Mid-Term: Question 4, Part 3a

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The loss function in this section can be written as:

$$\min_{\beta 1,\beta 2} L(\beta) = \min_{\beta 1,\beta 2} \sum_{i=1}^{n} \left(y_i - \frac{sin(\beta_1 x_i)}{\beta_2 x_i + 1} \right)^2$$

1 Part A

Decompose $L(\beta)$ such that $L(\beta) = g(\beta)^T g(\beta)$:

$$g(\beta) = \left[y_i - \frac{sin(\beta_1 x_i)}{\beta_2 x_i + 1} \right]_{n \times 1}$$

Derive the Jacobian $J_{g(\beta)}$:

$$J_{g(\beta)} = \left[\frac{\partial g(\beta)}{\partial \beta_1}, \frac{\partial g(\beta)}{\partial \beta_2} \right]_{n \times 2}$$

This Jacobian for our loss function can be written as:

$$J_{g(\beta)} = \left[-\frac{cos(\beta_1 x_i)}{\beta_2 x_i + 1}, \frac{\beta_2 sin(\beta_1 x_i)}{(\beta_2 x_i + 1)^2} \right]_{n \times 2}$$