ACM/IDS 104 - Problem Set 2 - MATLAB Problems

Before writing your MATLAB code, it is always good practice to get rid of any leftover variables and figures from previous scripts.

```
clc; clear; close all;
```

Problem 5 (10 points) Fundamental Matrix Subspaces

Your task for this problem is to write a function that takes a matrix `A` as its argument, and outputs four matrices: `K, I, cK` and `cI` where:

- Columns of `K` form a basis of the kernel of `A`. If $\ker A = \{0\}$, then `K` must be a zero vector of the appropriate dimension.
- Columns of `I` form a basis of the image of `A`. If $\mathrm{im} A = \{0\}$, then `I` must be a zero vector of the appropriate dimension.
- Columns of `cK` form a basis of the cokernel of `A`. If $\mathrm{coker} A = \{0\}$, then `cK` must be a zero vector of the appropriate dimension.
- Columns of `cI` form a basis of the coimage of `A`. If $\mathrm{coim} A = \{0\}$, then `cI` must be a zero vector of the appropriate dimension.

Move to the bottom of this livescript to write the function.

Now, let us test our function:

```
A = magic(6); % feel free to define A as you like
[K, I, cK, cI] = subspacer(A); % this is how you call a MATLAB function
disp(K);
```

```
   -0.4714
   -0.4714
    0.2357
    0.4714
    0.4714
   -0.2357
```

```
disp(I);
```

```
   -0.4082    0.5574    0.0456   -0.4182    0.3092
   -0.4082   -0.2312    0.6301   -0.2571   -0.5627
   -0.4082    0.4362    0.2696    0.5391    0.1725
   -0.4082   -0.3954   -0.2422   -0.4590    0.3971
   -0.4082    0.1496   -0.6849    0.0969   -0.5766
   -0.4082   -0.5166   -0.0182    0.4983    0.2604
```

```
disp(cK);
```

```
    0.5000
    0.0000
   -0.5000
   -0.5000
    0.0000
    0.5000
```

```
disp(cI);
```

```
   -0.4082    0.6234   -0.3116    0.2495   -0.2511
   -0.4082   -0.6282    0.3425    0.1753   -0.2617
   -0.4082   -0.4014   -0.7732   -0.0621    0.1225
   -0.4082    0.1498    0.2262   -0.4510   -0.5780
   -0.4082    0.1163    0.2996    0.6340    0.3255
   -0.4082    0.1401    0.2166   -0.5457    0.6430
```

START HERE by writing the function:

```
function [K, I, cK, cI] = subspacer(A)
%{
This is the MATLAB function syntax.
-> [K, I, cK, cI] are the outputs of the function.
-> "subspacer" is the name of the function. (you can change that if
                            you wish but make sure you change
                            every function call as well!)
-> A is the argument of the function.
%}
[m, n] = size(A);
r = rank (A);
%{
We start by finding out the dimensions and rank of A.
Let us consider the matrix K. There exist 2 cases:
1) The kernel is trivial i.e. kerA = {0}
2) The kernel is not trivial -> Hint: use null()
Complete the following if/else statement.
%}
if r == n % this condition is done for you
    K = zeros([n 1]);
else
    K = null(A);
end

%{
Now, let us consider the matrix cK.
As above, there exist 2 cases. Remember, you can use ' to
transpose a matrix.
Write a similar if/else statement to produce cK.
%}

if r == m % this condition is done for you
    cK = zeros([m 1]);
else
    cK = null(A');
end

%{
For the image I and coimage cI, there exists only 1 condition
we must test, and that is if rankA = 0. With this in mind,
```

```
complete the following if/else statement.
-> Hint: orth() is useful here.
%}
if r == 0
    I = zeros([n 1]);
    cI = zeros([m 1]);
else
    I = orth(A);
    cI = orth(A');


end
end
```