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Subject
HATYAN technical design

1 Introduction

This memo describes the technical design of HATYAN. This technical design will be used as a basis for reproducing the current HATYAN program and for improvements in a later stage.

The memo first gives an overview of all HATYAN routines and their relations. It continues with a detailed insight in some of the main and complementary routines, combined with some remarks on possible improvements. The memo ends with an insight in the current usage of HATYAN in practice.

2 HATYAN routines overview and workflow

The HATYAN package consists of 9 routines. The function of each routine is presented in Table 2.1. The two main routines, needed to go from an observed time-series to a predicted time series are the analysis routine (HATYAN30) and the prediction routine (HATYAN 40), which internally make use of the general libraries in HATYAN00. These two routines are the only routines that were previously used within the MATLAB wrapper created for the reproduction of the Getijtafels in 2016 (Irazoqui M., 2016). However, one can find several other complementary routines for pre/postprocessing and addition of intermediate steps in the workflow.

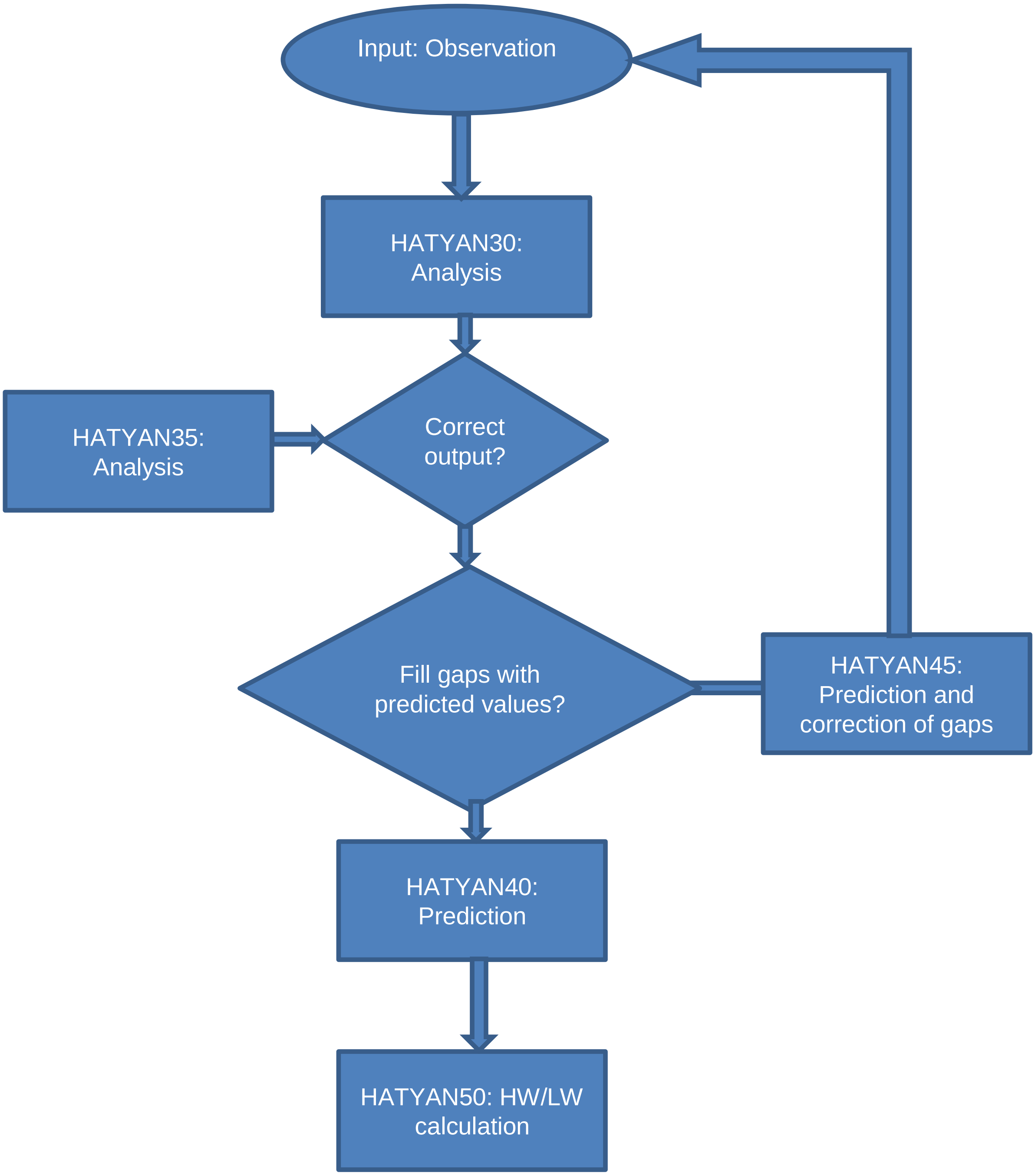
Table 2.1 List of routines forming the HATYAN package and their main function. Routines needing a technical description are further described in the subsequent sections.

	Main function	Technical description
HATYAN00	Collection of libraries used commonly and across the other routines	YES
HATYAN15	Quality control of time-series	YES
HATYAN20	Manipulation of hard coded HATYAN00 parameters	NO
HATYAN30	Harmonic analysis	YES
HATYAN35	Manipulation of results from	NO

	harmonic analysis	
HATYAN40	Prediction	YES
HATYAN45	Prediction and correction	YES
HATYAN50	HW/LW calculation	YES
HATYAN55	Display overview of HATYAN00 parameters	NO

The sequence and interaction between the different routines is represented by the workflow diagram in Figure 2.1. The routine HATYAN00 is common to all of the different routines in the workflow. The routine HATYAN15 is independent.

Figure 2.1 HATYAN workflow and interaction of different routines



3 Technical design of main routines for tidal prediction

In this section, a technical description of the functionalities within the main HATYAN routines mentioned above is provided. The technical description consists of both theoretical background and implementations related to computational optimization (e.g. speed, memory). It is noteworthy that the technical design presented in this section was carried out in 1981. Nowadays, the computational capacity and capabilities are such that a lot of these cumbersome procedures have become obsolete, and a lot simpler solutions are possible. Furthermore, the knowledge on sea-level science has also increased since the design time of HATYAN .

Therefore, a section with recommended improvements is available at the end of each routine description, including both practical recommendations (e.g. software performance-related) and recommendations related to the methodology itself.

3.1 HATYAN30- Analysis

Function	Input	Output
Tidal analysis of a time-series	• Time-series to be analysed • List of constituents for analysis • Nodal correction factors (given for 12 constituents only)	List of constituents with their respective harmonic constants (amplitude, phase)

3.1.1 Requirements for input

- Constant time-step
- Preferred length of one calendar year, since nodal factors are computed in the middle of the timeseries only.
- Analysis-related: Warnings will be displayed if one of the following criteria is not met:
  - $\Delta t < \pi/\omega_{max}$
  - $Rayleigh\ criterion \begin{cases} (\omega_i - \omega_j) < 2\pi/(t_{end} - t_{ini}) \\ \omega_{min} < 2\pi/(t_{end} - t_{ini}) \end{cases}$

3.1.2 Procedure

The harmonic synthesis with N constituents with tidal constants (i.e. amplitude A_i , phase φ_i) and frequency ω_i is given by the expression:

$$h(t) = H_0 + \sum_{i=1}^{N} f_i A_i \cos\{\omega_i t + (v_0 + u)_i - \varphi_i\} \quad (1)$$

Where:

$h(t)$= water-level at time *t, the resulting astro time series*

H_0= Mean sea level
$(f,u)_i$=nodal factor for amplitude and phase respectively. See section 4.1 for more details.
ω_i = the angular frequency per constituent, to be calculated with the phases/frequencies of each constituent
t = an amount of time with respect to a reference within the measurement series
$(v_0)_i$ = the phase angle at the time zero for the constituent *i*. This is calculated from orbital elements (e.g. mean longitude of moon), so equilibrium tide for the given timezone. See section 4.1 for more details.
φ_i = phase lag on the equilibrium tide phase. This use of the equilibrium tide as a reference for tidal analysis is one of its important functions. See section 4.1 for more details.
For a time-series of *m* points $h(t)=(y_1, y_2, \dots, y_m)$, equation (1) forms a set of *m* non-linear equations with *2N* unknowns (A_i, φ_i). These can be made linear by substitution of (2) and (3) in (1).

$$A_i f_i = \sqrt{{a_i}^2 + {b_i}^2} \tag{2}$$

$$(v_0 + u)_i - \varphi_i = -\arctan\left(\frac{b_i}{a_i}\right) \tag{3}$$

$$h(t) = \sum_{i=1}^{N} a_i \cos(\omega_i t) + b_i \sin(\omega_i t) \tag{4}$$

Which, for *m* discrete water-level points, reads:

$$\sum_{k=1}^{2N} x_{mk}\beta_k = y_m \tag{5}$$

This system of *m* linear equations can be expressed in matrix representation as follows:

$$\boldsymbol{x}\beta = \boldsymbol{y} \tag{6}$$

Where:

$$\boldsymbol{x} = \begin{pmatrix} \cos(\omega_1 t_1) & \dots & \cos(\omega_N t_1) & \sin(\omega_1 t_1) & \dots & \sin(\omega_N t_1) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \cos(\omega_1 t_m) & \dots & \cos(\omega_N t_m) & \sin(\omega_1 t_m) & \dots & \sin(\omega_N t_m) \end{pmatrix} \tag{7}$$

$$\beta = \begin{pmatrix} a_1 \\ \vdots \\ a_N \\ b_1 \\ \vdots \\ b_N \end{pmatrix} \tag{8}$$

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} \tag{9}$$

This represents a system with *m* equations and *2N* unknowns. Since $m > 2N$, this means the system is over determined. A way to solve the equations is through a least squares approximation, in which a solution that minimizes the sum of the squares of the residuals is determined. For a set of linear equation, this is equivalent to solving the following determined system of equations:

$$(\boldsymbol{x}^T\boldsymbol{x})\hat{\beta} = \boldsymbol{x}^T\boldsymbol{y} \tag{10}$$

where $\hat{\beta}$ is the coefficient vector of the least-squares hyperplane, which fit the equations “best”, and can be solved normally as:

$$\hat{\beta} = \left(\boldsymbol{x}^T\boldsymbol{x}\right)^{-1}\boldsymbol{x}^T\boldsymbol{y} \tag{11}$$

If we divide the matrix $\widehat{\boldsymbol{x}} = (\boldsymbol{x}^T\boldsymbol{x})$ into 4 quadrants of size *N*x*N as follows:*

$$\widehat{\boldsymbol{x}} = \begin{pmatrix} [\widehat{\boldsymbol{x}}_{11}] & [\widehat{\boldsymbol{x}}_{12}] \\ [\widehat{\boldsymbol{x}}_{21}] & [\widehat{\boldsymbol{x}}_{22}] \end{pmatrix} \tag{12}$$

The submatrices on the diagonal $\widehat{\boldsymbol{x}}_{11}, \widehat{\boldsymbol{x}}_{22}$ contain only cosine and sine terms, respectively, and the other two contain (cosine, sine) cross terms. The matrix $\widehat{\boldsymbol{x}}$ is symmetric.
The cross terms form a summation over odd functions. These functions have the property that, for a sufficiently long time-series with equidistant values, they will tend to zero.

According to the HATYAN documentation, there are two steps that are time-consuming: the generation of the matrix $\widehat{\boldsymbol{x}}$ and solving the system.

In HATYAN, there are two methods to solve this system. Todays solvers are more robust and accurate. This procedure is no longer needed and only adds to the complexity of the code:

1. If the source data has no missing values: In this case, the so called cross terms are disregarded for being small compared to the elements in the diagonal submatrices. Since the cross terms are odd functions, a time-shift is performed to the middle of the considered period in order to make these summations zero. This time-shift is therefore also applied in the right-hand side of equation (10). When making the cross-terms zero, the resulting matrix looks as follows:

$$\widehat{\boldsymbol{x}} = \begin{pmatrix} [\widehat{\boldsymbol{x}}_{11}] & 0 \\ 0 & [\widehat{\boldsymbol{x}}_{22}] \end{pmatrix} \tag{13}$$

   And therefore the equations associated to the submatrices in the diagonal are solved independently, in separate modules.

2. If gaps are found in the time-series, there is no possibility to shift the time origin so that the cross terms disappear. Therefore, the full $\hat{x}$ has to be solved. For the construction of the matrix, a classification of its term into “types of elements” is made to fill the matrix quicker. As for the solving, given that $\hat{x}$ is symmetric, the Choleski decomposition (LU) methodology is used in HATYAN to solve the system. A time-shift to the middle of the period considered is still performed, which is mentioned to result in a more stable matrix.

Once the system is solved, the coefficients (a_i, b_i) are translated back to $(A_i f_i, \varphi_i - vu_i)$ through the equations (2) and (4). Since we want to obtain the real tidal constants (A_i, φ_i), the effect of the nodal modulation is removed by using the nodal factors as specified in HATYAN00 (the so-called *F* and *UV*-factors).

<u>Nodal factors and correction</u>
The theoretical nodal factors f_i associated to relevant components are determined for the middle of the analysis period (using the equations in HATYAN00) and applied as depicted by equations (2) and (3) to derive the real amplitude and phase of the constituent. Since the

nodal factors vary slowly with time, 1 year series are considered short enough to take constant nodal factors.

However, the choice was made to correct these nodal factors at the Dutch coast due to non-linear shallow water interactions. Otherwise, they would lead to over-correction. This does lead to improvements for the Dutch coast, but the values have to be calibrated per area. The factors are currently station-independent. These are the so called empirical nodal factors fp_i, and a single set of correction factors is used for all stations in HATYAN. They are defined through the so called X-factors (X_{fac}) as follows:

$$fp_i = X_{fac}(f_i - 1) + 1 \tag{14}$$

Where by default, X_{fac} equals 1.

Some constituents do not have a direct dependency on the nodal cycle but still show a 18.61 year amplitude variation. The empirical nodal factors for these constituents are defined in relation to the nodal factor of the M2-tide as follows:

$$fp_i = X_{fac}(f_{M2} - 1) + 1 \tag{15}$$

The values for X_{fac}were derived in 1987 from a tidal analysis of hourly observations (1971 to 1986) for the locations of Vlissingen, Hoek van Holland, Ijmuiden, Den Helder, Harlingen and Delfzijl. In HATYAN, only nodal correction factors associated to amplitudes are implemented, and not for phase. According to Koos Doekes, the impact is of applying xfactors on phases is assumed to be rather limited.

The X_{fac} values are used in HATYAN for twelve constituents, defined in the input file:

'MU2'	'N2'	'NU2'	'M2'	'2MN2'	'S2'	'M4'	'MS4'	'M6'	'2MS6'	'M8'	'3MS8'
0	0	0.8	0.53	0.2	-0.82	0.7	0	0.75	0.2	0.7	0.6

Of these twelve, ‘S2’ is the only constituent for which the M2-exception is relevant, in the code this rule applies to all constituents with f equals 1 if an X_{fac} would be provided. Because of this implementation setting Xfac=1 for all constituents is not the same as not specifying the Xfac, so the default would apply for these constituents. This is most likely not what people would expect.

3.1.3 Recommended improvements:

<u>*Short-term improvements*</u>:

- Solve the whole matrix without neglecting the cross terms. Nowadays this operation takes seconds (SVD decomposition) to calculate.
- Deal with gaps in data in a robust way; Leave out the rows (times) of missing data. At the moment this is not possible because this would lead to an irregular time-series, which is not accepted by HATYAN.
- Handle regular and irregular time-series (e.g. time-series with different timesteps)
- Calculate nodal corrections inside the equations, with the $X_{fac}$and middle-of-the-period nodal factor. This enables in a later stage to easily substitute the currently constant nodal cycle factor value by a time-varying value (see Future improvements).

Future improvements

- Make analysis of unlimited length to replace vector averaging (for a trustworthy analysis, currently the maximum length of the time-series is one calendar year) This required the next item to be implemented as well for this to make sense.
- Implement nodal factors as a function of time
- $\omega_i t + (v_0)_i$ represents where the satellite *i* in question(e.g. “moon”) is at a certain time. In other words, it is the phase angle of the Equilibrium constituent *i* . Instead of taking v_0 and then adding $\omega_i t$, we can directly get the phase at the exact time.
- Only the 18.61 year nodal cycle is implemented in HATYAN, other nodal cycles are considered (e.g. lunar perigee 8.85 year cycle) to be implemented.
- The nodal factor computation now refers back to the old equilibrium tide computation of Schureman. This can be replaced with a more recent equilibrium tide computation. It would be useful to move the equilibrium tide data of Schureman to a configuration instead of being hardcoded like it is now.

Given the user-defined nodal correction factors per constituent, a “succespercentage” is computed given by the sum of the squared residuals between observed and synthesised time-series form the analysis results.

3.2 HATYAN40- Prediction

Function	Input	Output
Tidal prediction given a: • Analysis results (Phase=2) • Timeseries (Phase=1)	• Time-series to be analysed and predicted **or** List of constituents from analysis • Nodal correction factors	Predicted time-series

3.2.1 Procedure

There are two options:

- Phase1: Provided a time-series, do analysis and make a prediction. The input is then first subjected to HATYAN30 and then HATYAN40.
- Phase2: Make only the synthesis (prediction). Only HATYAN40 is used.

The procedure to derive the predicted time-series consists on solving equation (1) again, given now the values of all the unknowns in the right hand side of the equation. Hence, each predicted time $h_{pred}(t) = (y_1, y_2, \dots, y_m)_{pred}$, can be computed independently from the others. The nodal factor values in equation (1) used for the prediction are taken once again at the middle of the considered period.

There is an option to use the prediction routine to fill gaps in the observation data and perform the sequence in phase 1 (see above) again with a complete series. This option is possible using HATYAN45 (see section 4.3).

3.2.2 Recommended improvements:

Short-term improvements:

- Avoid having a separate routine for filling the gaps with predicted values. If no regular time-step is needed as proposed above, then this change is not needed. Interpolation of gaps will most likely be worse than removing the appropriate terms from the last-squares estimation

Future improvements

- Implement nodal factors as a function of time (same as for analysis). This prevents the current jump in the astro prediction at each new year, because there is a step in reference date in the middle of one year to the middle of the next year.

4 Complementary routines

In this section, a brief explanation of the (technical) operations performed under each of the remaining routines from Table 2.1 that entail some theoretical background is given.

4.1 HATYAN00

In HATYAN00 the following hard-coded values are stored. These values are calculated using Schureman (1941). A distinction between 3 types of components is made: harmonic components, shallow water components, and special components. The V, F, U numbers for shallow water components are derived from (a combination of) the harmonic and/or special components that they originate from. For the special components, special formulas are used.

- Identification number of constituent (code, up to 195 constituents). These are sorted from lower to higher frequency

- Phase angle at the time zero for the constituent *i*. $(v_0)_i$. These (called V factors in the tables) are calculated from the formulas in Schureman for the orbital elements (e.g. mean longitude of moon/sun, longitude of lunar perigee, etc). These orbital elements are calculated as a function of the number of Julian centuries, with reference time 31st December 1899 midday (12.00am). This is because the constants appearing in these formulas correspond to this reference date, which was suitable for the century in which Schureman was written (20th). Other parameters like the eccentricity of the earth's orbit and the obliquity of the ecliptic are given for the same époque (Jan 1st 1900). These are reported to change 0.000042 and 0.013 of a degree per century.

- Frequencies: The formulas for the frequencies are not given in the documentation. However, the same formulas as for the (v_0) apply, but substituting the orbital elements by their time derivatives.

- Nodal factors $(\mathrm{f,u})_\mathrm{i}$. These (called F and U factors in the tables) are calculated based on orbital elements that are dependent on the distance of the ascending node of the moon to the vernal equinox, which at the same time is dependent on time (number of Julian centuries). The nodal factors for shallow-water constituents are assumed to follow the factors for the constituents involved in their generation. These formulas are explicitly given for each component in HATYAN00, making the origin of the compound tides and overtides clear. When special formulas apply, a number between parenthesis is written on the tables which refer to the equation with the corresponding number in the Schureman book.

4.1.1 Recommended improvements:

Future improvements

- Have an “equilibrium tide solution” input file. At the moment is based on Schureman(1941) and hard-coded in HATYAN00. By having an input file, we can change how complicated or simple the used equilibrium tide solution is. Related tables can be re-computed based on this input file (e.g. Doodson tables).
- Have nodal factor values, Doodson numbers and combination of constituents from which the Doodson numbers come from printed in separate files instead of hardcoded inside the body of the code. This makes the code more transparent for the user.
- Get rid of the use and calculation of v_0 phases (see Future improvements in section 3.1.2)
- Consider updating values which were taken for the previous century and investigate more modern equilibrium tide solutions.
- Calculating the 19 year A and phi values for SA and SM was done in the past, but these results are still used in nowadays predictions. Back then, a 19 year timeseries was used as input, with f and u in the middle of the period and v at the start. This analysis would be better with nodal factors as a function of time or if the years would have been analysed separately and the results vector averaged.

4.2 HATYAN15

This module checks for the quality of the input time-series in several aspects (format, gaps, etc) and displays an error code for each.

There are three types of control-procedures:

1. Tolerance-exceedance: An error is displayed if the difference between the to-be-controled value and the expected value (calculated or interpolated) exceeds 20 cm or a tolerance value given by the user. The calculated/interpolated value at a certain time is derived through fitting a 5th degree polynomial to the 3 precedent and 3 subsequent water-level values with a timestep of 1 hour.
2. HW/LW period exceedance: When the period between two consecutive extremes is too short or too long (given a minimum and maximum) an error is displayed. Default values are 300 minutes (5 hours) and 420 minutes (7 hours) respectively.
3. Percentage of missing values.

4.3 HATYAN45

This synthesis-correction routine corrects the source/observed time-series by substituting missing values with predicted values.

First, an analysis is performed using only the strongest constituents (which are meant to be less sensitive to missing data). Then, a prediction is made for the same period as the observation with the constituents derived from the analysis, and the gaps in the original time-series are filled in with the values from this prediction. Lastly, the whole procedure (analysis and prediction) as performed by HATYAN40 is followed. For a clear overview of the workflow, see Figure 2.1

4.4 HATYAN50

This routine explains how the calculation of the HW/LW times and values is performed, given a time-series.

4.4.1 Procedure

In the current HATYAN, the extremes in a given time-series are localized by differences. In order to do this effectively, the timestep between the series points has to be small enough. Since the time-series are normally produced with a time-step of 10 minutes and this is not enough for an accurate extreme detection through differences, the time-series is refined through a parabolic interpolation.

The method is as follows: The timeseries is investigated in gaps of $2\Delta t$ (3 data-points) and if an extreme is found inside, then a parabola is fitted through these points. A refinement of the timestep (halved) is done if the extreme is not found in the previous step.

Treatment of an “agger” or double high/low water: If three consecutive extremes are found in the form of LW-HW-LW (or HW-LW-HW), it is checked if this constitutes an “agger”. In order to be classified as an “agger”, the following criteria have to be met:

- Time between the 1st and 3rd extreme is longer than 2 hours but shorter than 4 hours
- The height difference between the 2nd extreme and the other two has to be higher than 5 cm (in both cases).

If the criteria are met, then this is identified as an “agger”. If the criteria are not met, then only the lowest LW (for LW-HW-LW case) or highest HW (for HW-LW-HW case) are selected as extremes.

Short term improvements
There are alternative methods of calculating extremes from timeseries. A possibility is to start by identifying the crossing of the timeseries with its mean. Positive crossings then define the start of a tidal period and within the separate tidal periods, the minimum and maximum can be calculated. A down side of this simple method of extremes calculation is that this method does not work if local min/max values occur around the mean of the time series, but this could be solved by specifying a minimal duration of the tidal period.

5 Current usage of HATYAN

This chapter provides some basic insight into how HATYAN is used at RWS when doing a prediction of astro series. These insights are based on information provided in meetings with RWS and in the development of the HATYAN 2.0 prototype in Python. It does not go into details like input and output files, this information will be provided in the Functional Design of HATYAN 2.0.

For each prediction, an analysis of measurement data has to be made. There are two sets of analysis data used for each prediction, one of approximately 19 years and one of 4 years. The 19-year analysis is used for the low-frequency components SA and SM and the 4 year analysis for the rest of the components. This analysis result can be used to make tidal predictions.

The 4-year analysis is calculated with four years of relatively recent measurement data and a new analysis period is chosen when the data gets too old. For example, in order to make tide predictions for 2016, measurement data from 2009 to 2012 was used for most stations. In the analysis, each year of data is analysed separately with f and u in the middle of the year and v in the beginning. The results of the separate years are vector averaged in order to obtain amplitudes and phases of the four year period.

The 19 year data was analysed in the past for several stations and the results are still used for the components SA and SM. For most stations, this 19 year period ranges from 1976 to 1994 (exceptions are for instance Roompot Buiten and IJmuiden buiten). These old values for component amplitude and phases, are manually copied to replace the SA and SM values from the 4-year analysis. When doing the analysis of this measurement data, f and u were calculated in the middle of the period and v in the beginning of the 19 year period.

Currently all timeseries are reduced to a 60 minute interval when doing an analysis, which means that nowadays five of every six measurements are not used. When the 19 year analysis was done, it was the only way of analysing data, because the first 12 years of the 19 year period only contained 60 minute values and HATYAN only accepts regular timeseries. Since 1987/1988 however, water level measurements are stored for every 10 minutes, which could be used for the 4-year analysis.

6 References

Bogaard T, 1411-0103-m-Werkprocess getijanalyse Rijkswaterstaat_FINAL, december 2014

Bogaard T, 1220116-000-ZKS-0001-m-Optimalisatie knoopfactorcorrecties voor de Nederlandse kust, march 2016

Irazoqui Apecechea M., 1230074-000-ZKS-0004-m-Getijtafels_v1.0, december 2016

Schureman P, Manual of harmonic analysis and prediction of tides, U.S. Coast and Geodetic Survey, Special Publication No. 98, Washington 1941.