Converting a Zero-Sphere-of-Influence Flyby to a Non-Zero-Sphere-of-Influence Flyby in EMTG

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Abstract

This document describes the method used by EMTG to convert a zero-sphere-of-influence flyby to a non-zero-sphere-of-influence flyby. EMTG implements this capability in the PyEMTG/HighFidelity Python scripts.

Contents

1 ZSOI Flyby to Periapse Position and Velocity Vectors

The principle is to take the known quantities from the feasible zero sphere of influence (ZSOI) flyby to produce initial guesses for quantities used to parameterize the non-ZSOI flyby. The known quantities are:

- $\boldsymbol{v}_\infty^-$
- $\boldsymbol{v}_\infty^+$
- r_p
- r_{SOI} (set by the user for the flyby body in the Evolutionary Mission Trajectory Generator (EMTG) universe file)

The above data may be converted into a position and velocity at periapse of the flyby in the following manner. A unit vector pointing in the direction of periapse is obtained by

$$\hat{\boldsymbol{r}}_p = \frac{\boldsymbol{v}_\infty^- - \boldsymbol{v}_\infty^+}{||\boldsymbol{v}_\infty^- - \boldsymbol{v}_\infty^+||} \tag{1}$$

The angular momentum direction is

$$\hat{\boldsymbol{h}} = \frac{\boldsymbol{v}_\infty^- \times \boldsymbol{v}_\infty^+}{||\boldsymbol{v}_\infty^- \times \boldsymbol{v}_\infty^+||} \tag{2}$$

The velocity direction at periapse is

$$\hat{\boldsymbol{v}}_p = \frac{\hat{\boldsymbol{h}} \times \hat{\boldsymbol{r}}_p}{||\hat{\boldsymbol{h}} \times \hat{\boldsymbol{r}}_p||} \tag{3}$$

From energy considerations, the velocity magnitude at periapse is

$$v_p = \left(\frac{2\mu}{r_p} + \boldsymbol{v}_\infty^{-T}\boldsymbol{v}_\infty^+\right)^{1/2} \tag{4}$$

With the direction and magnitude of position and velocity known, we finally have

$$\boldsymbol{r}_p = r_p \hat{\boldsymbol{r}}_p \tag{5}$$
$$\boldsymbol{v}_p = v_p \hat{\boldsymbol{v}}_p \tag{6}$$

The preceding procedure is implemented in EMTG in EphemerisPeggedFlybyOut::calculate_flyby_periapse_state(). The flyby state is output for an ephemeris-pegged flyby in the section of the .emtg output file for the phase following the flyby.

2 Periapse Position and Velocity Vectors to Decision Variable Guesses

The Python converter scripts have the task of taking data obtained from the .emtg and .emtgopt files and converting them to guesses for the state at the sphere of influence (SOI) boundaries, the state at periapse, and the time of flights (TOFs) between the SOI boundaries and periapse (incoming and outgoing).

The periapse position and velocity vectors, which are reported in the .emtg file, are converted to Keplerian orbital elements. The true anomalies at the incoming and outgoing SOI boundaries, as well as the times of flight between the SOI boundaries and periapse, are calculated assuming two-body motion.

$$H_{SOI} = \text{acosh}\left[\frac{1}{e}\left(\frac{r_{SOI}}{-a} + 1\right)\right] \tag{7}$$

$$N = e\sinh H_{SOI} - H_{SOI} \tag{8}$$

$$\Delta t_{SOI} = \frac{N}{\left[\frac{\mu}{-(a^3)}\right]^{1/2}} \tag{9}$$

$$\nu_{SOI} = 2\text{atan}\left[\sqrt{\frac{e+1}{e-1}}\tanh\left(\frac{H_{SOI}}{2}\right)\right] \tag{10}$$

2.1 Incoming SOI Boundary

$\boldsymbol{v}_\infty^-$ for the ZSOI flyby is obtained from the .emtgopt decision variables. The angular direction of $\boldsymbol{v}_\infty^-$ is

$$v_{RA} = \text{atan2}\left(\boldsymbol{v}_{\infty,y}^-, \boldsymbol{v}_{\infty,x}^-\right) \tag{11}$$

$$v_{DEC} = \text{asin}\left(\frac{\boldsymbol{v}_{\infty,z}^-}{v_\infty^-}\right) \tag{12}$$

The Cartesian position vector at the incoming SOI boundary (calculated from the Keplerian elements, based on the periapse state) is used to calculate the spherical coordinates at the incoming SOI boundary:

$$r_{RA} = \text{atan2}\left(\boldsymbol{r}_{SOI,y}, \boldsymbol{r}_{SOI,x}\right) \tag{13}$$

$$r_{DEC} = \text{asin}\left(\frac{\boldsymbol{r}_{SOI,z}}{r_{SOI}}\right) \tag{14}$$

The TOF for the journey from the SOI to periapse is taken from Eq. (9). The same amount of time is *subtracted* from the TOF of the journey that ends at the incoming SOI boundary in order to keep the overall mission TOF consistent.

These actions are performed in HighFidelityJourney.CreateInitialGuess().

2.2 Periapse

The initial guess for the spacecraft state at periapse is taken directly from the periapse state written in the .emtg file. This action is performed in HighFidelityFlybyIn.CreateInitialGuess().

2.3 Outgoing SOI Boundary

$\boldsymbol{v}_\infty^+$ for the ZSOI flyby is obtained from the .emtgopt decision variables. The angular direction of $\boldsymbol{v}_\infty^+$ is

$$v_{RA} = \text{atan2}\left(\boldsymbol{v}_{\infty,y}^+, \boldsymbol{v}_{\infty,x}^+\right) \tag{15}$$

$$v_{DEC} = \text{asin}\left(\frac{\boldsymbol{v}_{\infty,z}^+}{v_\infty^+}\right) \tag{16}$$

The Cartesian position vector at the outgoing SOI boundary (calculated from the Keplerian elements, based on the periapse state) is used to calculate the spherical coordinates at the outgoing SOI boundary:

$$r_{RA} = \text{atan2}\left(\boldsymbol{r}_{SOI,y}, \boldsymbol{r}_{SOI,x}\right) \tag{17}$$

$$r_{DEC} = \text{asin}\left(\frac{\boldsymbol{r}_{SOI,z}}{r_{SOI}}\right) \tag{18}$$

The TOF for the journey from periapse to the SOI is taken from Eq. (9). The same amount of time is *subtracted* from the TOF of the journey that starts at the outgoing SOI boundary in order to keep the overall mission TOF consistent.

These actions are performed in HighFidelityFlybyOut.CreateInitialGuess().