Math1510 Mid Semester Test, 2021 - **Practice Paper**

There are 6 questions, each worth 5 marks. Time: 90 min writing + 20 min upload. Clearly identify each question. Include all working, and your name/student number. If you get stuck on one question, go on to others and come back to it later. When finished, scan/photograph, concatenate with "cheat sheet" and upload a single file.

1. (a) Simplify: $\{1,2,3,4,5\} \cap \{5,6,7,8,9\}$

   (b) Let $A = \mathbb{Z} \setminus \{0\}$ and let $B = \{0\}$. Simplify $A \cup B$.

   (c) What is 3×4, modulo 5?

   (d) Let $f : \{-1,0,1\} \to \{0,1\}$ by the rule $f(x) = x^2$. Why is f not injective?

   (e) Let $g : \{0,1\} \to \{1,2,3\}$ by the rule $g(x) = x+1$. Why is g not surjective?

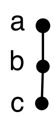
2. Let p_1 be the premise: If there is sun and water, flowers will grow. Let p_2 be the premise: There is sun and water. Let c be the conclusion: Flowers will grow. Prove that the argument with premises p_1 and p_2 and conclusion c is valid.

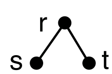
3. Consider the graph G with adjacency matrix $A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

   (a) Draw G

   (b) Is G connected? *Why/why not?*

   (c) Does G have any bridges or cut vertices? *Why/why not?*

4. The graphs 

and 

are isomorphic as trees but not as rooted trees.

   Prove the isomorphism as trees by stating an explicit isomorphism between the vertices and checking edge relations are preserved. Prove the non-isomorphism as rooted trees by using an invariant.

5. Draw a graph which is isomorphic to K_5 and label its vertices. Give an example of (i) a Hamiltonian cycle on your graph, and (ii) an Eulerian cycle on your graph.

6. A *perfect binary tree*, T_h, of height h, is a complete binary tree in which all leaves are on level h (i.e. the leaves are all at the deepest level in the tree). Let P_h be the claim: "A perfect binary tree of height h has 2^h leaves." Check this claim for the first three cases by drawing graphs, then prove it in general using Induction.