Aug. 18 | Introduction | Class : 01

Book: An Introductory method of Numerical Analysis, S S Sastry

methods of solution
(i) Analytical
(ii) Graphical
(iii) Numerical

Numerical methods:

Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations.

Non computer methods:

Analytical vs. Numerical methods

Need for Numerical Methods:

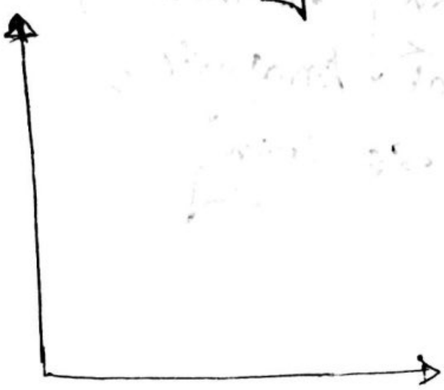
Reasons to study numerical Analysis

Mathematical Modeling

Complex mathematical model

Analytical Solution to Newton's Second Law

Comparison between Analytical vs Numerical Solution



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Class: 02

Approximations and Errors

Computer Based Solutions

Accuracy and Precision

Accuracy: Accuracy is related to the closeness to the true value.

Precision: Precision is related to the closeness to other estimated values.

Bias: Bias refers to systematic deviation of values from the true value.

Significant Figures

Rules for identifying sig. figures:

-
-
-

Scientific notation:

Why measure errors?

- To determine the accuracy of numerical results
- To develop . . . .

Error Defination

Error

- Inherent
- Truncation
- Round-off

Round-off Errors

True Error (E_t)

Approximate Error

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Class:

Root finding problems

Given a continuous function $f(x)$ find the value r such that $f(r) = 0$,

These problems are called root finding problems

A number r that satisfies an equation is called a root of the equation.

The equation: $x^4 - 3x^3 - 7x^2 + 15x = -18$

Zeros of a Function

Graphical Interpretation of zeros

Simple Zeros

Multiple Zeros

Solution Methods

Several ways to solve nonlinear equations are possible.

- Analytical Solutions

- Graphical Solutions

- Numerical Solutions
  - Open methods
  - Bracketing methods

Numerical Iterative Methods

Iterative Methods (continued)

- Bracketing methods (Interpolation methods)
- Open end methods (Extrapolation methods).

Starting an iterative process

Search bracket

Bisection Method

Intermediate Value Theorem

if a function is continuous and f(a) and f(b) have different signs then....

Bisection Method

If the function is continuous on [a,b] and f(a) and f(b) have different signs. Bisection method obtain...

Assumptions:

Bisection Algorithm

Flow Chart of Bisection Method

Bisectional Method: Example 1

Find the root of the equation: $x^3+4x^2-1=0$

Solution:

Let, $a=0$, and $b=1$

$$f(0)=(0)^3+4(0)^2-1=-1<0 \text{ and}$$

$$f(1)=(1)^3+4(1)^2-1=4>0$$

i.e $f(a)$ and $f(b)$ has opposite signs.

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Class: 04

Best Estimate and Error Level

Questions:

What is the best estimate of the zero of $f(x)$?

What is the error level in the obtained estimate?

Stopping Criteria

Convergence Analysis

Example

Exp: Use Bisection method to find a root of the equation $x = \cos(x)$ with absolute error < 0.02
(Assume the initial interval $[0.5, 0.9]$)

Q1: What is $f(x)$?

Q2: Are the assumptions satisfied?

Q3: How many iteration are needed?

Q4: How to compute the .... ?

Bisection Method

Advantages:

Simple and easy to implement

One function evaluation per iteration

Disadvantage:

Slow to coverage

Good intermidiate approximation may be discarded

Iteration Method

Suppose we have an equation in the form $g(x)=0$

Rewrite the equation in the form $x=f(x)$.

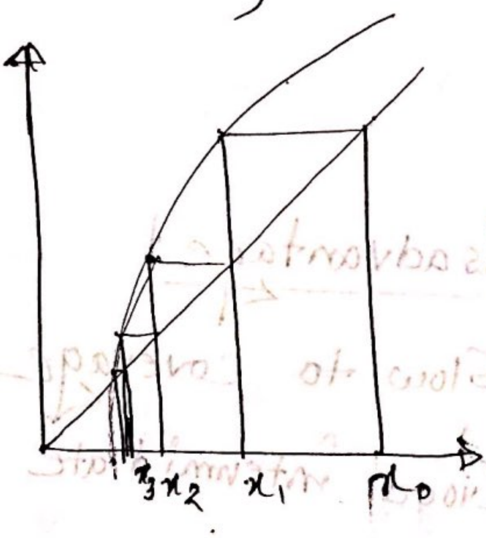
Iteration Method: Convergence Conditions

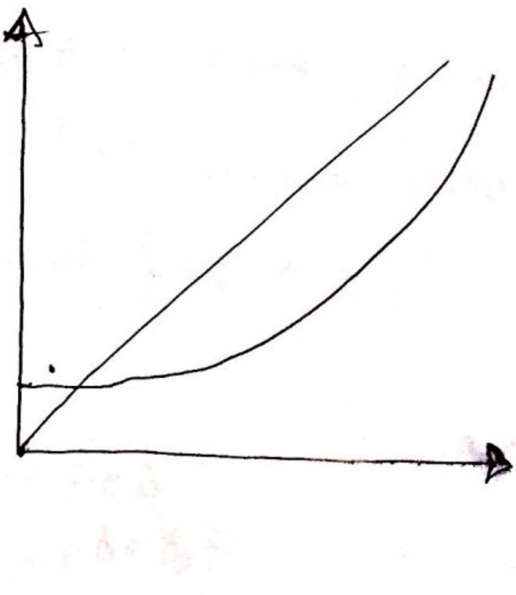
Any arbitrary approximation x_0, x_1, x_2 does not assure that it will converge to the actual root x of the equation

$$\underline{x_0}$$

$$x_1 = f(x_0)$$

$$x_2 = f(x_1)$$





Iteration Method: Example

Solve, $x = 2 + \sin(x)/2$

Find the real root of the equation:

$$g(x) = x^3 + x^2 - 1 = 0$$

Iteration Method: Drawbacks

We need an approximate initial guesses x_0.

It is also a slower method to find the root

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Class: 05

The Method of False Position or Regula Falsi

Like the bisection method, Method of False Position requires two initial guesses x_a and x_b such that

The Method of False Position: Geometric Significance

Ex: Find the real root of the equation till 2 decimal place.

$$f(x) = x^3 - 2x - 5 = 0$$

We observe that $f(2) = -1$ and $f(3) = 16$

And hence a root lies between 2 and 3. Then,

Newton-Rapson Method

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Iterative Method: Drawbacks

Convergence Criteria of Iteration Method

Acceleration of Convergence: Aitken's Δ^2 Process

Explain the acceleration of iterative method, how can ~~can process~~ find root fast.

Class test: Approximation error, root ~~value~~ finding method upto newton rapson method.

Problems:

Example 2.7

Chap: 1

Numerical method কী ?

" " এর প্রয়োজনীয়তা.

Balagurswami → Numerical method (Error)

Taxonomy of errors

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Newton-Rapsan Method: Drawbacks

The Newton-Rapson method requires the calculation of the derivative.

Generalized Newton's Method

$$x_{n+1} = x_n - p \frac{f(x)}{f'(x)}$$

$$f(x) = x^3 - x^2 + x + 1$$

$p = 3$

$$f'(x) = 8x^2 - 2x + 1$$

$p = 2$, $(p-1)$

$$f''(x) = 6x - 2$$

$p = 1$, $(p-2)$

Find a double root of the equation.

$$f(x) = x^3 - x^2 - x + 1 = 0$$

Here, $f'(x) = 3x^2 - 2x - 1$, and $f''(x) = 6x - 2$ with $x_0 = 0.8$

We obtain

$$x_0 - 2\frac{f(x_0)}{f'(x_0)} = 0.8 - 2\frac{0.072}{-(0.68)} = 1.012$$

and

$$x_0 - \frac{f'(x_0)}{f''(x_0)} = 0.8 - \left(-\frac{0.68}{2.8}\right) = 1.043$$

<u>Secant Method:</u>

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

if x_i and x_{i-1} are two initial points:

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{(x_i - x_{i-1})}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{\frac{f(x_i) - f(x_{i-1})}{(x_i - x_{i-1})}} = x_i - f(x_i)\frac{(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Geometrical representation of the secant method

Derivation of Secant Method

$$\frac{AB}{AE}$$

Apply Secant Method in the floating point problem:

$$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4}$$

Let us assume the initial guesses of the root of $f(x) = 0$ as $x_1 = 0.02$ and $x_0 = 0.05$

$$x_1 = x_0 - \quad f(x_0)\ (x_0 - x_{-1})$$

Secant method:

Floating ball problem

Iteration 3 for the floating ball problem (Secant method)

$x_3 = 0.06238$

The absolu

Flow Chart of Secant's Method:

Advantages of Secant Method:

1. It converges at faster than a linear rate, so that

2.

Disadvantage of Secant Method:

1.

2.

Next: Interpolation