CDS110 **October 8, 2024**

Pset 01

Problem 1

a) For Type 1 diabetes, the actuation of the pancreas is the most directly compromised function. In this scenario, the chemicals secreted into the blood by the pancreas is the actuation. It is the mechanism that gives the body the means to regulate its system (i.e. regulate its glucose level). If the pancreas cannot create chemicals at full capacity as in Type 1 diabetes, then this would means that its actuation is compromised.

b) For Type 2 diabetes, the actuation of the liver is the most directly compromised function. Although the liver may have a reduction in insulin receptors, its sensing for glucose levels is not completely compromised (i.e. sensing can still be done or at least to some extent). As stated in the problem, with Type 2 diabetes the liver does not process excess glucose effectively, which suggests an the actuation function of the liver being compromised. This is because the actuation of the liver is the liver's ability process excess glucose in the blood supply when the glucose level is too high, and release its glucose stores when its glucose level is too low. Hence the function most directly compromised is the actuation of the liver.

c) In the scenario given, the actuation of the artificial pancreas is the most directly compromised function. The artificial pancreas is still be able to sense glucose levels, and compute the required insulin and/or glucagon to regulate the sensed glucose level. In other words, the computation and sensing functionality of the artificial pancreas are still intact. Therefore, the actuation of the artificial pancreas is the most directly compromised function.

Problem 2

a)

$$\ddot{y}(t) + 4\dot{y}(t) + 8y(t) = 2\dot{u}(t) + 3u(t)$$

As given in the problem let $u(t) = u_\epsilon(t)$. Taking the limit we have:

$$\lim_{\epsilon \to 0} u_\epsilon(t) = \begin{cases} \infty & t = 0 \\ 0 & otherwise \end{cases} = \delta(t)$$

In other words, as $\epsilon \to 0$, u(t) becomes the Dirac Delta function $\delta(t)$.
With this in mind, taking the Laplace transform, given that y(0) = 0 and $\dot{y}(0) = 0$ we have:

$$\mathscr{L}\{\ddot{y}\} = s^2Y(s) - sy(0) - \dot{y}(0) = s^2Y(s)$$

$$\mathscr{L}\{\dot{y}\} = sY(s) - y(0) = sY(s)$$

$$\mathscr{L}\{y\} = Y(s)$$

$$\mathscr{L}\{\dot{u}\} = \mathscr{L}\{\dot{\delta}\} = s$$

$$\mathscr{L}\{u\} = \mathscr{L}\{\delta\} = 1$$

Substituting we get:

$$\mathscr{L}\{\ddot{y}(t) + 4\dot{y}(t) + 8y(t)\} = \mathscr{L}\{2\dot{u}(t) + 3u(t)\}$$
$$s^2Y(s) + 4sY(s) + 8Y(s) = 2s + 3$$
$$Y(s) = \frac{2s+3}{s^2+4s+8} = \frac{2(s+2)}{(s+2)^2+2^2} - \frac{1}{2}\cdot\frac{2}{(s+2)^2+2^2}$$

Solving for $y(t)$ by taking the inverse Laplace:

$$y(t) = \mathscr{L}^{-1}\{Y(s)\} = 2\,\mathscr{L}^{-1}\left\{\frac{(s+2)}{(s+2)^2+2^2}\right\} - \frac{1}{2}\,\mathscr{L}^{-1}\left\{\frac{2}{(s+2)^2+2^2}\right\}$$

$$y(t) = 2e^{-2t}cos(2t) - \frac{1}{2}e^{-2t}sin(2t)$$

Hence we have that as $\epsilon \to 0$ the solution $y(t)$ becomes the following:

$$y(t) = 2e^{-2t}cos(2t) - \frac{1}{2}e^{-2t}sin(2t) = e^{-2t}(2cos(2t) - \frac{1}{2}sin(2t))$$

b)

$$\ddot{x} + 4\dot{x} + 8x(t) = 0$$

i)

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} \implies \begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = A\begin{bmatrix}x\\ \dot{x}\end{bmatrix}$$

Solving for A yields:

$$A = \begin{bmatrix}0 & 1\\ -8 & -4\end{bmatrix}$$

Thus:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} \implies \begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -8 & -4\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix}$$

ii) Solving for eigenvalues:

$$A - \lambda I = \begin{bmatrix}0 & 1\\ -8 & -4\end{bmatrix} - \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix} = \begin{bmatrix}-\lambda & 1\\ -8 & -4-\lambda\end{bmatrix}$$
$$det(A-\lambda I) = -(4+\lambda)(-\lambda) - (-8) = \lambda(4+\lambda) + 8 = 0$$
$$\lambda^2 + 4\lambda + 8 = 0$$
$$\lambda = \frac{-4 \pm \sqrt{16-32}}{2} = \frac{-4 \pm 4i}{2}$$

Thus the two eigenvalues are :

$$\lambda_1 = -2 + 2i$$
$$\lambda_2 = -2 - 2i$$

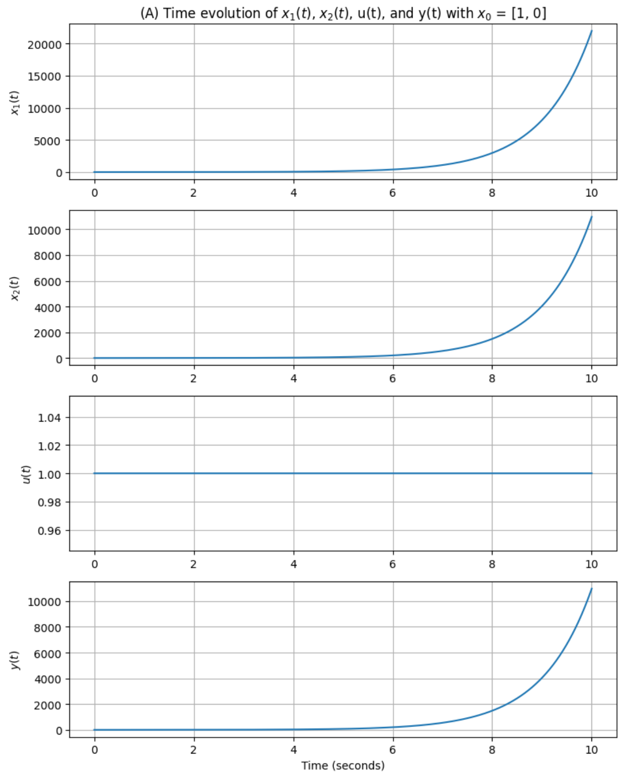
These two eigenvalues suggest a solution in the same form as given in part a. That is, these eigenvalues would yield a solution in the form shown below:

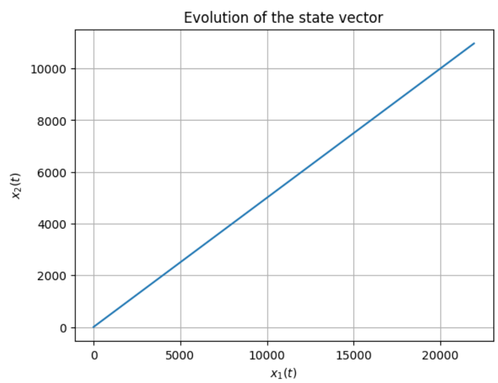
$$y(t) = e^{-2t}(c_1 \cdot cos(2t) + c_2 \cdot sin(2t))$$

where $c_1, and c_2$ are some constants.

Problem 3

System A
Code: `https://colab.research.google.com/drive/1PhEuivsyCTNbRtoFHH7Nv33uEciu5C2J?usp=sharing`
Plots shown below for $x_0 = [1, 0]$:



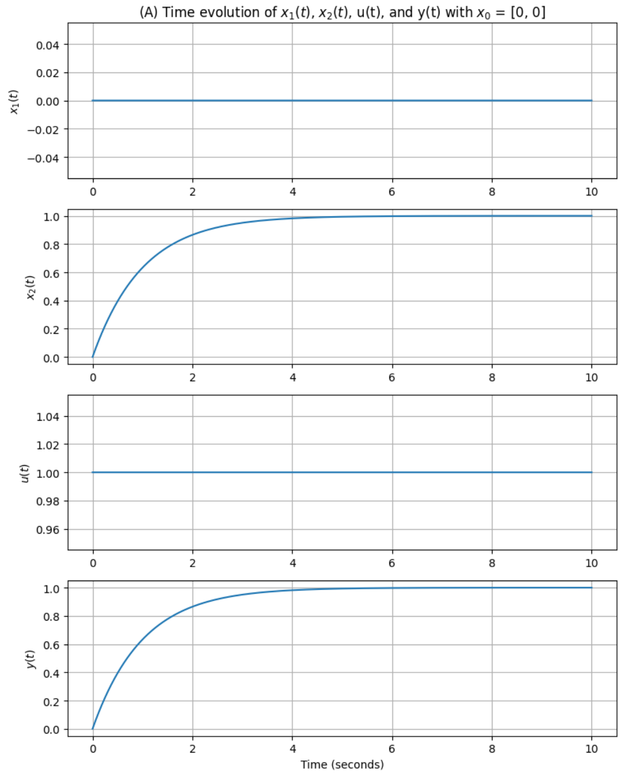


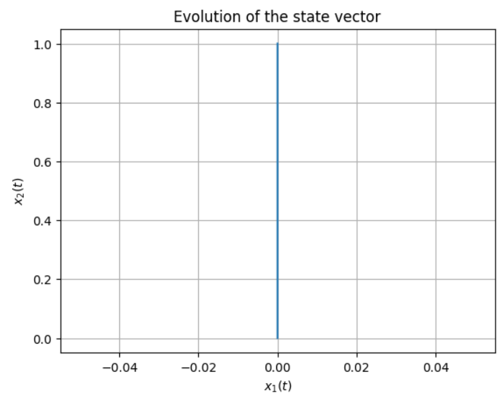
For the time observed in the plots, we can see that both $x_1(t)$ and $x_2(t)$ are growing in an exponential manner. The input of the system $u(t)$ is constant. The output of the system $y(t)$ is also growing in an exponential manner.

System A (cont.)

Code: `https://colab.research.google.com/drive/1PhEuivsyCTNbRtoFHH7Nv33uEciu5C2J?usp=sharing`

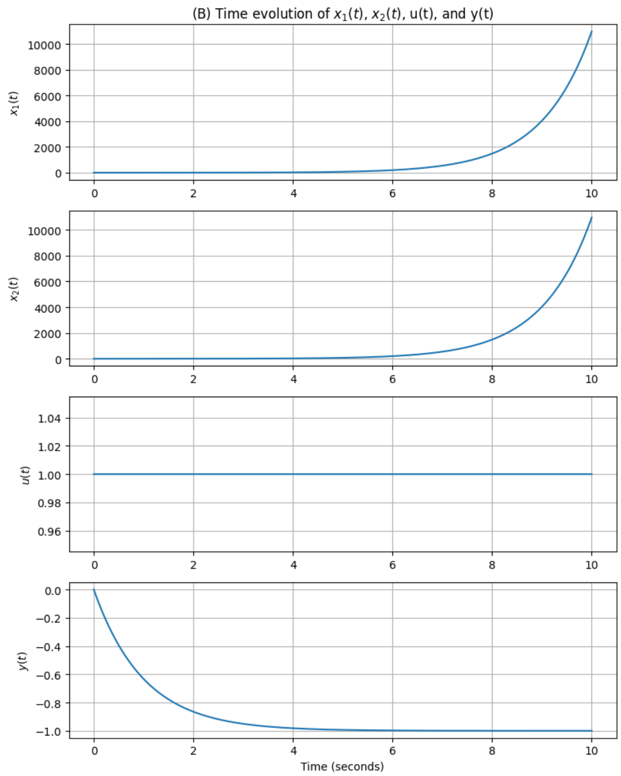
Plots shown below for $x_0 = [0, 0]$:

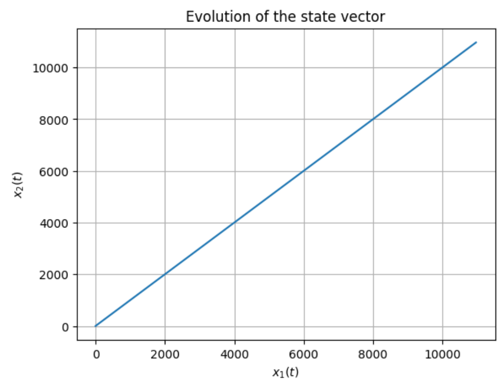




For the time observed in the plots, we can see that $x_1(t)$ is constant. On the other hand, we see $x_2(t)$ increasing at a decreasing rate (in other words, reaching a steady-state). As before, the input of the system $u(t)$ is constant. The output of the system $y(t)$ is increasing at decreasing rate (in other words, reaching a steady-state).

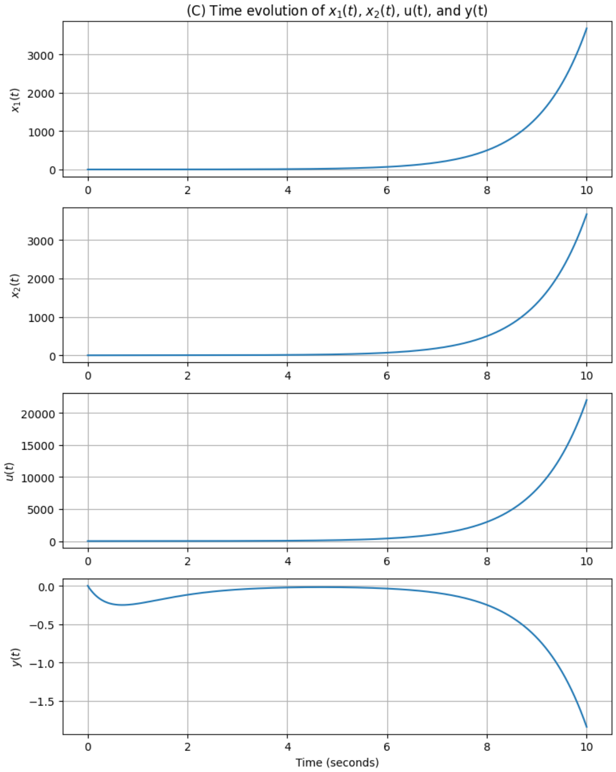
System B
Code: `https://colab.research.google.com/drive/1PhEuivsyCTNbRtoFHH7Nv33uEciu5C2J?usp=sharing`
Plots shown below:

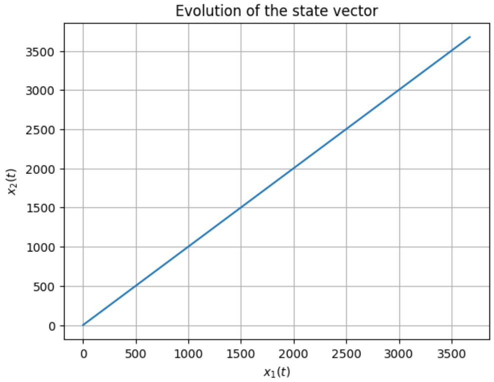




For the time observed in the plots, we can see that system B has a similar behavior as system A (with the exception of the output). That is, we can see that both $x_1(t)$ and $x_2(t)$ are growing in an exponential manner and the he input of the system $u(t)$ is constant. Unlike system A, however the output of the system $y(t)$ is decreasing at decreasing rate (in other words, reaching a steady-state).

System C
Code: `https://colab.research.google.com/drive/1PhEuivsyCTNbRtoFHH7Nv33uEciu5C2J?usp=sharing`
Plots shown below:





For the time observed in the plots, we can see that both $x_1(t)$ and $x_2(t)$ are growing in an exponential manner. We can also see that system C is unlike system A and B in terms of its input $u(t)$ and output $y(t)$. The input $u(t)$ is growing in an exponential manner. The output $y(t)$ seems to at first undershoot then reach a steady state for a few seconds and then eventually starts to decrease at an increasing rate.