№1

$$(y-x)\,dx + (y+x)\,dy = 0$$

$$\frac{dy}{dx} = -\frac{y-x}{y+x} = -\frac{\frac{y}{x}-1}{\frac{y}{x}+1}$$

$$\frac{y}{x} = t \Rightarrow y' = t + xt'$$

$$t + xt' = \frac{1-t}{1+t}$$

$$\frac{x\,dt}{dx} = xt' = \frac{1-t-t-t^2}{1+t} = \frac{1-2t-t^2}{1+t}$$

$$\int \frac{dx}{x} = \int \frac{(1+t)\,dt}{(1-2t-t^2)} = \frac{(1+t)\,dt}{((t+1)^2-2)}$$

$$\ln x = -\frac{1}{2}\ln|-t^2-2t+1| + \ln C$$

$$x = \frac{C}{\sqrt{|\ln|-t^2-2t+1|}} = \frac{C}{\sqrt{|\ln|-(\frac{y}{x})^2-\frac{2y}{x}+1|}}$$

Лучше

$$-\frac{1}{2}\ln|-t^2-2t+1| = \ln x + \ln C$$

$$\ln|-t^2-2t+1| = -2\ln x - 2\ln C$$

$$-t^2-2t+1 = \frac{1}{x^2C^2} - 1$$

$$t^2+2t-\left(1-\frac{1}{x^2C^2}\right) = 0$$

$$t = \frac{-2+\sqrt{4+4+\frac{4}{x^2C^2}}}{2} = -1+\sqrt{1+\frac{1}{x^2C^2}}$$

$$t = -1-\sqrt{1-\frac{1}{x^2C^2}}$$

$$y = -x \pm x\sqrt{1-\frac{1}{x^2C^2}}$$

Ответ: 0,5

№2

$y' \cos(x) - y \sin(x) = 2x \qquad y(0)=0$

$y' = \frac{2x}{\cos(x)} + \frac{y \sin(x)}{\cos(x)}$ — ЛНДУ $\qquad \cos(x)=0$, $x=\frac{\pi}{2}$

$\frac{dy}{dx} = \frac{y \sin(x)}{\cos(x)}$

$\int \frac{dy}{y} = \int \frac{dx \sin(x)}{\cos(x)}$

$\int \frac{dy}{y} = -\int \frac{d\cos(x)}{\cos(x)}$

$\ln|y| = -\ln|\cos(x)| + C$

$y = \frac{C}{\cos(x)}$ — общее решение

$y' = \frac{C' \cdot \cos(x) + C \sin(x)}{\cos^2(x)}$

$\frac{C' \cdot \cos(x)}{\cos(x)} + \frac{C \sin(x)}{\cos(x} - \frac{C \sin(x)}{\cos(x)} = 2x$

$C' = 2x$

$C = \int 2x\,dx = x^2 + C_1$

$y = \frac{x^2 + C_1}{\cos(x)} \qquad y(0)=0 \qquad 0 = \frac{0 + C_1}{\cos(0)} \Rightarrow C_1 = 0$

$y(2\pi) = \frac{4\pi^2}{2} = 4\pi^2$

Ответ: 4

№ 3

$$4(y')^2 - 9x = 0$$

$$] \; y' = p = \frac{dy}{dx}$$

$$(y')^2 = \frac{9}{4}x$$

$$y' = \pm \frac{3}{2}\sqrt{x}$$

$$\frac{dy}{dx} = \pm \frac{3}{2}\sqrt{x}$$

$$dy = \pm \frac{3}{2}\sqrt{x}\,dx$$

$$y = \int \pm \frac{3}{2}\sqrt{x}\,dx + C_1$$

$$y = \pm \sqrt{x^3} + C_1$$

$$y = \pm \sqrt{x^3} + 0 \cdot x^2 + C_1$$

Ответ: 0

✓4

$y''(x+2)^5 = 1 \qquad y(-1) = \frac{1}{12} \qquad y'(-1) = \frac{-1}{4}$

$\frac{d^2y}{dx^2} = \frac{1}{(x+2)^5}$

$\frac{d^2y}{dx^2} = (x+2)^{-5}$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = (x+2)^{-5}$

$\int 1 \cdot d\left(\frac{dy}{dx}\right) = \int (x+2)^{-5}dx + C_1$

$\frac{dy}{dx} = \frac{-1}{4}(x+2)^{-4} + C_1$

$\frac{dy}{dx}(-1) = \frac{-1}{4} + C_1 = \frac{-1}{4} \quad |\Rightarrow C_1 = 0$

$\int dy = \frac{-1}{4}\int (x+2)^{-4}dx + \int C_1\,dx + C_2$

$y(x) = \frac{1}{12}(x+2)^{-3} + C_1 x + C_2$

$y(-1) = \frac{1}{12} + C_2 = \frac{1}{12} \quad |\Rightarrow C_2 = 0$

$y(x) = \frac{1}{12}(x+2)^{-3}$

$\frac{1}{12}(x+2)^{-3} = \frac{1}{96}$

$\frac{1}{(x+2)^3} = \frac{12}{96}$

$\frac{1}{(x+2)^3} = \frac{1}{8}$

$x + 2 = 2$

$x = 0$

Ответ: 0

5

$y'' - y' = e^{2x} \cdot \cos e^x$

$t = y'$

$t' - t = e^{2x} \cos e^x$

$t = C_1 \cdot e^x$

$C_1' = e^x \cdot \cos e^x$

$\Downarrow$

$C_1 = \sin(e^x) + C_2$

$t = C_1 e^x + e^x \sin e^x$

$\Downarrow$

$y = C_2 + C_1 \cdot e^x - \cos(e^x)$

Ответ: -1

№6

$$\left\{\begin{aligned} \lambda_1 &= -1 \Rightarrow y_1 = e^{-x} \\ \lambda_2 &= -1 \Rightarrow y_2 = x \cdot e^{-x} \end{aligned}\right.$$

$y'' + 2y' + y = e^{-x}(3-5x)$

$f(x) = e^{-x}(3-5x)$

$y_{o.o} = C_1 e^{-x} + C_2 \cdot e^{x} \cdot x$

$y_1 = x^2(Ae^{-x} + Be^{-x}x)$

$y_1' = -x^2 \cdot A \cdot e^{-x} + 2A \cdot e^{-x} - Be^{-x} \cdot x^3 + 3B \cdot e^{-x} \cdot x^2$

$y_1'' = A2e^{x} + e^{-x}x^2 A - 4e^{-x}xA + Be^{-x}x^3 - 6e^{-x}x^2 B + B6e^{-x}x$

Тогда:

$2Ae^{-x} + 6Be^{-x}x = 3e^{-x} - 5e^{-x}x$

$A = 3/2$

$B = -5$

$y_1(x) = \frac{-5}{6}e^{-x}x^3 + \frac{3}{2}e^{-x}x^2$

$y_{o.н} = e^{-x}\left(\frac{-5}{6}x^3 + \frac{3}{2}x^2 + C_2 x + C_1\right)$

№ 7

$$\begin{cases} \frac{dx}{dt} = -x + y + z & x(0) = 1 \\ \frac{dy}{dt} = x - y + z & y(0) = 1 \\ \frac{dz}{dt} = x + y - z & z(0) = 1 \end{cases}$$

$$A = \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$\begin{vmatrix} -1-\lambda & 1 & 1 \\ 1 & -1-\lambda & 1 \\ 1 & 1 & -1-\lambda \end{vmatrix} = 0$$

$$-\lambda^3 - 3\lambda^2 + 4 = 0$$

$$\lambda_1 = 1$$

$$\lambda_{2,3} = -2$$

1. $\lambda = \lambda_1 = 1$ $\qquad (A - \lambda E) \cdot \Gamma_1 = 0$

$$\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$\begin{cases} -2x + y + z = 0 \\ -1{,}5y + 1{,}5z = 0 \end{cases}$$

$y = z$

$x = z$ $\qquad \Gamma_1^0(1) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$z = ?$

$$\begin{cases} x^{(1)} = e^{1t} \\ y^{(1)} = e^{1t} \\ z^{(1)} = e^{1t} \end{cases}$$

$\exists \ \lambda = -2$ — кратность 2

$$\begin{cases} x = (A_1 t + A_2) e^{-2t} \\ y = (B_1 t + B_2) e^{-2t} \\ z = (C_1 t + C_2) e^{-2t} \end{cases}$$

$$\begin{cases} A_1 \cdot \cancel{e^{-2t}} - 2(A_1 t + A_2)\cancel{e^{-2t}} = -(A_1 t + A_2)\cdot\cancel{e^{-2t}} + (B_1 t + B_2)\cancel{e^{-2t}} + \\ + (C_1 t + C_2)\cancel{e^{-2t}} \\ B_1 \cancel{e^{-2t}} - 2(B_1 + B_2)\cancel{e^{-2t}} = (A_1 t + A_2)\cancel{e^{-2t}} - (B_1 t + B_2)\cancel{e^{-2t}} + (C_1 t + C_2)\cancel{e^{-2t}} \\ C_1 \cancel{e^{-2t}} - 2(C_1 t + C_2)\cancel{e^{-2t}} = (A_1 t + A_2)\cancel{e^{-2t}} + (B_1 t + B_2)\cancel{e^{-2t}} - (C_1 t + C_2)\cancel{e^{-2t}} \end{cases}$$

$$t^0 \begin{cases} A_1 - 2A_2 = -A_2 + B_2 + C_2 \\ B_1 - 2B_2 = A_2 - B_2 + C_2 \\ C_1 - 2C_2 = A_2 + B_2 - C_2 \end{cases}$$

$$t^1 \begin{cases} -2A_1 = -A_1 + B_1 + C_1 \\ -2B_1 = A_1 - B_1 + C_1 \\ -2C_1 = A_1 + B_1 - C_1 \end{cases}$$

$$\begin{cases} A_1 + B_1 + C_1 = 0 \\ A_1 + B_1 + C_1 = 0 \\ A_1 + B_1 + C_1 = 0 \end{cases}$$

$$\begin{cases} A_2 + B_2 + C_2 = A_1 \\ A_2 + B_2 + C_2 = B_1 \\ A_2 + B_2 + C_2 = B_1 \end{cases} \Bigg| \Rightarrow A_2 + B_2 + C_2 = 0$$

$$\begin{cases} A_1 - B_1 - C_1 = 0 \\ A_2 + B_2 + C_2 = 0 \end{cases} \qquad \begin{cases} A_2 = -C_1 - C_2 \\ B_2 = C_2 \\ C_2 = C_1 \end{cases}$$

$$x^{\lambda}(t) = \begin{pmatrix} -C_1 - C_2 \\ C_1 \\ C_2 \end{pmatrix} \cdot e^{-2t}$$

рассмотрим α и β - const

$$x(t) = \alpha e^{t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta e^{-2t} \begin{pmatrix} -C_1 - C_2 \\ C_1 \\ C_2 \end{pmatrix}$$

рассмотрим начальные усл

$$\begin{cases} \alpha e^{0} \cdot 1 + \beta(-C_1 - C_2) = 1 \\ \alpha \cdot e^{0} \cdot 1 + \beta \cdot C_1 = 1 \\ \alpha + \beta C_2 = 1 \end{cases}$$

$3\alpha = 3 \quad \Rightarrow \alpha = 1$

$$\begin{cases} \beta(-C_1 - C_2) = 0 \\ \beta C_1 = 0 \\ \beta C_2 = 0 \end{cases}$$

$x(t) = e^{t}$

$y(t) = e^{t}$

$z(t) = e^{t}$

$x(7) = e^{7}$

$y(7) = e^{7}$

$z(7) = e^{7}$

$\ln(3 \cdot e^{7}) - \ln(3) = 7$

Ответ: 7