CS 133a	**October 4, 2023**

Problem Set 1

Problem 1 (DOFs of Objects) - 14 points

(a) A point (no volume) movable in 2D (planar) space can either move vertically or horizontally to have an (x,y) coordinate, so it has 2 DOFs (degrees of freedom).

(b) A rigid body with finite size has 3 DOFs in 2D (planar) space. Like the point it can move in two directions to have and (x,y) coordinate. Unlike the point, the rigid body can have heading θ, therefore it has 3 DOFs.

(c) A circle of variable radius has 3 DOFs in 2D (planar) space. Like a point it can move in two directions, so it can have an (x,y) coordinate. Furthermore, since it is of variable radius, then it can become bigger or smaller providing another degree of freedom. Note that since it is a circle its heading will not change how the circle appears in 2D space. Thus it only has 3 DOFs.

(d) A rigid body (finite size in all directions) movable in 3d space has 6 DOFs. Three come from its position in 3d space (x,y,z). The other three come from its orientation in space (row, yaw, pitch).

(e) A line segment (no thickness, fixed finite length) movable in 3D space has 5 DOFs. It can be moved in 3 different directions to have a coordinate (x,y,z) and can be rotated in two different manners to have an orientation (θ, ϕ).

(f) A 2D plane (with infinite size) movable in 3D space has 3 DOFs. The plane can be moved along the direction perpendicular to it (along the direction of its normal vector), and it can be rotated in two different manners to have an orientation (θ, ϕ).

(g) A torus of variable radii movable in 3D space has 7 DOFs. It can be moved in 3 different directions to have a coordinate (x,y,z), and be rotated in two different manners to have an orientation (θ, ϕ). Furthermore, its inner radius and outer radius can be changed to change its size providing two more degrees of freedom. Thus there are a total of 7 DOFs.

Problem 2 (Towing in 2D Plane) - 16 points

1. When towing the trailer, the system of connected objects will have 4 degrees of freedom. The car whose hitch is attached to the trailer has 3 degrees of freedom, as the car can have an (x,y) coordinate along with a heading θ on a flat surface. Since the trailer is attached to the car via a hitch, the connection between the car and trailer will act a revolute, providing an extra degree of freedom. Thus 4 total.

2. Without a trailer (car attached via a rope), the system will have 5 degrees of freedom. Like before, three degrees of freedom come from the car that is driving, due to being able to have an any (x,y) position along with a heading θ. The end of the rope on this car's hitch will act as a revolute, providing another degree of freedom. Since the rope is also attached to the tow point of the car being towed, then this end of the rope will also act as a revolute providing an extra degree of freedom. Thus the total degrees of freedom that this system has is 5.

Problem 3 (Human Hand) - 18 points

Each finger (excluding the thumb) has 4 degrees of freedom. Each finger has two joints that each act like a revolute (providing two degrees of freedom) and a third joint (the one closest to the palm of the hand) that can rotate in two different manners (providing another two degrees of freedom).
The thumb has 5 degrees of freedom. It has one joint (the one closest to the nail) that acts as a revolute and provide a degree of freedom, and two other joints (one in the other knuckle and one in the palm) and that each provide two degrees of freedom (each can be rotated in two different manners). Thus the human hand has 21 DOFs (4*4+5=21).

Problem 4 (Riding a Stationary Bike) - 18 points:

(a) Seating securely on the seat will lead to having 5 degrees of freedom in total. The position of the crank on the bicycle provides one degree of freedom. Each bike pedal provides a degree of freedom as they allow for the ankle to be rotated forward and backward (so 2 in total as there are 2 bike pedals). Lastly, each knee provide an additional degree of freedom as each is able to be moved inward and outwards (or in other words, toward and away from the center of the bike). Therefore there are 5 DOFs in total.

(b) Standing up on pedals will allow for 11 DOFs. The 5 DOFs from part a are still avaliable, but now since the hip is able to move freely then we have 6 additional DOFs. Hence 11 DOFs in total. Note that the hip has 6DOFS because it has 3 rotational DOFs and 3 translational DOFs.

Problem 5 (C-space of 4-bar Linkage) - 30 points:

First we get the coordinates (B_x, B_y) of B relative to the origin depicted in the figure given in the problem set:

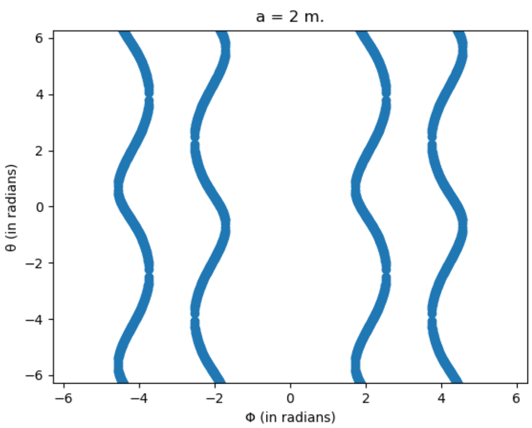
$B_x = d + c \cdot cos(\phi)$
$B_y = c \cdot sin(\phi)$

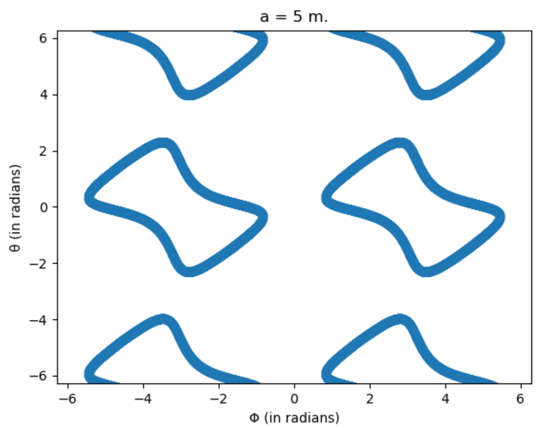
Next we get the value of γ using atan2: $\gamma = atan2(B_y, B_x)$

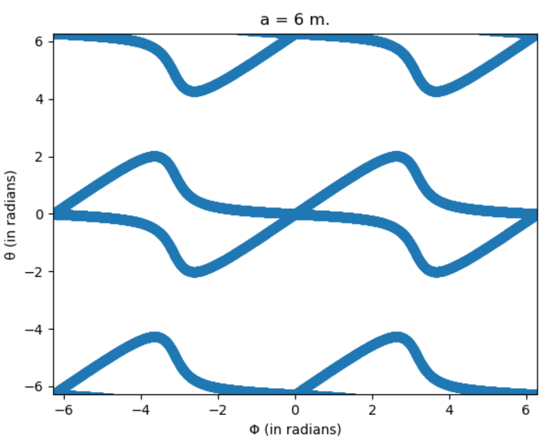
The value of r can be calculated using the coordinates of B: $r = \sqrt{B_x^2 + B_y^2}$.

Now with the value of r, the value of β can be calculated using the law of cosines (note that there are two possible values): $\beta = \pm cos^{-1}(\frac{a^2+r^2-b^2}{2ar})$

Since $\theta = \gamma + \beta$, then given ϕ we can find the value of θ. (Note $\theta + 2\pi$ and $\theta - 2\pi$ are also solutions). Using this method yields the plots shown below (the three cases for the value of a are shown below):







Problem 6 (Time Spent) - 4 points:

Approximately how much time did you spend on this homework? Any particular bottlenecks? This helps us better gauge how things are going and whether we should make any adjustments.

I spent about 5 hours on the homework. It was somewhat difficult to visualize the degrees of freedom pertaining to the bicycle question (problem 4), but other than that there weren't any particular bottlenecks in this set.