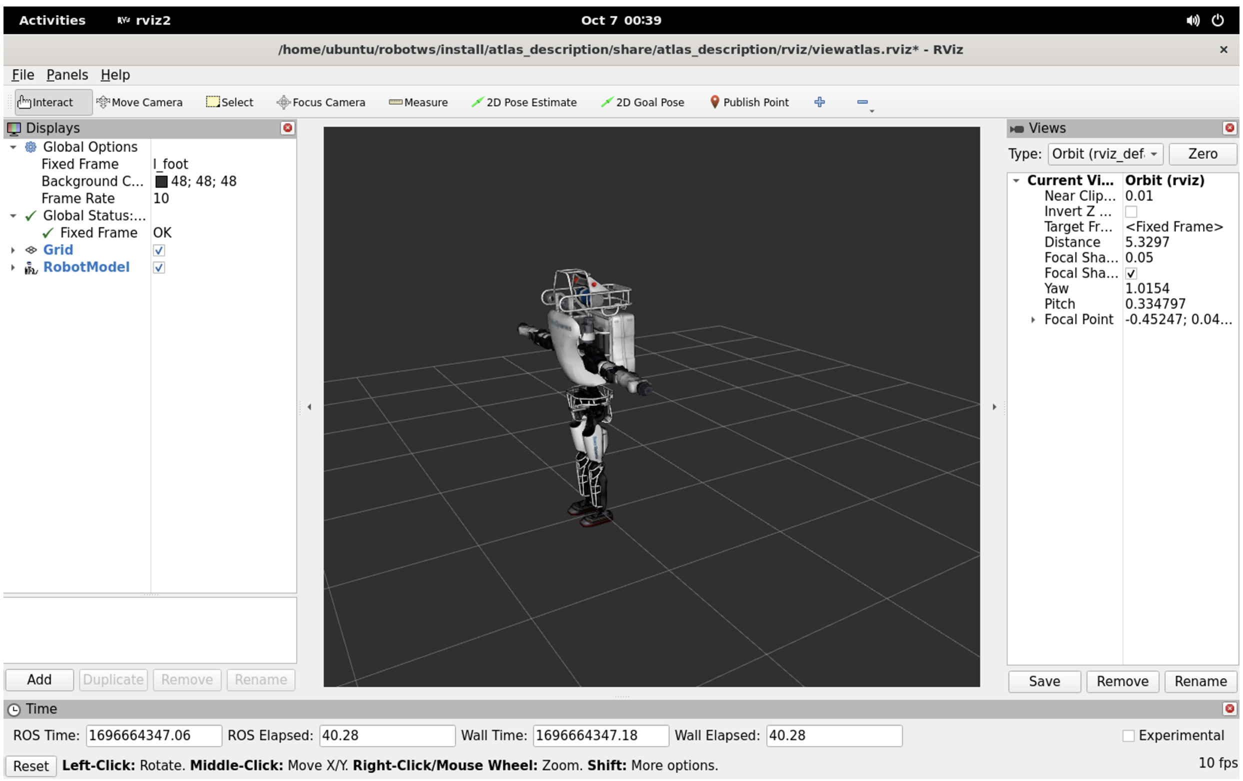
CS 133a **October 12, 2023**

Problem Set 2

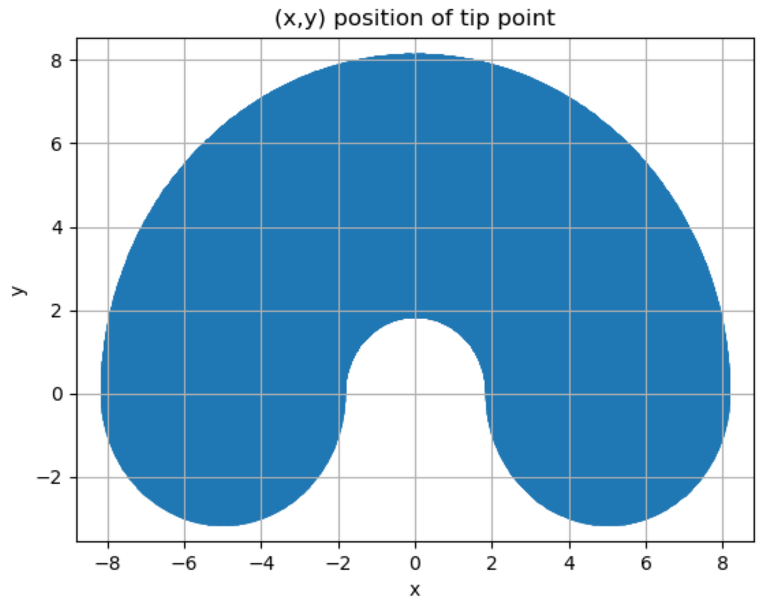
Problem 1 (Install Ubuntu 22.04 Jammy Jellyfish, ROS Humble Hawksbill) - 10 points



Problem 2 (Workspace of Planar 3R Robot) - 14 points

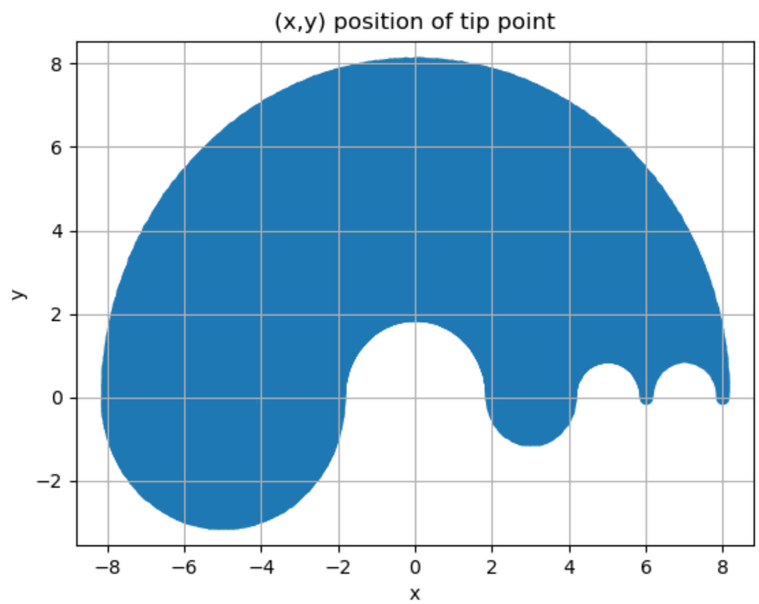
part a

$0 \leq \theta_1 \leq \pi$ with θ_2 unlimited and θ_3 unlimited

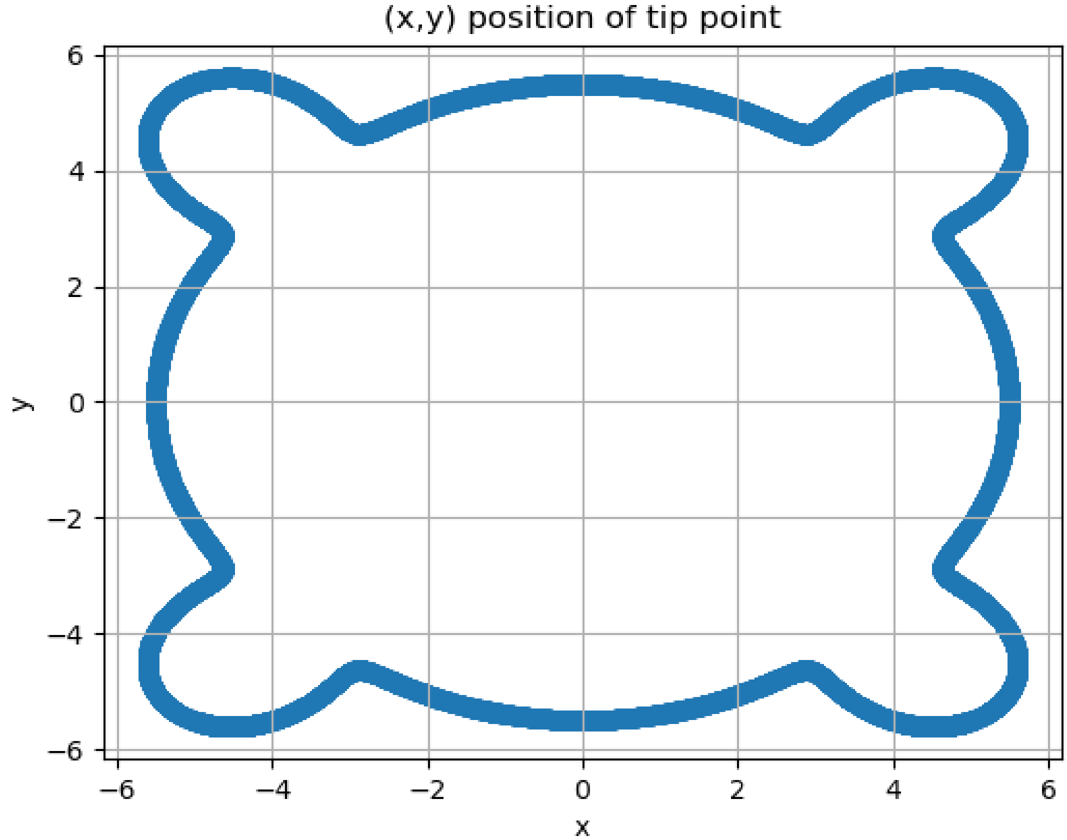


part b

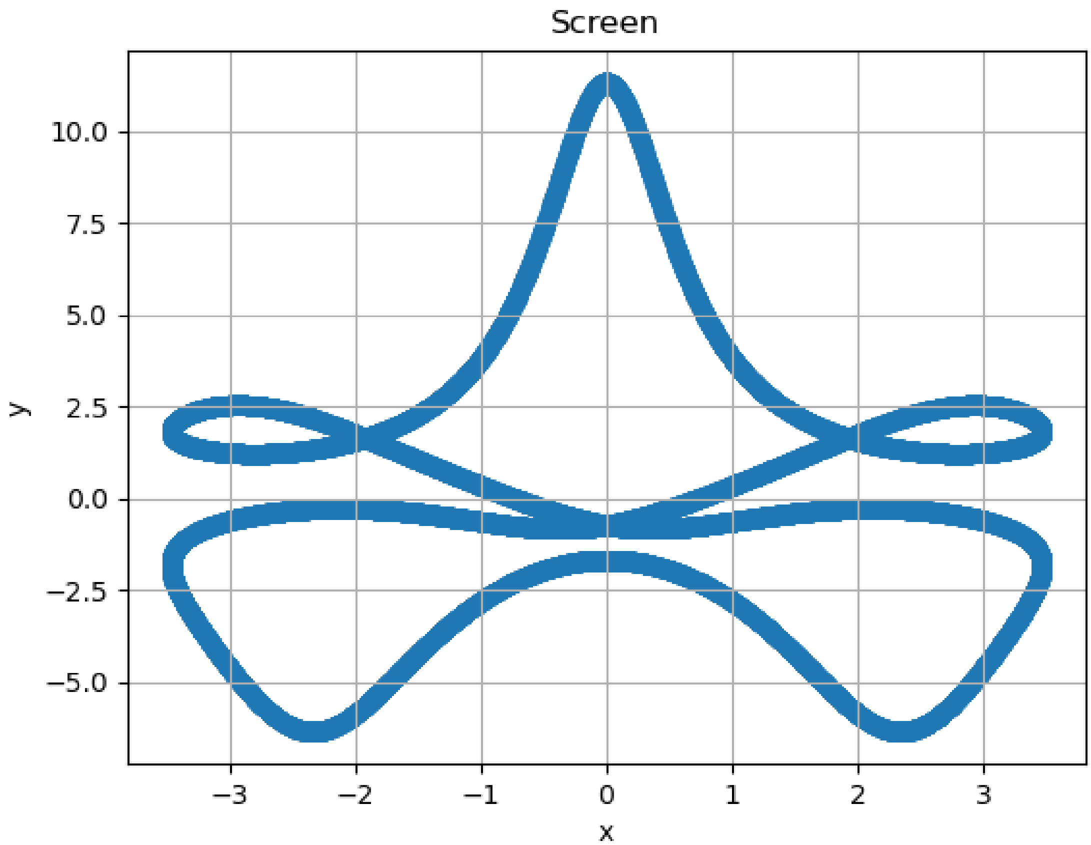
$0 \leq \theta_1 \leq \pi$ with $0 \leq \theta_2 \leq \pi$ and $0 \leq \theta_3 \leq \pi$ unlimited



Problem 3 (Tip Motion of Planar 3R Robot) - 14 points



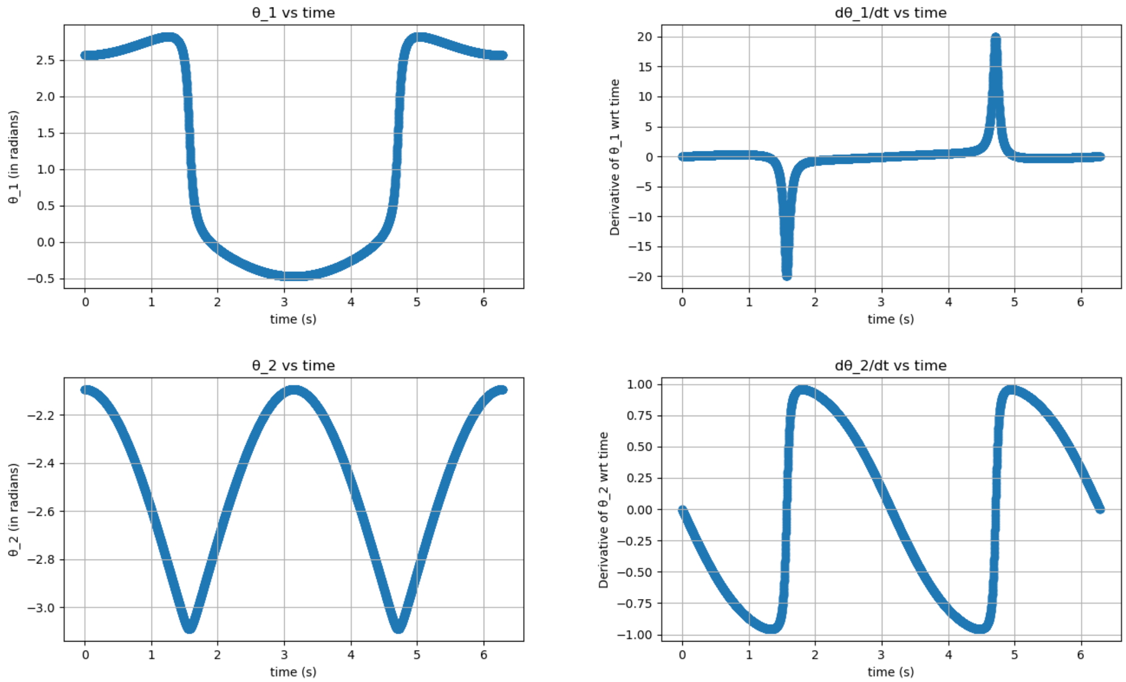
Problem 4 (Laser Pointer on 2DOF Pan/Tilt Gimbal) - 20 points



Problem 5 (Joint Velocities near Singularities) - 16 points:

part a and part b

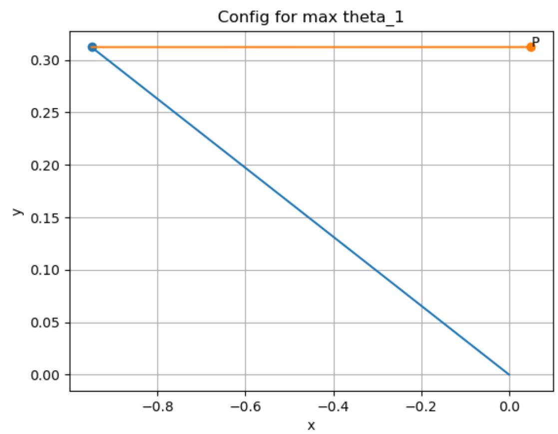
Part A and part b are shown below, the graphs on the right column are the derivatives of the graph to its left. Each is titled corresponding to the angle it represents over time θ_1 and θ_2.



part c

The max value of θ_1 is 2.824 radians with a corresponding $\theta_2 = -2.824$ radians

The robot configuration is shown below where the blue line corresponds to l_1 and the orange line corresponds to l_2.



In this case, the max is reached when the second link (the orange one above with length l_2) is parallel to the x axis. What characterizes this configuration is the the length of the links and the value of d (eg. if d is negative or positive). We know that $P = (d, y)$, and since the orange link is parallel to the x axis in this configuration then we have its joint position is $(d-1, y)$. Because this position is the end of the first link (the one with magnitude l_1), and since $l_1 = 1$ then the following must also be true:

$$1^2 = (d-1)^2 + y^2 \Rightarrow y = \pm\sqrt{1-(d-1)^2}$$

Given that $l_1 = l_2 = 1$, then for a positive d, we have that $y = \sqrt{1-(d-1)^2}$ would give us a greater value of θ_1. From the notes we have that:

$$\theta_2 = \pm cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$

Given that we must satisfy $\theta_2 < 0$ then

$$\theta_2 = -\left|cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)\right|$$

Plugging in for y and $l_1 = l_2 = 1$, we have that:

$$\theta_2 = -\left|cos^{-1}\left(\frac{d^2+(1-(d-1)^2)-2}{2}\right)\right|$$

$$\theta_1 = atan2(\sqrt{1-(d-1)^2},\ d) - atan2(sin(\theta_2), 1+cos(\theta_2))$$

part d

From the geometry (by the law of cosines) we have that (for the general case):

$$r^2 = x^2 + y^2$$

$$\theta_2 = \pm cos^{-1}\left(\frac{r^2 - l_1^2 - l_2^2}{2l_1l_2}\right)$$

Given $y = 0$, $x = d$, $l_1 = l_2 = 1$ we have that:

$$\theta_2 = \pm cos^{-1}\left(\frac{d^2 - 2}{2}\right)$$

Since $\theta_2 < 0$ for the motion described in the problem then we have that

$$\theta_2 = -\left|cos^{-1}\left(\frac{d^2 - 2}{2}\right)\right|$$

Now solving for θ_1 we get that:

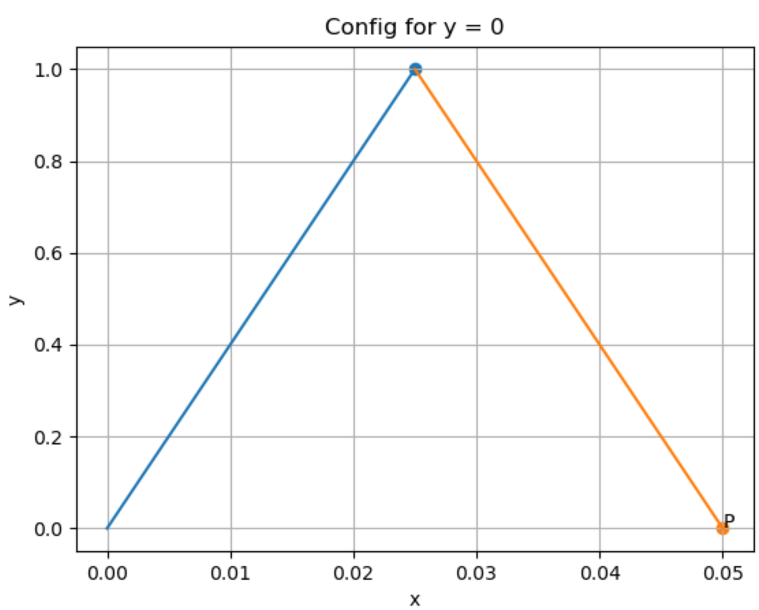
$$\theta_1 = atan2(y, x) - atan2(l_2sin(\theta_2), l_1 + l_2cos(\theta_2))$$

Given $y = 0$, $x = d$, $l_1 = l_2 = 1$ we have that:

$$\theta_1 = atan2(0, d) - atan2(sin(\theta_2), 1 + cos(\theta_2))$$

Therefore since θ_2 is dependent only on d, then θ_1 is also only dependent on d (as θ_1 only depends on θ_2). With $d = 0.05$ then we have that $\theta_2 = -3.0916$ radians and $\theta_1 = 1.5458$ radians.

The robot configuration is shown below where the blue line corresponds to l_1 and the orange line corresponds to l_2.



Problem 6 (Tip Position of 3DOF Robot) - 22 points:

part a

For simplicity let us focus on the yz plane, so set $\theta_{pan} = 0$. Now given θ_1 and θ_2 the yz coordinate of tip point $\vec{P'}$ can be retrieved. In particular, we would have the following:

$$P'_x = 0$$

$$P'_y = l_1 \cdot cos(\theta_1) + l_2 \cdot cos(\theta_1 + \theta_2)$$
$$P'_z = l_1 \cdot sin(\theta_1) + l_2 \cdot sin(\theta_1 + \theta_2)$$

Now for the general case (for any θ_{pan}), we can simply apply a rotation to the vector $\vec{P'}$ so that it rotates about the z axis to find the tip point $\vec{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Using a rotation matrix we have that:

$$\vec{P} = R_z(\theta_{pan})\vec{P'} = \begin{bmatrix} cos(\theta_{pan}) & -sin(\theta_{pan}) & 0 \\ sin(\theta_{pan}) & cos(\theta_{pan}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P'_x \\ P'_y \\ P'_z \end{bmatrix}$$

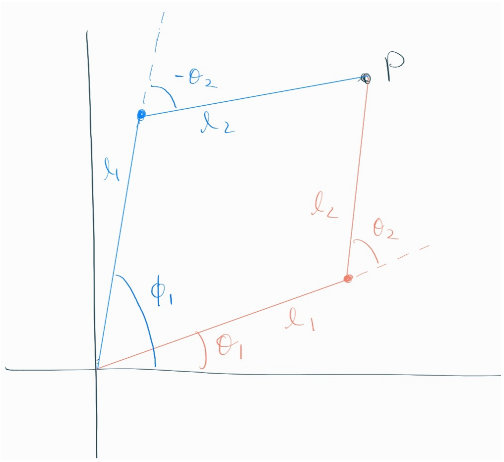
In other words we have:

$$x = P'_x \cdot cos(\theta_{pan}) - P'_y \cdot sin(\theta_{pan}) = -sin(\theta_{pan}) \cdot (l_1 \cdot cos(\theta_1) + l_2 \cdot cos(\theta_1 + \theta_2))$$

$$y = P'_x \cdot sin(\theta_{pan}) + P'_y \cdot cos(\theta_{pan}) = cos(\theta_{pan}) \cdot (l_1 \cdot cos(\theta_1) + l_2 \cdot cos(\theta_1 + \theta_2))$$
$$z = P'_z = l_1 \cdot sin(\theta_1) + l_2 \cdot sin(\theta_1 + \theta_2)$$

part b

Generally, there are 4 multiplicities in total. Let the red solution (shown below) have the configuration $(\theta_{pan}, \theta_1, \theta_2)$, then there is another corresponding configuration $(\theta_{pan} \pm \pi, \pi - \theta_1, -\theta_2)$ that reaches point P. Let the blue solution (shown below) have the configuration $(\theta_{pan}, \phi_1, -\theta_2)$ then there is another corresponding configuration $(\theta_{pan} \pm \pi, \pi - \phi_1, \theta_2)$ that reaches point P.
For the special case when $\theta_2 = k_1\pi$ and $\theta_1 \neq \pm\frac{\pi}{2} + 2k_2\pi$ where $k_1, k_2 \in \mathbb{Z}$ then there are only two solutions. For the last case where the links are along the z axis or in other words when $\theta_2 = k_1\pi$ and $\theta_1 = \pm\frac{\pi}{2} + 2k_2\pi$ where $k_1, k_2 \in \mathbb{Z}$ then θ_{pan} becomes arbitrary.



part c
Let $\vec{P} = (x, y, z)$ Then we have that (since θ_{pan} is the angle about the yz plane):

$$\theta_{pan} = atan2(-x, y)$$

Note that the distance between P and the z-axis is $\sqrt{x^2 + y^2}$,and the distance between P and the xy plane is simply z. Therefore we have (this follows from the geometry explained in the notes):

$$\theta_2 = \pm acos\left(\frac{(x^2 + y^2 + z^2) - l_1^2 - l_2^2}{2l_1 l_2}\right)$$

By geometry we also have that (this also follows from the notes):

$$\theta_1 = atan2(z, \sqrt{x^2 + y^2}) - atan2(l_2 \cdot sin(\theta_2), l_1 + l_2 \cdot cos(\theta_2))$$

Putting this all together, for $\vec{P} = (x, y, z)$ we have:

$$\theta_{pan} = atan2(-x, y) + 2k_1\pi$$

$$\theta_2 = \pm acos\left(\frac{(x^2 + y^2 + z^2) - l_1^2 - l_2^2}{2l_1 l_2}\right)$$

$$\theta_1 = atan2(z, \sqrt{x^2 + y^2}) - atan2(l_2 \cdot sin(\theta_2), l_1 + l_2 \cdot cos(\theta_2)) + 2k_3\pi$$

where $k_1, k_2, k_3 \in \mathbb{Z}$. As mentioned in problem b there are 4 solutions. The equations above provide us with 2 configurations of the form $(\theta_{pan}, \theta_1, \theta_2)$. For each configuration of the form $(\theta_{pan}, \theta_1, \theta_2)$, there is another solution $(\theta_{pan} \pm \pi, \pi - \theta_1, -\theta_2)$. Thus we have four solutions in total as described in part b.

Note that the equations above are all functions of the coordinates of P (with l_1 and l_2 given). θ_2 is a function of the xyz coordinates of P. Since θ_{pan} is a function of the x and y coordinates of P. Lastly, θ_1 is a function of θ_2, and since θ_2 is a function of the xyz coordinates of P, then so is θ_1.

Edge cases
Let $P = (x, y, z)$

The first edge case to consider is when the links are along the z axis. More specifically when the coordinates $x = y = 0$. In this case if $|P| = l_1 + l_2$ then $\theta_2 = \pm 2k_2\pi$, otherwise $\theta_2 = \pm(2k_2 - 1)\pi$ with $k_2 \in \mathbb{N}$. If $z < 0$ then $\theta_1 = -\frac{\pi}{2} + 2k_1\pi$, otherwise $\theta_1 = \frac{\pi}{2} + 2k_1\pi$ with $k_1 \in \mathbb{Z}$. Lastly, θ_{pan} becomes arbitrary and can be any value for this edge case.

The other edge case to consider is when $|P| = l_1 + l_2$ or $|P| = |l_1 - l_2|$and with either or both x and y being non-zero. This gives us two solutions. The first one being:

$$\theta_{pan} = atan2(-x, y) + 2k_1\pi$$

$$\theta_2 = 2k_2\pi$$

$$\theta_1 = atan2(z, \sqrt{x^2 + y^2}) + 2k_3\pi$$

And the second one being:

$$\theta_{pan} = atan2(-x, y) + \pi + 2k_1\pi$$

$$\theta_2 = 2k_2\pi$$

$$\theta_1 = \pi - atan2(z, \sqrt{x^2 + y^2}) + 2k_3\pi$$

where $k_1, k_2, k_3 \in \mathbb{Z}$ in both instances.

1 Problem 7 (Time Spent) - 4 points:

The set took me about 6 hours. Fortunately, I did not have any issues installing ROS on my mac. Though I know quite a few people had issues with that particular section.