Question 1

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1 Part A

When $x = \varepsilon$, then $(x - \varepsilon)^3 = 0$.
Additionally, $(x - \varepsilon)^3_+$ implies that the result must be positive. If that value is not positive, it is reduced to zero. This is similar to ridge regression.
Therefore, if $x <= \varepsilon$, then $(x - \varepsilon)^3_+ = 0$.
Therefore, $f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$
$f_1(x) = a_1 + b_1 x + c_1 x^2 + d_1 x^3$
Therefore, $\beta_0 = a_1; \beta_1 = b_1; \beta_2 = c_1; \beta_3 = d_1$

2 Part B

When $x > \varepsilon$, then we only need to consider ε as part of the coefficients.
$\beta_4(x - \varepsilon)^3 = \beta_4 x^3 - \beta_4 3x^3\varepsilon + \beta_4 3x\varepsilon^2 - \beta_4\varepsilon^3$
Therefore, $f_2(x) = (\beta_0 - \beta_4\varepsilon^3) + (\beta_1 + 3\beta_4\varepsilon^2)x + (\beta_2 - 3\beta_4\varepsilon)x^2 + (\beta_3 + \beta_4)x^3$
Therefore, $a_2 = \beta_0 - \beta_4\varepsilon^3; b_2 = \beta_1 + 3\beta_4\varepsilon^2; c_2 = \beta_2 - 3\beta_3\epsilon; d_2 = \beta_3 + \beta_4$

3 Part C

$f_1(\varepsilon) = \beta_0 + \beta_1\varepsilon + \beta_2\varepsilon^2 + \beta_3\varepsilon^3$
$f_2(\varepsilon) = (\beta_0 - \beta_4\varepsilon^3) + (\beta_1 + 3\beta_4\varepsilon^2)\varepsilon + (\beta_2 - 3\beta_4\varepsilon)\varepsilon^2 + (\beta_3 + \beta_4)\varepsilon^3$
$f_2(\varepsilon) = \beta_0 + \beta_1\varepsilon + \beta_2\varepsilon^2 + \beta_3\varepsilon^3$
Therefore, $f_1(x) = f_2(x)$ when $x = \varepsilon$

4 Part D

$f_1'(x) = \beta_1 + 2\beta_2 x + 3\beta_3 x^2$
$f_1'(\varepsilon) = \beta_1 + 2\beta_2\varepsilon + 3\beta_3\varepsilon^2$
$f_2'(x) = (\beta + 3\beta_4\varepsilon^2) + 2(\beta_2 - 3\beta_4\varepsilon)x + 3(\beta_3 + \beta_4)x^2$
$f_2'(\varepsilon) = (\beta + 3\beta_4\varepsilon^2) + 2(\beta_2 - 3\beta_4\varepsilon)\varepsilon + 3(\beta_3 + \beta_4)\varepsilon^2$
$f_2'(\varepsilon) = \beta_1 + 2\beta_2\varepsilon + 3\beta_3\varepsilon^2$
Therefore, $f_1(x) = f_2(x)$ when $x = \varepsilon$

5 Part E

$f_1''(x) = 2\beta_2 + 6\beta_3 x$
$f_1''(\varepsilon) = 2\beta_2 + 6\beta_3 \varepsilon$
$f_2''(x) = 2\beta_2 - 6\beta_4 \varepsilon + 6(\beta_3 + \beta_4)x$ $f_2''(\varepsilon) = 2\beta_2 - 6\beta_4 \varepsilon + 6(\beta_3 + \beta_4)\varepsilon$
$f_2''(\varepsilon) = 2\beta_2 + 6\beta_3 \varepsilon$
Therefore, $f_1''(x) = f_2''(x)$ when $x = \varepsilon$