Given the velocity field: $\underline{v} = xt\underline{i} + \underline{j}$. Determine:

1. The stream lines
2. The path line (Parametric and explicit)
3. The streak lines (Parametric and explicit)

0.1 Stream lines

$$\frac{dx}{u} = \frac{dy}{v} => \frac{dx}{xt} = \frac{dy}{1}$$

$$\int \frac{dx}{x} = \int tdy$$
$$ln(x) = ty + c_1$$
$$x = e^{ty+c_1} = C_2 e^{ty}$$

0.2 Path Lines

Parametric form:

$$\frac{dX}{dt} = Xt$$
$$\int \frac{dX}{x} = \int tdt$$
$$ln(X) = \frac{t^2}{2} + C_3$$
$$X = C_4 e^{\frac{t^2}{2}}$$

At $t = 0$, $X = X_0$. Therefor $C_4 = X_0$

$$X = X_0 e^{\frac{t^2}{2}} \tag{1}$$

Calculating the Y component:

$$\frac{dY}{dt} = 1$$
$$\int dY = \int dt$$
$$Y = t + C_5$$

At $t = 0$, $Y = Y_0$. Therefor $C_5 = Y_0$

$$Y = t + Y_0 \tag{2}$$

Explicit Form: Eliminate t, $t = Y - Y_0$

$$X = X_0 e^{0.5(Y-Y_0)^2}$$

$$\ln(\frac{X}{X_0}) = \frac{1}{2}(Y - Y_0)^2$$

$$Y = Y_0 + \sqrt{2\ln(\frac{X}{X_0})} \tag{3}$$

0.3 Streak lines

Parametric Form: Calculating the X component:

$$X = X_0 e^{0.5t^2} \tag{4}$$

$$X^* = X_0 e^{0.5\tau^2} \tag{5}$$

$$X_0 = X^* e^{-0.5\tau^2} \tag{6}$$

Substitute (**??**) into (**??**). Resulting in:

$$X = X^* e^{0.5(t^2-\tau^2)} \tag{7}$$

Calculating the Y component:

$$Y = Y_0 + t \tag{8}$$

$$Y^* = Y_0 + \tau \tag{9}$$

$$Y_0 = Y^* - \tau \tag{10}$$

Substitute (**??**) into (**??**). Resulting in:

$$Y = Y^* + t - \tau \tag{11}$$

Explicit Form: Using (**??**) to eliminate τ:

$$\ln(\frac{X}{X^*}) = \frac{1}{2}(t^2 - \tau^2)$$

$$\tau = \sqrt{t^2 - 2\ln(\frac{X}{X^*})} \tag{12}$$

Substitute (**??**) into (**??**), which results in the explicit form:

$$Y = Y^* + t - \sqrt{t^2 - 2\ln(\frac{X}{X^*})} \tag{13}$$