HDDA Final Exam: Question 4 Part 1

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1 Part 1

Consider the Robust PCA problem below:

$$min\{||L|_* + \lambda||S||_1\} \qquad s.t. \quad M = L + S$$

where M is the original matrix, L is the recovered low-rank matrix, and S is the sparse outlier matrix. Set $\lambda = 0.01$.

We can rewrite the problem as:

$$\underset{L,S}{argmin}\{||L|_* + \lambda||S||_1\} \qquad s.t. \quad M = L + S$$

Using the Augmented Lagrangian Multiplier Form (ADMM) of the minimization problem:

$$l(L, S, Y; \mu) = ||L||_* + \lambda||S||_1 + \frac{\mu}{2}\Big||M - L - S + \frac{Y}{\mu}|\Big|_F^2 + \frac{\mu}{2}\Big||\frac{Y}{\mu}|\Big|_F^2$$

This can then be solved by ADMM using the below procedure:

Robust Principal Component Analysis

Initialize $S_0, Y_0 \in \mathbf{R}^{n \times n}$ and $S_0 = Y_0 = 0$
Initialize $\mu > 0$
Initialize $\lambda = 0.01$
while not converged **do**
    Compute $L_{k+1} = \mathscr{D}_\mu(M - S_k - \mu^{-1}Y_k)$
    Compute $S_{k+1} = S_{\lambda\mu}(M - L_{k+1} - \mu^{-1}Y_k)$
    Compute $Y_{k+1} = Y_k + \mu(M - L_{k+1} - S_{k+1})$
end while
output: L, S

The more detailed procedure can be written as:

Robust Principal Component Analysis

01. Initialize $S_0, Y_0 \in \mathbf{R}^{n \times n}$ and $S_0 = Y_0 = 0$
02. Initialize $\mu > 0$
03. Initialize $\lambda = 0.01$
04. **while** not converged **do**
05. $\quad X = M - S_k + \frac{Y}{\mu}$
06. $\quad L_{k+1} = \text{TruncatedSVD}_{\frac{1}{\mu}}(X)$
07. $\quad X = M - L_{k+1} + \frac{Y}{\mu}$
08. $\quad S_{k+1} = sgn(X) \times max(|X| - \frac{\lambda}{\mu}, 0)$
09. $\quad Y_{k+1} = Y_k + \mu(M - L_{k+1} - S_{k+1})$
11. **end while**
12. **output**: L, S