Research Progress Report

Yuchen Sun

Ocean University of China

January 2, 2024

Table of Contents

High Order Fully Actuated Approach

Consider this strict feedback nonlinear system:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + G_1 x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + G_2 x_3 \\ \quad \vdots \\ \dot{x}_{n-1} = f_{n-1}(x_1, \ldots, x_{n-1}) + G_{n-1} x_n \\ \dot{x}_n = f_n(x_1, \ldots, x_n) + G_n u \end{cases}$$

This strict feedback system can be transformed into the HOFA form:

$$x_1^{(n)} - \sum_{i=1}^{n} (G_1 G_2 \cdots G_{i-1}) f_i^{(n-i)} [x_1, h_2(\cdot), h_3(\cdot), \cdots, h_i(\cdot)] = (G_1 G_2 \cdots G_n) u$$

High Order Fully Actuated Approach

Let's make a simple deduction:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + G_1 x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + G_2 x_3 \\ \dot{x}_3 = f_2(x_1, x_2, x_3) + G_3 u \end{cases}$$

High Order Fully Actuated Approach

This is a text in first column.

$$E = mc^2$$

- First item
- Second item

This text will be in the second column and on a second tought this is a nice looking layout in some cases.

Table of Contents

Riemann Hypothesis

In mathematics, the Riemann hypothesis is a conjecture that the Riemann zeta function has its zeros only at the negative even integers and complex numbers with real part $\frac{1}{2}$. Many consider it to be the most important unsolved problem in pure mathematics. It is of great interest in number theory because it implies results about the distribution of prime numbers. It was proposed by Bernhard Riemann (1859), after whom it is named.

Riemann Hypothesis

Riemann ζ function

$$\zeta(s) = -\frac{\Gamma(1-s)}{2\pi i}\int_C \frac{(-z)^{s-1}}{e^z-1}\mathrm{d}z$$

The real part of every nontrivial zero of the Riemann zeta function is $\frac{1}{2}$

Basic Blocks

Standard Block

This is a standard block.

Alert Message

This block presents alert message.

An example of typesetting tool

Example: MS Word, LaTeX

Mathematical Environment Blocks

Definition

This is a definition.

Theorem

This is a theorem.

Lemma

This is a proof idea.

Mathematical Environment Blocks-Continued

Proof.

This is a proof. $\square$

Corollary

This is a corollary

Example

This is an example

Time-Varying Model 1: Duan 5[1]

This paper considers the control of the following uncertain HOFA system:

$$x^{(n)} = f(x^{0\sim n-1}) + \Delta f(x^{0\sim n-1}) + H^T(x^{0\sim n-1})\theta + L(x^{0\sim n-1})u$$

where $x, u \in \mathbb{R}^r$ are the state vector and the control input vector, respectively,
$f(x^{0\sim n-1}) \in \mathbb{R}^r$ is a continuous vector function,
$H(x^{0\sim n-1}) \in \mathbb{R}^{m\times r}$ and $L(x^{0\sim n-1}) \in \mathbb{R}^{r\times r}$ are two continuous matrix functions, and
$L(x^{0\sim n-1})$ satisfies the following fully actuated assumption:
Assumption A1: $\det L(x^{0\sim n-1}) \neq 0, \forall x(i) \in \mathbb{R}^r, i = 0, 1, \ldots, n-1.$
Furthermore, $\Delta f(x^{0\sim n-1}) \in \mathbb{R}^r$ is an uncertain nonlinearity of the system satisfying the following assumption:
Assumption A2: There exists a non-negative continuous scalar function $\rho(x^{0\sim n-1})$, such that

$$\|\Delta f(x^{0\sim n-1})\| \leq \rho(x^{0\sim n-1}).$$

While $\theta = \theta(t) \in \mathbb{R}^m$ is an unknown time-varying parameter vector with a pre-estimate $\hat{\theta}_0 = \hat{\theta}_0(t) \in \mathbb{R}^m$, and satisfies
Assumption A3: $\|\theta - \hat{\theta}_0\| \leq \delta_0$, $\|\dot{\theta} - \dot{\hat{\theta}}_0\| \leq \delta_1$, $\forall t \geq 0$, with δ_0 and δ_1 being two non-negative real numbers.

[1] **Duan G. High-order fully actuated system approaches: Part V. Robust adaptive control[J]. International Journal of Systems 2021, 52(10): 2129-2143..**

Time-Varying Model 2: Time-Varying Delay[2]

Consider the n-th order fully actuated discrete-time nonlinear system with time-varying input delay described as

$$\begin{cases} y(t+1) = f\left(y^{(n-1)}(t), w^{(m-1)}(t-1)\right) + g\left(y^{(n-1)}(t), w^{(m-1)}(t-1), w(t)\right), \\ w(t) = u(t-d_t), \end{cases}$$

where

$$y^{[n-1]}(t) = (y(t), y(t-1), \ldots, y(t-n+1))$$
$$w^{[m-1]}(t-1) = (w(t-1), w(t-2), \ldots, w(t-m))$$

the initial conditions of the system are given by $y(t) = \phi(t)$, $u(t) = \psi(t)$, $t \leq 0$, $y(t) \in \mathbb{R}^p$ is the output vector, $u(t) \in \mathbb{R}^p$ the control input vector, $w(t) \in \mathbb{R}^p$ the intermediate variable vector, $d_t \in \mathbb{R}$ the time-varying input delay (an integer), $f(\cdot) \in \mathbb{R}^p$ and $g(\cdot) \in \mathbb{R}^p$ nonlinear function vectors, $\phi(t)$ and $\psi(t)$ the initial function vectors, and n, m, p are positive integers. Also, it assumes that $f(\cdot)$, $g(\cdot)$, d_t, $\phi(t)$, $\psi(t)$, n, m and p are known.

[2] **Liu G P. Predictive control of high-order fully actuated nonlinear systems with time-varying delays[J]. Journal of Systems Sc 2022, 35(2): 457-470..**

References

[1] Duan G. High-order fully actuated system approaches: Part V. Robust adaptive control[J]. International Journal of Systems Science, 2021, 52(10): 2129-2143.

[2] Liu G P. Predictive control of high-order fully actuated nonlinear systems with time-varying delays[J]. Journal of Systems Science and Complexity, 2022, 35(2): 457-470.

Thanks!