.08E-03	2.52E-03	5.40E-03	9.89E-01	9.80E-01	9.66E-01	9.44E-01	9.13E-01	8.72E-01	8.20E-01
25	5.86E-04	1.42E-03	3.14E-03	6.47E-03	9.88E-01	9.78E-01	9.62E-01	9.40E-01	9.08E-01	8.66E-01
26	3.13E-04	7.82E-04	1.80E-03	3.84E-03	7.62E-03	9.86E-01	9.75E-01	9.59E-01	9.35E-01	9.03E-01
27	1.64E-04	4.26E-04	1.02E-03	2.24E-03	4.60E-03	8.84E-03	9.84E-01	9.73E-01	9.56E-01	9.31E-01
28	8.55E-05	2.29E-04	5.64E-04	1.29E-03	2.73E-03	5.43E-03	1.01E-02	9.82E-01	9.70E-01	9.52E-01
29	4.39E-05	1.22E-04	3.10E-04	7.29E-04	1.60E-03	3.28E-03	6.32E-03	9.88E-01	9.80E-01	9.67E-01
30	2.23E-05	6.39E-05	1.68E-04	4.07E-04	9.21E-04	1.95E-03	3.87E-03	7.27E-03	9.87E-01	9.78E-01
31	1.13E-05	3.32E-05	8.98E-05	2.25E-04	5.24E-04	1.14E-03	2.34E-03	4.52E-03	8.28E-03	9.86E-01
32	5.61E-06	1.70E-05	4.75E-05	1.23E-04	2.94E-04	6.60E-04	1.39E-03	2.77E-03	5.22E-03	9.34E-03
33	2.77E-06	8.67E-06	2.49E-05	6.61E-05	1.63E-04	3.77E-04	8.17E-04	1.67E-03	3.24E-03	5.96E-03
34	1.36E-06	4.37E-06	1.29E-05	3.52E-05	8.95E-05	2.12E-04	4.74E-04	9.96E-04	1.98E-03	3.75E-03
35	6.61E-07	2.19E-06	6.63E-06	1.86E-05	4.85E-05	1.18E-04	2.71E-04	5.86E-04	1.20E-03	2.32E-03
36	3.19E-07	1.08E-06	3.38E-06	9.73E-06	2.61E-05	6.53E-05	1.54E-04	3.40E-04	7.14E-04	1.42E-03
37	1.53E-07	5.33E-07	1.71E-06	5.04E-06	1.39E-05	3.56E-05	8.59E-05	1.95E-04	4.21E-04	8.59E-04
38	7.29E-08	2.60E-07	8.55E-07	2.59E-06	7.31E-06	1.92E-05	4.76E-05	1.11E-04	2.45E-04	5.13E-04
39	3.45E-08	1.26E-07	4.25E-07	1.32E-06	3.81E-06	1.03E-05	2.61E-05	6.24E-05	1.41E-04	3.02E-04
40	1.62E-08	6.08E-08	2.10E-07	6.67E-07	1.98E-06	5.46E-06	1.42E-05	3.47E-05	8.03E-05	1.76E-04
41	7.57E-09	2.91E-08	1.03E-07	3.35E-07	1.01E-06	2.87E-06	7.64E-06	1.91E-05	4.53E-05	1.02E-04
42	3.52E-09	1.39E-08	5.00E-08	1.67E-07	5.17E-07	1.50E-06	4.08E-06	1.04E-05	2.53E-05	5.81E-05
43	1.63E-09	6.55E-09	2.42E-08	8.26E-08	2.62E-07	7.76E-07	2.16E-06	5.65E-06	1.40E-05	3.29E-05

Table 4c. Gamma Cumulative Probability Function

Note: $\Gamma(k, \lambda x)/\Gamma(k)$ above double lines, $1 - \Gamma(k, \lambda x)/\Gamma(k)$ below double lines

λx	$k = 21$	22	23	24	25	26	27	28	29	30
10	1.59E-03	7.00E-04	2.96E-04	1.20E-04	4.69E-05	1.77E-05	6.42E-06	2.25E-06	7.64E-07	2.51E-07
11	4.67E-03	2.25E-03	1.04E-03	4.64E-04	1.99E-04	8.21E-05	3.27E-05	1.26E-05	4.68E-06	1.69E-06
12	1.16E-02	6.07E-03	3.05E-03	1.47E-03	6.86E-04	3.08E-04	1.33E-04	5.58E-05	2.26E-05	8.87E-06
13	2.50E-02	1.41E-02	7.62E-03	3.97E-03	1.99E-03	9.66E-04	4.52E-04	2.04E-04	8.94E-05	3.79E-05
14	4.79E-02	2.88E-02	1.67E-02	9.33E-03	5.02E-03	2.61E-03	1.31E-03	6.35E-04	2.98E-04	1.36E-04
15	8.30E-02	5.31E-02	3.27E-02	1.95E-02	1.12E-02	6.18E-03	3.31E-03	1.72E-03	8.61E-04	4.18E-04
16	1.32E-01	8.92E-02	5.82E-02	3.67E-02	2.23E-02	1.31E-02	7.46E-03	4.11E-03	2.19E-03	1.13E-03
17	1.95E-01	1.39E-01	9.53E-02	6.33E-02	4.06E-02	2.52E-02	1.52E-02	8.83E-03	4.98E-03	2.73E-03
18	2.69E-01	2.01E-01	1.45E-01	1.01E-01	6.83E-02	4.46E-02	2.82E-02	1.73E-02	1.03E-02	5.94E-03
19	3.53E-01	2.75E-01	2.07E-01	1.51E-01	1.07E-01	7.31E-02	4.86E-02	3.13E-02	1.95E-02	1.18E-02
20	4.41E-01	3.56E-01	2.79E-01	2.13E-01	1.57E-01	1.12E-01	7.79E-02	5.25E-02	3.43E-02	2.18E-02
21	5.29E-01	4.42E-01	3.60E-01	2.84E-01	2.18E-01	1.62E-01	1.17E-01	8.25E-02	5.64E-02	3.74E-02
22	6.13E-01	5.28E-01	4.44E-01	3.63E-01	2.88E-01	2.23E-01	1.68E-01	1.23E-01	8.71E-02	6.02E-02
23	6.90E-01	6.11E-01	5.28E-01	4.45E-01	3.65E-01	2.92E-01	2.28E-01	1.73E-01	1.27E-01	9.15E-02
24	7.57E-01	6.86E-01	6.08E-01	5.27E-01	4.46E-01	3.68E-01	2.96E-01	2.32E-01	1.77E-01	1.32E-01
25	8.15E-01	7.53E-01	6.82E-01	6.06E-01	5.27E-01	4.47E-01	3.71E-01	3.00E-01	2.37E-01	1.82E-01
26	8.61E-01	8.10E-01	7.48E-01	6.79E-01	6.04E-01	5.26E-01	4.48E-01	3.73E-01	3.03E-01	2.41E-01
27	8.99E-01	8.56E-01	8.05E-01	7.44E-01	6.76E-01	6.02E-01	5.26E-01	4.49E-01	3.75E-01	3.07E-01
28	9.27E-01	8.94E-01	8.52E-01	8.00E-01	7.40E-01	6.73E-01	6.00E-01	5.25E-01	4.50E-01	3.77E-01
29	9.49E-01	9.23E-01	8.90E-01	8.47E-01	7.96E-01	7.36E-01	6.70E-01	5.99E-01	5.25E-01	4.51E-01
30	9.65E-01	9.46E-01	9.19E-01	8.85E-01	8.43E-01	7.92E-01	7.33E-01	6.67E-01	5.97E-01	5.24E-01
31	9.76E-01	9.62E-01	9.42E-01	9.16E-01	8.81E-01	8.39E-01	7.88E-01	7.29E-01	6.65E-01	5.95E-01
32	9.84E-01	9.74E-01	9.59E-01	9.39E-01	9.12E-01	8.77E-01	8.34E-01	7.84E-01	7.26E-01	6.62E-01
33	9.90E-01	9.82E-01	9.72E-01	9.57E-01	9.36E-01	9.08E-01	8.73E-01	8.30E-01	7.80E-01	7.23E-01
34	6.75E-03	9.88E-01	9.81E-01	9.70E-01	9.54E-01	9.33E-01	9.05E-01	8.69E-01	8.27E-01	7.76E-01
35	4.30E-03	7.58E-03	9.87E-01	9.79E-01	9.68E-01	9.51E-01	9.30E-01	9.01E-01	8.66E-01	8.23E-01
36	2.70E-03	4.88E-03	8.46E-03	9.86E-01	9.78E-01	9.65E-01	9.49E-01	9.26E-01	8.98E-01	8.62E-01
37	1.67E-03	3.10E-03	5.50E-03	9.37E-03	9.85E-01	9.76E-01	9.63E-01	9.46E-01	9.23E-01	8.94E-01
38	1.02E-03	1.94E-03	3.53E-03	6.16E-03	9.90E-01	9.83E-01	9.74E-01	9.61E-01	9.43E-01	9.20E-01
39	6.17E-04	1.20E-03	2.24E-03	3.99E-03	6.85E-03	9.89E-01	9.82E-01	9.72E-01	9.59E-01	9.41E-01
40	3.68E-04	7.34E-04	1.40E-03	2.56E-03	4.48E-03	7.57E-03	9.88E-01	9.81E-01	9.71E-01	9.57E-01
41	2.17E-04	4.43E-04	8.65E-04	1.62E-03	2.90E-03	5.00E-03	8.32E-03	9.87E-01	9.79E-01	9.69E-01
42	1.27E-04	2.65E-04	5.28E-04	1.01E-03	1.85E-03	3.26E-03	5.55E-03	9.10E-03	9.86E-01	9.78E-01
43	7.35E-05	1.57E-04	3.19E-04	6.23E-04	1.17E-03	2.10E-03	3.65E-03	6.12E-03	9.91E-03	9.84E-01
44	4.21E-05	9.17E-05	1.91E-04	3.80E-04	7.28E-04	1.34E-03	2.38E-03	4.06E-03	6.72E-03	9.89E-01
45	2.39E-05	5.31E-05	1.13E-04	2.30E-04	4.49E-04	8.44E-04	1.53E-03	2.67E-03	4.50E-03	7.34E-03
46	1.34E-05	3.05E-05	6.62E-05	1.38E-04	2.74E-04	5.26E-04	9.71E-04	1.73E-03	2.98E-03	4.95E-03
47	7.48E-06	1.73E-05	3.84E-05	8.15E-05	1.66E-04	3.24E-04	6.11E-04	1.11E-03	1.95E-03	3.30E-03
48	4.14E-06	9.78E-06	2.21E-05	4.78E-05	9.92E-05	1.98E-04	3.80E-04	7.04E-04	1.26E-03	2.18E-03
49	2.27E-06	5.47E-06	1.26E-05	2.78E-05	5.88E-05	1.20E-04	2.34E-04	4.42E-04	8.06E-04	1.42E-03
50	1.24E-06	3.04E-06	7.13E-06	1.60E-05	3.45E-05	7.16E-05	1.43E-04	2.75E-04	5.11E-04	9.17E-04
51	6.67E-07	1.67E-06	4.00E-06	9.16E-06	2.01E-05	4.25E-05	8.64E-05	1.69E-04	3.20E-04	5.86E-04
52	3.58E-07	9.13E-07	2.23E-06	5.19E-06	1.16E-05	2.50E-05	5.17E-05	1.03E-04	1.99E-04	3.70E-04

Table 5. Standard normal cumulative probability $\Phi(x) = (2\pi)^{-1/2} \int_{-\infty}^x \exp(-u^2/2) du$

Note: $\Phi(-x) = 1 - \Phi(x)$

x	0.00	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	-0.08	-0.09
-4.9	4.80E-07	4.56E-07	4.33E-07	4.12E-07	3.91E-07	3.72E-07	3.53E-07	3.35E-07	3.18E-07	3.02E-07
-4.8	7.94E-07	7.56E-07	7.19E-07	6.84E-07	6.50E-07	6.18E-07	5.88E-07	5.59E-07	5.31E-07	5.05E-07
-4.7	1.30E-06	1.24E-06	1.18E-06	1.12E-06	1.07E-06	1.02E-06	9.69E-07	9.22E-07	8.78E-07	8.35E-07
-4.6	2.11E-06	2.02E-06	1.92E-06	1.83E-06	1.74E-06	1.66E-06	1.58E-06	1.51E-06	1.44E-06	1.37E-06
-4.5	3.40E-06	3.24E-06	3.09E-06	2.95E-06	2.82E-06	2.68E-06	2.56E-06	2.44E-06	2.33E-06	2.22E-06
-4.4	5.42E-06	5.17E-06	4.94E-06	4.72E-06	4.50E-06	4.30E-06	4.10E-06	3.91E-06	3.74E-06	3.56E-06
-4.3	8.55E-06	8.17E-06	7.81E-06	7.46E-06	7.13E-06	6.81E-06	6.51E-06	6.22E-06	5.94E-06	5.67E-06
-4.2	1.34E-05	1.28E-05	1.22E-05	1.17E-05	1.12E-05	1.07E-05	1.02E-05	9.78E-06	9.35E-06	8.94E-06
-4.1	2.07E-05	1.98E-05	1.90E-05	1.81E-05	1.74E-05	1.66E-05	1.59E-05	1.52E-05	1.46E-05	1.40E-05
-4.0	3.17E-05	3.04E-05	2.91E-05	2.79E-05	2.67E-05	2.56E-05	2.45E-05	2.35E-05	2.25E-05	2.16E-05
-3.9	4.81E-05	4.62E-05	4.43E-05	4.25E-05	4.08E-05	3.91E-05	3.75E-05	3.60E-05	3.45E-05	3.31E-05
-3.8	7.24E-05	6.95E-05	6.67E-05	6.41E-05	6.15E-05	5.91E-05	5.67E-05	5.44E-05	5.22E-05	5.01E-05
-3.7	1.08E-04	1.04E-04	9.96E-05	9.58E-05	9.20E-05	8.84E-05	8.50E-05	8.16E-05	7.84E-05	7.53E-05
-3.6	1.59E-04	1.53E-04	1.47E-04	1.42E-04	1.36E-04	1.31E-04	1.26E-04	1.21E-04	1.17E-04	1.12E-04
-3.5	2.33E-04	2.24E-04	2.16E-04	2.08E-04	2.00E-04	1.93E-04	1.85E-04	1.79E-04	1.72E-04	1.65E-04
-3.4	3.37E-04	3.25E-04	3.13E-04	3.02E-04	2.91E-04	2.80E-04	2.70E-04	2.60E-04	2.51E-04	2.42E-04
-3.3	4.83E-04	4.67E-04	4.50E-04	4.34E-04	4.19E-04	4.04E-04	3.90E-04	3.76E-04	3.62E-04	3.50E-04
-3.2	6.87E-04	6.64E-04	6.41E-04	6.19E-04	5.98E-04	5.77E-04	5.57E-04	5.38E-04	5.19E-04	5.01E-04
-3.1	9.68E-04	9.36E-04	9.04E-04	8.74E-04	8.45E-04	8.16E-04	7.89E-04	7.62E-04	7.36E-04	7.11E-04
-3.0	1.35E-03	1.31E-03	1.26E-03	1.22E-03	1.18E-03	1.14E-03	1.11E-03	1.07E-03	1.04E-03	1.00E-03
-2.9	1.87E-03	1.81E-03	1.75E-03	1.69E-03	1.64E-03	1.59E-03	1.54E-03	1.49E-03	1.44E-03	1.39E-03
-2.8	2.56E-03	2.48E-03	2.40E-03	2.33E-03	2.26E-03	2.19E-03	2.12E-03	2.05E-03	1.99E-03	1.93E-03
-2.7	3.47E-03	3.36E-03	3.26E-03	3.17E-03	3.07E-03	2.98E-03	2.89E-03	2.80E-03	2.72E-03	2.64E-03
-2.6	4.66E-03	4.53E-03	4.40E-03	4.27E-03	4.15E-03	4.02E-03	3.91E-03	3.79E-03	3.68E-03	3.57E-03
-2.5	6.21E-03	6.04E-03	5.87E-03	5.70E-03	5.54E-03	5.39E-03	5.23E-03	5.08E-03	4.94E-03	4.80E-03
-2.4	8.20E-03	7.98E-03	7.76E-03	7.55E-03	7.34E-03	7.14E-03	6.95E-03	6.76E-03	6.57E-03	6.39E-03
-2.3	1.07E-02	1.04E-02	1.02E-02	9.90E-03	9.64E-03	9.39E-03	9.14E-03	8.89E-03	8.66E-03	8.42E-03
-2.2	1.39E-02	1.36E-02	1.32E-02	1.29E-02	1.25E-02	1.22E-02	1.19E-02	1.16E-02	1.13E-02	1.10E-02
-2.1	1.79E-02	1.74E-02	1.70E-02	1.66E-02	1.62E-02	1.58E-02	1.54E-02	1.50E-02	1.46E-02	1.43E-02
-2.0	2.28E-02	2.22E-02	2.17E-02	2.12E-02	2.07E-02	2.02E-02	1.97E-02	1.92E-02	1.88E-02	1.83E-02
-1.9	2.87E-02	2.81E-02	2.74E-02	2.68E-02	2.62E-02	2.56E-02	2.50E-02	2.44E-02	2.39E-02	2.33E-02
-1.8	3.59E-02	3.51E-02	3.44E-02	3.36E-02	3.29E-02	3.22E-02	3.14E-02	3.07E-02	3.01E-02	2.94E-02
-1.7	4.46E-02	4.36E-02	4.27E-02	4.18E-02	4.09E-02	4.01E-02	3.92E-02	3.84E-02	3.75E-02	3.67E-02
-1.6	5.48E-02	5.37E-02	5.26E-02	5.16E-02	5.05E-02	4.95E-02	4.85E-02	4.75E-02	4.65E-02	4.55E-02
-1.5	6.68E-02	6.55E-02	6.43E-02	6.30E-02	6.18E-02	6.06E-02	5.94E-02	5.82E-02	5.71E-02	5.59E-02
-1.4	8.08E-02	7.93E-02	7.78E-02	7.64E-02	7.49E-02	7.35E-02	7.21E-02	7.08E-02	6.94E-02	6.81E-02
-1.3	9.68E-02	9.51E-02	9.34E-02	9.18E-02	9.01E-02	8.85E-02	8.69E-02	8.53E-02	8.38E-02	8.23E-02
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414