Applied 354 Lecture Notes

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1 Notes: 02 Aug

Calculate Divergance Theorem Given the vector field, where $z = 0$:

$$\underline{v} = 3y^2 z\underline{i} + 2y^2\underline{j} + yz^2\underline{k} \tag{1}$$

$$\underline{v} = 2y^2\underline{j} \tag{2}$$

The divergance theorem is given by:

$$\iint_A \nabla \cdot \underline{v} dA = \oint \underline{n} \cdot \underline{v} dr \tag{3}$$

Calculating the inside of the LHS:

$$\nabla \cdot \underline{v} = \underline{j}\frac{\partial}{\partial y} \cdot 2y^2\underline{j} = 4y \tag{4}$$

Therefor calculating LHS:

$$\iint_A \nabla \cdot \underline{v} \, dA = \int_2^3 \int_2^4 4y \, dxdy \tag{5}$$

$$\iint_A \nabla \cdot \underline{v} \, dA = \left| 8\frac{y^2}{2} \right|_2^3 = 20 \tag{6}$$

Calculating the RHS, using the graphic finding the normal vectors to each side. Only two terms are present as i dot j will be zero, so the verticals are zero.

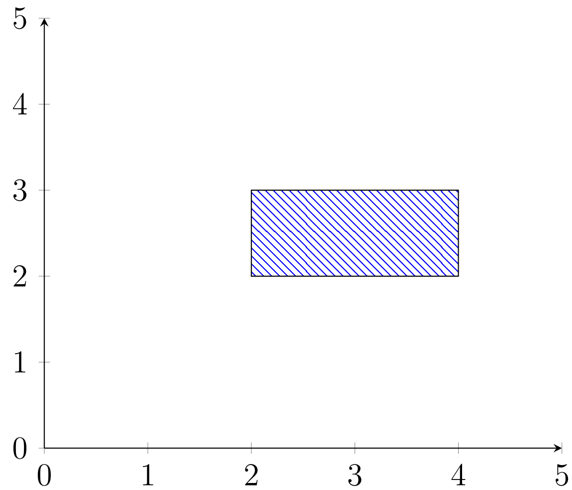
$$\oint \underline{n} \cdot \underline{v} dr = \int_2^4 -\underline{j} \cdot 2y^2 \underline{j} dx + \int_2^4 \underline{j} \cdot 2y^2 \underline{j} dx \tag{7}$$

Using Figure:1 replacing the y values from the graphic, 2 and 3 respectivly

$$= \int_2^4 -8dx + \int_2^4 18dx = 20 \tag{8}$$

Therefor RHS = LHS

Figure 1: Vector Field Figure



Calculate Stokes Integral Theorem for the velocity field $v = x\underline{j}$.
Stokes integral is given by:

$$\oint_{\partial S} \underline{v} \cdot dr = \iint_S \underline{n} \cdot \nabla \times \underline{v} \, dS \tag{9}$$

Calculating the LHS:

$$\oint_{\partial S} \underline{v} \cdot dr = \int (x\underline{j}) \cdot (dx \cdot \underline{i} + dy \cdot \underline{j}) \tag{10}$$

$$= \int xdy \tag{11}$$

The only non zero term is in the vertical because the dot product is 0 for non-equal unit vectors [1]

[1] Split integrals if more than one direction in velocity field

Based on Figure: 2 respect the rotations when picking the bounds for the integrals:

$$LHS = \int_2^3 xdy + \int_3^2 xdy \tag{12}$$
$$= \int_2^3 4dy + \int_3^2 2dy \tag{13}$$
$$= 2 \tag{14}$$

Calculating the RHS: The normal vector using the right hand rule, based on the rotation is $\underline{k}$

$$\iint_S \underline{n} \cdot \nabla \times \underline{v}\, dS = \iint \underline{k} \cdot \underline{k}\, dS \tag{15}$$
$$= \int_2^3 \int_2^4 dxdy \tag{16}$$
$$= 2 \tag{17}$$

The inside cross product between the Del and V:

$$\nabla \times \underline{v} = \underline{i}\frac{\partial}{\partial x} \times (x\underline{j}) = \underline{k} \tag{18}$$

The partial cancels the x term and i cross j by convention is k. Only the x partial is included as all the other partials would be zero.
Therefor LHS = RHS

Figure 2, has arrows indicating the force and magnitude of different positions, they indicate that the rotation is anti-clockwise

Figure 2: Stokes Integral Figure