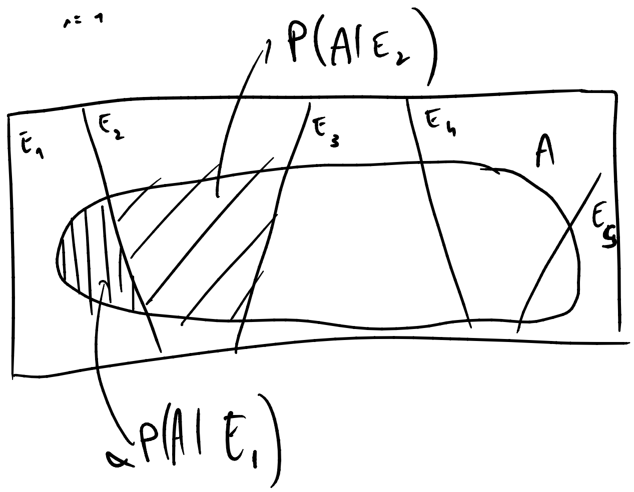
Mentor Session 4 (PA FB 2208)

Wednesday, August 10, 2022 10:52 AM

TOTAL PROBABILITY RULE

IF E_i, $i = 1, \dots, n$ ARE MUTUALLY EXCLUSIVE AND COLLECTIVELY EXHAUSTIVE EVENTS, THEN

$$P(A) = \sum_{i=1}^{n} P(A \mid E_i)\, P(E_i)$$

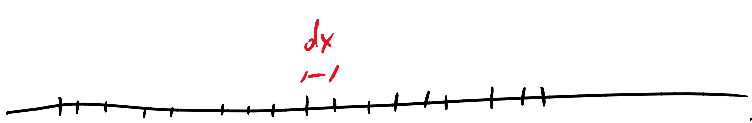


TOTAL PROBABILITY OF AN EVENT OVER A RANDOM VARIABLE

FOR DISCRETE R.V. HAVING PMF $p(x)$

$$P(E) = \sum_{\text{ALL } x} \underbrace{P(E \mid x)}_{P(E \mid X = x)}\, p(x)$$

FOR A CONTINUOUS r.v., WITH PDF $f(x)$



DISCRETIZATION OF SAMPLE SPACE

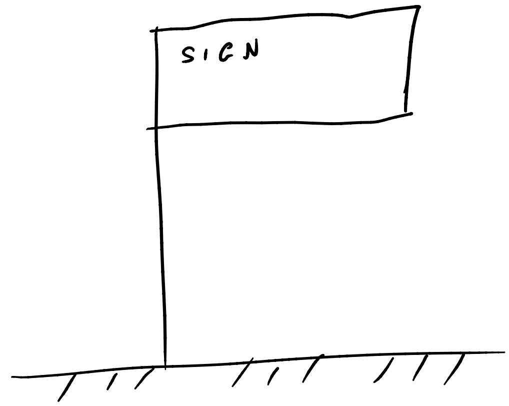
$$P(E) = \sum_{\text{ALL INCREMENTS}} \underbrace{P(E \mid x < X < x+dx)}_{P(E \mid x)}\, \underbrace{P(x < X < x+dx)}_{f(x)\,dx}$$

$$P(E) = \int P(E|x) f(x)\, dx$$

EXAMPLE

WE HAVE A SIGN POST

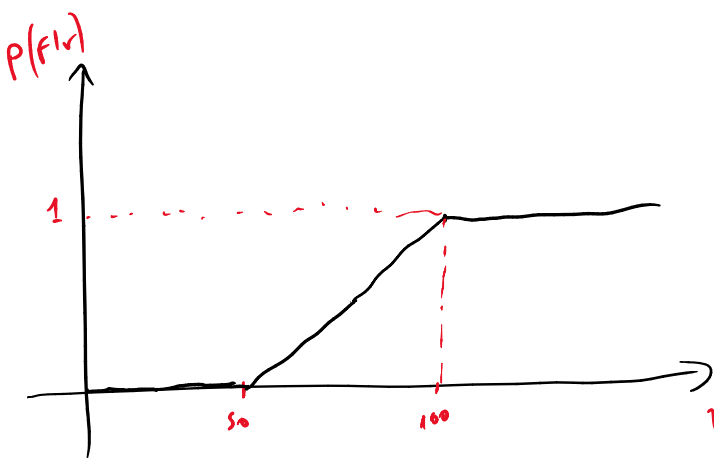
WE WANT THE ANNUAL PROBABILITY OF FAILURE DUE TO WIND SPEED v



- SIGNPOST CAPACITY
- WIND FORCE

IT HAS TO DO WITH THE FRAGILITY UNCERTAINTY IN FAILURE GIVEN A CERTAIN WIND VELOCITY

$$\begin{cases} P(\text{FAILURE}|v) = 0 & v < 50 \text{ Km/h} \\ P(\text{FAILURE}|v) = \dfrac{v-50}{50} & 50 < v < 100 \\ P(\text{FAILURE}|v) = 1 & v > 100 \text{ Km/h} \end{cases}$$



LET'S ASSUME THAT THE WIND SPEED FOLLOWS A TYPE II DISTRIBUTION

CDF $F(v) = e^{-\left(\frac{u}{v}\right)^k}$ $v>0$ u, k ARE PARAMETERS

CDF $\quad F(v) = e^{-\left(\frac{u}{v}\right)} \quad v > 0$ $\quad$ u, k ARE PARAMETERS

PDF $\quad f(v) = \frac{k}{u}\left(\frac{u}{v}\right)^{k+1} e^{-\left(\frac{u}{v}\right)^{k}}$

MEAN = 40 Km/h $\qquad$ c.o.v. $= 0.4 = \delta$

WE NEED TO GET u AND k BASED ON THIS INFO

$$\begin{cases} \mu = u\,\Gamma\left(1 - \frac{1}{k}\right) = 40 \\ \delta = \dfrac{\sqrt{\Gamma\left(1 - \frac{2}{k}\right) - \Gamma^2\left(1 - \frac{1}{k}\right)}}{\Gamma\left(1 - \frac{1}{k}\right)} = 0.4 \end{cases} \xRightarrow[\text{WITH A SOFTWARE}]{} \begin{aligned} u &= 34.4 \\ k &= 4.18 \end{aligned} \Rightarrow f(v)$$

WITH SOFTWARE

$$P(\text{FAILURE}) = \int P(\text{FAILURE} \mid v)\, f(v)\, dv$$

$$= \cancel{\int_0^{50} 0 \cdot f(v)\, dv} + \int_{50}^{100} \frac{v - 50}{50} f(v)\, dv + \int_{100}^{\infty} 1 \cdot f(v)\, dv = \boxed{0.0413}$$

CONDITIONAL DISTRIBUTION

FOR DISCRETE

$$p(x \mid E) = P(X = x \mid E) = \frac{P(X = x \cap E)}{P(E)}$$

FOR CONTINUOUS

$$f(x \mid E): \qquad f(x \mid E)\, dx = P(x < X < x + dx \mid E)$$

$$= \frac{P(x < X < x + dx \cap E)}{P(E)}$$

$$= \frac{P(X = x \cap E)}{P(E)}$$

BAYES' RULE

DISCRETE

$$\underbrace{p(x|E)}_{\text{POSTERIOR DISTRIBUTION}} = \frac{P(X = x \cap E)}{P(E)} = \frac{\overbrace{P(E|x)}^{\text{LIKELIHOOD}}}{P(E)} \underbrace{p(x)}_{\text{PRIOR}}$$

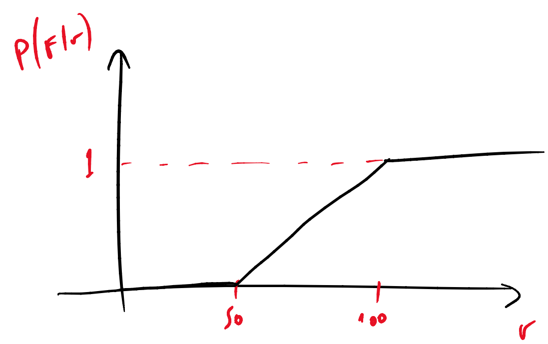
CONTINUOUS

$$\boxed{f(x|E) = \frac{P(E|x)}{P(E)} f(x)}$$

$$f(x|E)\,\cancel{dx} = P(x < X < x + dx \mid E) = \frac{P(E|x)}{P(E)} f(x)\,\cancel{dx}$$

SIGN POST EXAMPLE

$$P(\text{FAILURE}|v) = \begin{cases} 0 & v < 50 \\ \frac{v-50}{50} & 50 < v < 100 \\ 1 & v > 100 \end{cases}$$



$$f(r) = \frac{k}{u}\left(\frac{u}{r}\right)^{k+1} e^{-\left(\frac{u}{r}\right)^{k}} \qquad r > 0$$

$k = 4.18$

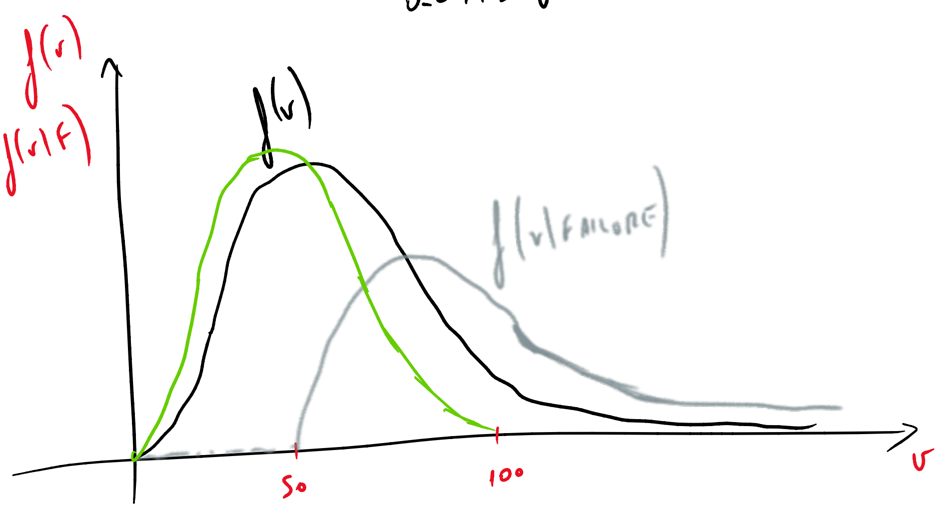
$u = 34.4$

$$P(\text{FAILURE}) = 0.0413$$

$$f(r|\text{FAILURE}) = \frac{1}{\underbrace{0.0413}_{P(E)}} \cdot P(\text{FAILURE}|r) \cdot f(r)$$

$P(F)$

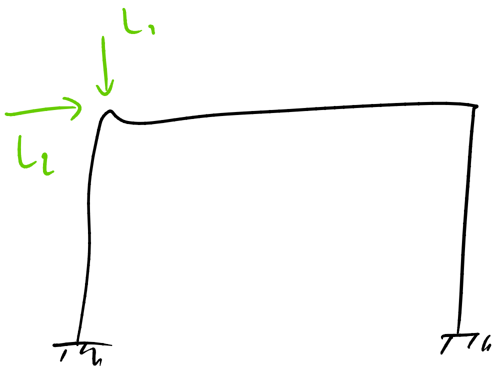
$$
= \begin{cases} 0 & v < 50 \\ \dfrac{1}{0.0413} \cdot \dfrac{v-50}{50} \cdot f(v) & 50 < v < 100 \\ \dfrac{1}{0.0413} f(v) & v > 100 \end{cases}
$$



$$
f(v \mid \text{survival}) = \underbrace{\frac{1}{P(\text{survival})}}_{1-P(F)} \cdot \underbrace{P(\text{survival} \mid v)}_{1-P(F \mid v)} f(v)
$$

$$
f(v \mid \text{survival}) = \begin{cases} \dfrac{1}{0.9587} (1) f(v) & 0 < v < 50 \\ \dfrac{1}{0.9587} \left(1 - \dfrac{v-50}{50}\right) f(v) & 50 < v < 100 \\ 0 \end{cases}
$$

SIMULATION EXAMPLE



$$L_1 = \mu_{L_1} \pm \sigma_{L_1}$$

$$L_2 = \mu_{L_2} \pm \sigma_{L_2}$$

$$\mathcal{N} = \left(\underline{M}_{L_1, L_2}, \ \underline{\Sigma}_{L_1 L_2} \right)$$

mvnrnd$(M, \Sigma, 10000)$

L_1	L_2	max M	max V

10,000 SIMULATIONS OF max M AND max V

YOU NEED TO FIND THE MAX M AND V THAT THE ELEMENTS CAN SUSTAIN

M	V
100	30
120	40
90	100
110	30

max M = 150

max V = 80

$$P(\text{FAILURE}) = \frac{\text{\# FAILURES}}{\text{\# SIMULATIONS}}$$