Homework #1
9/29/23
due Wednesday 10/4/23 11:59pm

We are starting the quarter with a shorter week and clearly haven't covered much material. But I would like us to settle into the normal weekly rhythm. Accordingly, this problem set is slightly shorter and only Problem 5 requires mathematical developments. So if you find yourself spending significant time, please take advantage of the office hours and/or post questions on Piazza.

For this and all other sets, please provide **enough explanation so we can follow your thinking**. If we can not understand your steps or process, we will not be able to assign full credit.

Problem 1 (DOFs of Objects) - 14 points:

To collect our thinking, how many degrees of freedom do the following objects have?

(a) a point (no volume) movable in 2D (planar) space
(b) a rigid body (fixed finite size in all directions) movable in 2D (planar) space
(c) a circle of variable radius (no line thickness) movable in 2D (planar) space

(d) a rigid body (fixed finite size in all directions) movable in 3D space
(e) a line segment (no thickness, fixed finite length) movable in 3D space
(f) a plane (no thickness, infinite size in other 2 axes) movable in 3D space
(g) a torus of variable radii (no surface thickness) movable in 3D space

Problem 2 (Towing in 2D Plane) - 16 points:

Your friend's car has broken down and you decide to help them move it to a repair shop. You consider the following two cases:

(a) You bring a trailer, load/secure their car on the trailer, and attach it to your hitch. That is, you tow a trailer carrying your friend's car.

(b) Without a trailer, you simply tow your friend's car. You attach a rope to your hitch and tie it firmly to the front of your friend's car (either at a tow point or securely fastened to one place on the bumper). The tension keeps the rope stretched to fixed length.

Everything moves on a flat surface without any vertical bouncing.

For both cases, how many degrees of freedom with the system of connected objects have? As always, please justify your answers.

Problem 3 (Human Hand) - 18 points:

The human hand is so amazing. How many DOFs are in your hand, not including the wrist?

Admittedly, this is a little tricky, both as your muscles may not be able to independently move each joint and as the ligaments holding the hand together are not perfectly rigid. When in doubt, relax the hand, grab a finger/thumb with the other hand, and externally push to see what moves. Consider movements of the bones, not skin stretching. And note movements may not be very far, but should be reasonably free/natural - no stretching ligaments or pulling joints apart!

Problem 4 (Riding a Stationary Bike) - 18 points:

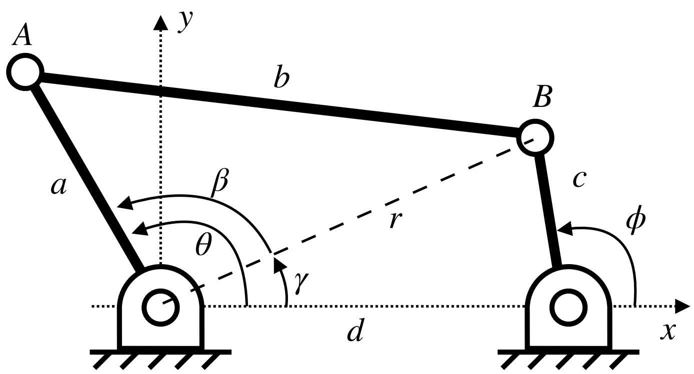
Imagine riding your favorite stationary bicycle (perhaps while working on this set?). Your feet are secured inside bike shoes (no toe wiggling) and clipped into the bike, so your feet are rigidly attached to the pedals. We will also ignore your head/arms/upper body and consider only your legs/ankles.

(a) If you sit comfortably but securely on the seat (hips/pelvis not moving), how many DOFs remain free to move in the system?

(b) If you want extra power and stand up on the pedals (hips/pelvis not attached to the seat), how many DOFS exist? Again ignore the upper body, even though you may be holding on.

Problem 5 (C-space of 4-bar Linkage) - 30 points:

You are given the 4-bar linkage



Please graph the configuration space of the linkage in the ϕ, θ plane, showing the range of $[-2\pi...2\pi]$ or $[-360°...360°]$ for both angles.

Hint: You may certainly solve the geometry however you like. But if you start with ϕ, compute the (x, y) location of B, you can focus on one triangle to determine r and γ. From r you can use the other triangle to get β. If this does not make sense, we will cover more such kinematics on Monday.

Remember there may be more than one solution for θ given ϕ or for ϕ given θ, beyond just adding 360°. And both angles may not be able to swing around fully, that is they may not reach all values in the requested range. Finally, you may want to solve this numerically, for example using Matlab.

Use the lengths b=6m, c=5m, d=7m. Repeat/graph for three cases, with

(a) $a = 2$m

(b) $a = 5$m

(c) $a = 6$m

Problem 6 (Time Spent) - 4 points:

Approximately how much time did you spend on this homework? Any particular bottlenecks? This helps us better gauge how things are going and whether we should make any adjustments.