Вариант 12 Патугин В.М (Р3214)

№5

$f(x) = x^3$; $[-\pi;\pi]$

$f(-x) = (-x)^3 = -x^3 \Rightarrow$ ф-я нечетная, разложим по синусам

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x^3 \sin nx\,dx = \begin{vmatrix} u = x^3 & du = 3x^2dx \\ dv = \sin nx\,dx & v = \frac{-1}{n}\cos nx \end{vmatrix} =$$

$$= \frac{2}{\pi}\cdot\frac{-1}{n}x^3\cos nx \Big|_0^{\pi} + \frac{6}{\pi n}\int_0^{\pi} x^2\cos nx\,dx =$$

$$= \begin{vmatrix} u = x^2 & du = 2x\,dx \\ dv = \cos nx\,dx & v = \frac{1}{n}\sin nx \end{vmatrix} = \frac{-2}{\pi n}\pi^3\cdot\cos\pi n +$$

$$+ \frac{6}{\pi n}\cdot\frac{1}{n}x^2\cdot\sin nx \Big|_0^{\pi} - \frac{6}{\pi n}\cdot\frac{2}{n}\int x\sin nx\,dx =$$

$$= \begin{vmatrix} u = x & du = dx \\ dv = \sin nx\,dx & v = \frac{-1}{n}\cos nx \end{vmatrix} = \frac{-2\pi^2}{n}\cdot(-1)^n -$$

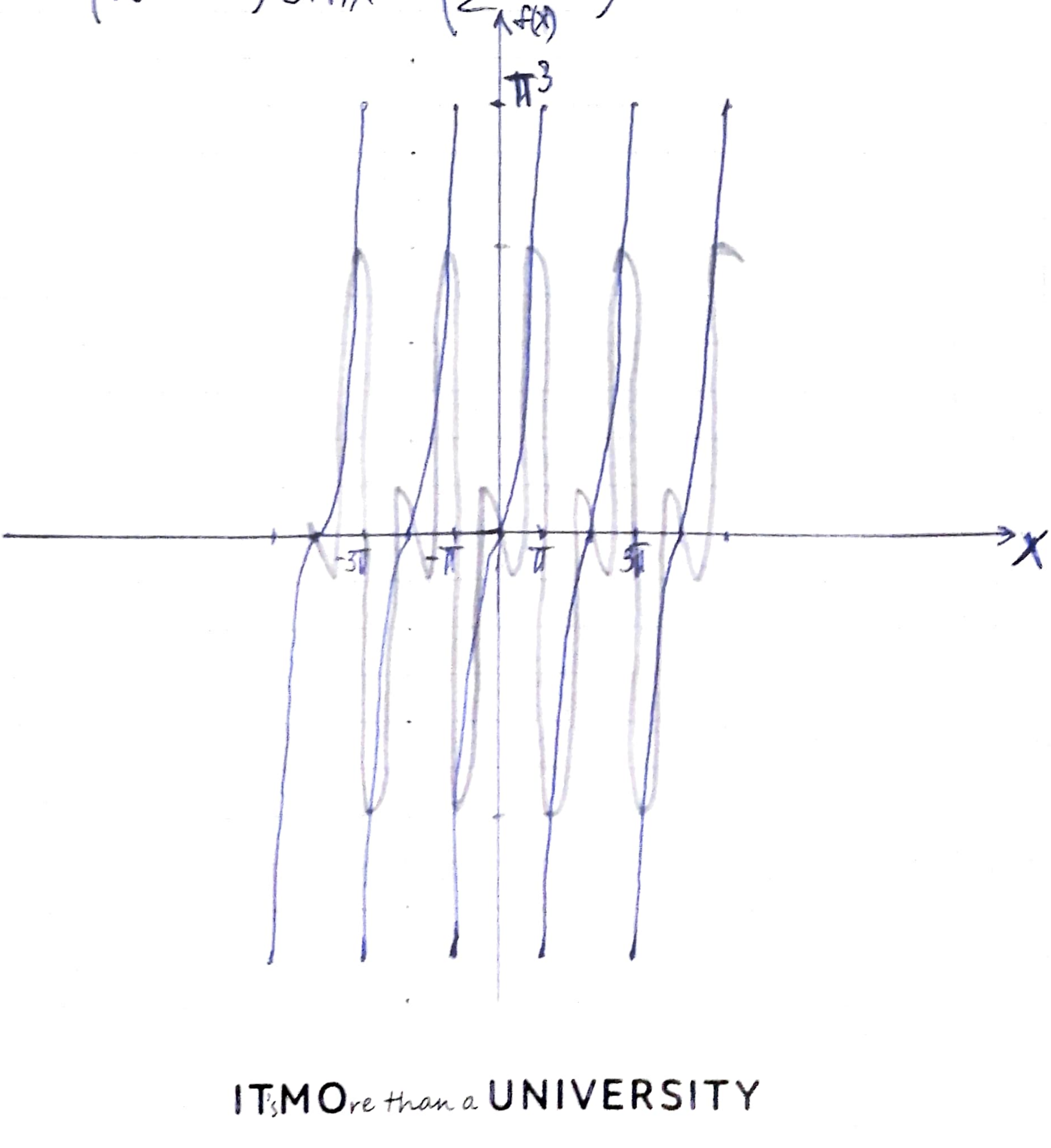
$$-\frac{12}{\pi n^2}\cdot\frac{-1}{n}x\cos nx\Big|_0^{\pi}-\frac{12}{\pi n^2}\cdot\frac{1}{n}\int\cos nx\,dx=$$

$$=\frac{-2\pi^2}{n}\cdot(-1)^n+\frac{12}{\pi n^3}\cdot\pi\cdot\cos\pi n-\frac{12}{\pi n^3}\cdot\frac{1}{n}\sin nx\Big|_0^{\pi}=$$

$$=\frac{2\pi^2\cdot(-1)^{n-1}}{n}+\frac{12\cdot(-1)^n}{n^3}$$

$$f(x)=\sum_{n=1}^{\infty}\left(\frac{12}{n^3}-\frac{2\pi^2}{n}\right)\cdot(-1)^n\cdot\sin nx$$

$$S_2=-(12-2\pi^2)\sin x+\left(\frac{3}{2}-\pi^2\right)\sin 2x$$



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