CDS110 **November 12, 2024**

Pset 06

Problem 1

Part A

First we will show that $\ddot{e}_1 = -\frac{C_y}{mv_x^\beta}\dot{e}_1 + \frac{C_y}{m}e_2 + \frac{C_y}{m}u_\delta - v_x^\beta\omega_d$ as stated in the system (2) given in the problem. By definition of $\dot{e}_1$ we have:

$$\ddot{e}_1 = \frac{d}{dt}(\dot{e}_1) = \frac{d}{dt}\left(v_y^\beta + v_x^\beta(\theta - \theta_d)\right)$$

Treating v_x^β as a constant we have:

$$\ddot{e}_1 = \dot{v}_y^\beta + v_x^\beta(\dot{\theta} - \dot{\theta}_d) = \dot{v}_y^\beta + v_x^\beta(\omega - \omega_d)$$

Substituting $\dot{v}_y^\beta$ we have:

$$\ddot{e}_1 = -\frac{C_y}{mv_x^\beta}v_y^\beta - v_x^\beta\omega + \frac{C_y}{m}u_\delta + v_x^\beta(\omega - \omega_d)$$

Simplifying we have:

$$\ddot{e}_1 = -\frac{C_y}{mv_x^\beta}v_y^\beta + \frac{C_y}{m}u_\delta - v_x^\beta\omega_d$$

Now we compare the equation above with the equation given in system (2) by substituting $\dot{e}_1$ and e_2:

$$\ddot{e}_1 = -\frac{C_y}{mv_x^\beta}\dot{e}_1 + \frac{C_y}{m}e_2 + \frac{C_y}{m}u_\delta - v_x^\beta\omega_d$$

$$\ddot{e}_1 = -\frac{C_y}{mv_x^\beta}(v_y^\beta + v_x^\beta(\theta - \theta_d)) + \frac{C_y}{m}(\theta - \theta_d) + \frac{C_y}{m}u_\delta - v_x^\beta\omega_d$$

$$\ddot{e}_1 = -\frac{C_y}{mv_x^\beta}v_y^\beta - \frac{C_y}{mv_x^\beta}v_x^\beta(\theta - \theta_d) + \frac{C_y}{m}(\theta - \theta_d) + \frac{C_y}{m}u_\delta - v_x^\beta\omega_d$$

Thus because $-\frac{C_y}{mv_x^\beta}v_x^\beta(\theta - \theta_d) + \frac{C_y}{m}(\theta - \theta_d) = 0$ then we have:

$$\ddot{e}_1 = -\frac{C_y}{mv_x^\beta}\dot{e}_1 + \frac{C_y}{m}e_2 + \frac{C_y}{m}u_\delta - v_x^\beta\omega_d = -\frac{C_y}{mv_x^\beta}v_y^\beta + \frac{C_y}{m}u_\delta - v_x^\beta\omega_d$$

Secondly we will show that $\ddot{e}_2 = -\frac{L^2C_y}{2I_zv_x^\beta}(\omega - \omega_d) + \frac{LC_y}{2I_z}u_\delta - \frac{L^2C_y}{2I_zv_x^\beta}\omega_d$ as stated in the system (2) given in the problem. By definition of e_2 we have:

$$\dot{e}_2 = \dot{\theta} - \dot{\theta}_d = \omega - \omega_d$$

$$\ddot{e}_2 = \frac{d}{dt}(\dot{e}_2) = \dot{\omega} - \dot{\omega}_d$$

Assuming $\dot{\omega}_d = 0$ Then we have $\ddot{e}_2 = \dot{\omega} = -\frac{L^2C_y}{2I_zv_x^\beta}\omega + \frac{LC_y}{2I_z}u_\delta$. Simplifying the equitation for $\ddot{e}_2$ as given by (2) we have:

$$\ddot{e}_2 = -\frac{L^2C_y}{2I_zv_x^\beta}(\omega - \omega_d) + \frac{LC_y}{2I_z}u_\delta - \frac{L^2C_y}{2I_zv_x^\beta}\omega_d = -\frac{L^2C_y}{2I_zv_x^\beta}\omega + \frac{LC_y}{2I_z}u_\delta - \frac{L^2C_y}{2I_zv_x^\beta}\omega_d + \frac{L^2C_y}{2I_zv_x^\beta}\omega_d$$

Thus we have:

$$\ddot{e}_2 = -\frac{L^2 C_y}{2 I_z v_x^\beta}(\omega - \omega_d) + \frac{L C_y}{2 I_z} u_\delta - \frac{L^2 C_y}{2 I_z v_x^\beta}\omega_d = -\frac{L^2 C_y}{2 I_z v_x^\beta}\omega + \frac{L C_y}{2 I_z} u_\delta = \dot{\omega}$$

Thus we have shown that the equations provided for $\ddot{e}_1$ and $\ddot{e}_2$ in (2) are correct. Lastly, the equations for $\dot{e}_1$ and $\dot{e}_2$ from (2) are $\dot{e}_1 = \dot{e}_1$ and $\dot{e}_2 = \dot{e}_2$ which are correct by definition.

Part B

Using Python's SymPy library, the rank of the reachability matrix is 4. This indicates that the system is reachable.

Part C

Refer to the code submitted. The plots are shown there. From the plots we see that there is steady state error for e_1 and e_2. We also see that the car does not 'perfectly' overlap the circle, it seems to be a bit off in certain sections of the circle.

Part D

Refer to the code submitted. The plots are shown there.

Part E

Refer to the code submitted. The plots are shown there.

Part F

From the geometry of the trajectory where R is the radius of the circle and L is the wheelbase length we have that the feed forward term is $\alpha = \tan^{-1}\left(\frac{L}{R}\right)$. Refer to the code submitted. The plots are shown there.