CDS110 **November 19, 2024**

Pset 07

Problem 1

Adjusting the values of q and r we see that q corresponds to the weight placed on reaching the final state while r corresponds to the weight placed on minimizing the sum of the squares of the inputs (as shown in the equation for J as given in the problem set). In other words, if q is larger than r we would prioritize reaching the final state over minimizing J (sum of the squares of the inputs). And on the other hand, if r is larger than q we would prioritize minimizing J over reaching the final state within the specified time. As given in the code for this problem, q is larger than r and we are able to reach the final state within the specified final time t_f. However if we decrease q and increase r, such that $q \leq r$ we will see that the final state may not be reached within the specified final time t_f, but the absolute value of the control input becomes smaller.

Problem 2

Part A

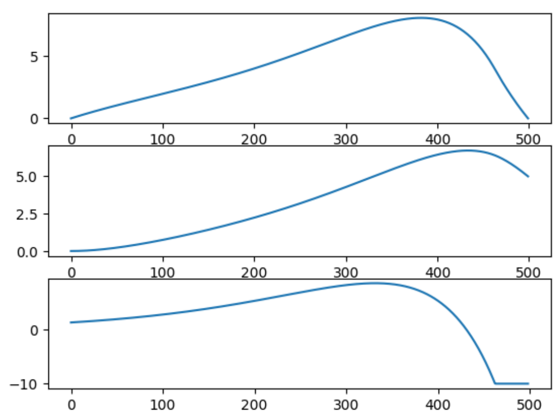
Discretizing we have:

$$\begin{bmatrix} m_{k+1} \\ p_{k+1} \end{bmatrix} = \begin{bmatrix} m_k \\ p_k \end{bmatrix} + \begin{bmatrix} \dot{m}_k \\ \dot{p}_k \end{bmatrix} \Delta t$$

Where:

$$\begin{bmatrix} \dot{m}_k \\ \dot{p}_k \end{bmatrix} = \begin{bmatrix} -\delta & 0 \\ \kappa & \gamma \end{bmatrix} \begin{bmatrix} m_k \\ p_k \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_k + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \alpha_0$$

Part B



The upper plot shows that $m_0 = 0$ and $m_f = 0$. The middle plot shows $p_0 = 0$ and $p_f = 5$. Lastly the lower plot shows that the input is always between -10 and 10.

Problem 3

Part A
Discretizing we have:

$$x_{k+1} = x_k + \dot{x}_k \Delta t$$
$$y_{k+1} = y_k + \dot{y}_k \Delta t$$
$$\theta_{k+1} = \theta_k + \dot{\theta}_k \Delta t$$
$$v_{x_{k+1}} = v_{x_k} + \dot{v}_{x_k} \Delta t$$

Where:

$$\dot{x}_k = cos(\theta_k) v_{x_k}$$
$$\dot{y}_k = sin(\theta_k) v_{x_k}$$
$$\dot{\theta}_k = \frac{v_{x_k}}{L}\, tan(u_{\delta_k})$$
$$\dot{v}_{x_k} = u_{v_k}$$

Part B
Let $S = (S_x, S_y)$ be the rear point of the bicycle and $F = (F_x, F_y)$ be the front point of the bicycle. Now lets consider a circle centered at point C with some radius r. Lets define vector $\vec{SF} = F - S$ and vector $\vec{SC} = C - S$. To find the closest point of the line segment to the center of the circle we project $\vec{SC}$ onto $\vec{SF}$. Thus the closest point of the bicycle to the center is:

$$proj_{\vec{SF}} \vec{SC} = S + clip\left(\frac{\vec{SC} \cdot \vec{SF}}{||\vec{SF}||^2}\right) \vec{SF}$$

Where $clip(value)$ clips the value between 0 and 1. In this case, this means that if $\frac{\vec{SC} \cdot \vec{SF}}{||\vec{SF}||^2} > 1$ then we clip $\frac{\vec{SC} \cdot \vec{SF}}{||\vec{SF}||^2}$ to 1 and if $\frac{\vec{SC} \cdot \vec{SF}}{||\vec{SF}||^2} < 0$ we clip $\frac{\vec{SC} \cdot \vec{SF}}{||\vec{SF}||^2}$ to 0. This is because the bicycle has finite length L, so the closest point must be on that finite bicycle segment. Thus we would have the constraint $||C - proj_{\vec{SF}} \vec{SC}|| > r$, which means that the closest point to the center C from the bicycle has to be a distance greater than r (the radius of the circle) away from the center C of the circle.
Generalizing this, for the Kth time step we would have:

$$S_k = \begin{bmatrix} x_k - (L/2)cos(\theta_k) \\ y_k - (L/2)sin(\theta_k) \end{bmatrix}$$

$$F_k = \begin{bmatrix} x_k + (L/2)cos(\theta_k) \\ y_k + (L/2)sin(\theta_k) \end{bmatrix}$$

And we would add the constraint $||C_i - proj_{\vec{S_k F_k}} \vec{S_k C_i}|| > r_i$ n circular obstacles centered at $C_i = (x_i, y_i)$ with radius r_i. Note that in the constraint $proj_{\vec{S_k F_k}}$ is defined as above (i.e. using clip()).

Part C

$$
\begin{aligned}
x[k]^*, u[k]^* = \arg \min_{x[k],u[k]} \quad & (x[N]-x_d)^T Q_f (x[N]-x_d) + \sum_{k}^{N-1} u[k]^T R u[k] \\
\text{subject to} \quad & x[k+1] = f(x[k], u[k]) \\
& x[0] = x_0 \\
& ||C_i - proj_{\vec{S_k F_k}} \vec{S_k C_i}|| > r_i \ \forall \ i \in [0, n] \text{ and } \forall \ k \in [1, N]
\end{aligned}
$$

Note, $||C_i - proj_{\vec{S_k F_k}} \vec{S_k C_i}|| > r_i$ is defined as in Part B, n is the number of circular obstacles, and N is the number of points in the trajectory.

Part D