Proceedings of the 2001 IEEE
International Conference on Robotics & Automation
Seoul, Korea • May 21-26, 2001

A Counter Example to the Theory of Simultaneous Localization and Map Building

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Abstract

This paper analyzes the properties of the full covariance simultaneous map building problem (SLAM). We prove that, even for the special case of a stationary vehicle (with no process noise) which uses a range-bearing sensor and has non-zero angular uncertainty, the full-covariance SLAM algorithm always yields an inconsistent map. We also show, through simulations, that these conclusions appear to extend to a moving vehicle with process noise. However, these inconsistencies only become apparent after several hundred beacon updates.

1 Introduction

Autonomous Guided Vehicle (AGV) technology plays a prominent role in a wide variety of scientific, industrial, and military applications. In all of these applications there is a critical need for *localization* — the AGV must be able to accurately and repeatedly estimate its own position. The most general localization strategy is to equip the AGV with the capability to construct its own map of beacons (which can be artificial or naturally-occurring). The AGV maintains a continuous estimate of its absolute position as it re-observes the dynamically mapped beacons. This strategy is referred to as Simultaneous Localization And Map building (SLAM).

The seminal work on the rigorous application of the Kalman filter to the SLAM problem was carried out in the late 1980s by Smith, Self and Cheeseman [1] who introduced the notion of a *stochastic map*. Previous researchers addressed the SLAM problem by assuming that the beacon and vehicle estimates could be propagated independently of one another. What [1] showed was that the vehicle and beacon estimates are not independent of one another and a complete joint covariance matrix comprising the vehicle and all beacon estimates *must* be maintained. Failure to do so causes the filter to believe that it has more information than is really available and, as demonstrated in [2], this leads to an erroneous map and an inconsistent vehicle estimate.

It has been widely assumed that the stochastic mapping approach of [1] is theoretically sound. Consequently, most recent research has focused on practical issues such as reducing the computational resources, which scale quadratically with map size, required by the optimal algorithm [3–6].

In this paper we re-examine the properties of the stochastic map and consider the problem of a vehicle which possesses a sensor which is capable of measuring range and bearing to beacons in the environment. We show that, when the vehicle is stationary and no process noise acts on it, the joint system is guaranteed to be inconsistent. Furthermore, simulation studies show that a moving vehicle with process noise exhibits a similar type of behavior. However, the time required for the map to become visibly inconsistent is of the order of several hundred time steps, which is longer than most experimental runs presented in the literature.

The structure of this paper is as follows. The full covariance SLAM algorithm is described in Section 2. The behavior of a stationary vehicle is examined in Section 3 and we derive a theory of a necessary condition which must be met by a map building algorithm. Section 4 describes a simple system and shows that it does not build a consistent map for either a stationary or moving vehicle. Conclusions are drawn in Section 5.

2 Simultaneous Localization and Map Building

The structure of a joint vehicle-beacon system is as follows. The state of the vehicle at time step k is $\mathbf{x}_v(k)$ and the state of the ith beacon is $\mathbf{p}_i(k)$. The complete state space for a system which comprises of n beacons is

$$\mathbf{x}_n(k) = \left[\mathbf{x}_v(k)\ \mathbf{p}_1^T(k)\ \dots\ \mathbf{p}_n^T(k)\right]. \tag{1}$$

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The mean and covariance of this estimate are

$$\hat{\mathbf{x}}_n(k \mid k) = \left[\hat{\mathbf{x}}_v^T(k \mid k) \ \ldots \ \hat{\mathbf{p}}_n^T(k \mid k)\right] \tag{2}$$

$$\mathbf{P}_n(k \mid k) = \tag{3}$$

$$\begin{pmatrix} \mathbf{P}_{vv}(k \mid k) & \mathbf{P}_{v1}(k \mid k) & \ldots & \mathbf{P}_{vn}(k \mid k) \\ \mathbf{P}_{1v}(k \mid k) & \mathbf{P}_{11}(k \mid k) & \ldots & \mathbf{P}_{1n}(k \mid k) \\ \mathbf{P}_{2v}(k \mid k) & \mathbf{P}_{21}(k \mid k) & \ldots & \mathbf{P}_{2n}(k \mid k) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{nv}(k \mid k) & \mathbf{P}_{n1}(k \mid k) & \ldots & \mathbf{P}_{nn}(k \mid k) \end{pmatrix}$$

where $\mathbf{P}_{vv}(k \mid k)$ is the covariance of the AGV's position estimate, $\mathbf{P}_{ii}(k \mid k)$ is the covariance of the position estimate of the ith beacon and $\mathbf{P}_{ij}(k \mid k)$ is the cross-correlation between the estimate of i and j.

By assumption, all of the beacons are stationary and no process noise acts upon them. Therefore, the process model is of the form

$$\mathbf{f}\left[\mathbf{x}_n(k), \mathbf{u}(k), \mathbf{v}(k)\right] = \begin{pmatrix} \mathbf{f}_v\left[\mathbf{x}_v(k), \mathbf{u}(k), \mathbf{v}(k)\right] \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix} \tag{4}$$

where $\mathbf{u}(k)$ is the control input and $\mathbf{v}(k)$ is the process noise. Similarly, the process noise covariance matrix is

$$\mathbf{Q}_n(k+1) = \begin{bmatrix} \mathbf{Q}_v(k+1) & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix}. \tag{5}$$

The observation model and observation model Jacobian for when the vehicle observes the ith beacon is

$$\begin{aligned} \mathbf{z}_i(k) &= \mathbf{h}_i\left[\mathbf{x}_v(k), \mathbf{p}_i(k), \mathbf{w}(k)\right] \\ \nabla\mathbf{h}_i &= \left[\nabla\mathbf{h}_i^x \ \ -\nabla\mathbf{h}_i^p\right] \end{aligned} \tag{6}$$

where the negative sign on $\nabla\mathbf{h}_i^p$ denotes the fact that the beacon estimates often enter with the opposite sign to the vehicle estimates. Whenever the vehicle observes a beacon which is in the map, the joint system is updated using the Kalman filter update rule. If the beacon is not in the map, a new beacon estimate is created and inserted into the map. To initialize the beacon position, the inverse of the beacon observation equation is used:

$$\mathbf{p}_i(k) = \mathbf{g}_i\left[\mathbf{x}_v(k), \mathbf{z}_i(k), \mathbf{w}(k)\right].$$

The new beacon estimate is appended to the map, and the new mean and covariance become:

$$\hat{\mathbf{x}}_{n+1}(k \mid k) = \begin{bmatrix} \hat{\mathbf{x}}_n(k \mid k) \\ \mathbf{g}_n\left[\hat{\mathbf{x}}_v(k \mid k), \mathbf{z}_n(k), \mathbf{0}\right] \end{bmatrix}, \tag{7}$$

$$\mathbf{P}_{n+1}(k \mid k) = \tag{8}$$

$$\begin{pmatrix} \mathbf{P}_n(k \mid k) & \mathbf{P}_n(k \mid k)\nabla^T\mathbf{g}_n^x \\ \nabla\mathbf{g}_n^x\mathbf{P}_n(k \mid k) & \nabla\mathbf{g}_n^x\mathbf{P}_n(k \mid k)\nabla^T\mathbf{g}_n^x + \nabla\mathbf{g}_n^w\mathbf{R}(k)\nabla^T\mathbf{g}_n^w \end{pmatrix}.$$

3 Mapping From A Stationary AGV

Consider the following scenario. An AGV is placed at an unknown location in its environment with a specified mean $\hat{\mathbf{x}}_v(0 \mid 0)$ and covariance $\mathbf{P}_{vv}(0 \mid 0)$. The AGV then uses a sensor (such as a laser range finder) to measure the position of the beacons relative to the AGV. The AGV is assumed to remain stationary, so no process noise is injected as beacons are initialized and updated. Because the AGV's state is unchanging, and because beacon positions are only measured relative to the AGV, the AGV's position estimate should not change. Therefore, $\hat{\mathbf{x}}_v(k \mid k) = \hat{\mathbf{x}}_v(0 \mid 0)$ for all timesteps k. This condition is satisfied if the following theorem holds:

Theory 1. *If an AGV estimate is initialized with a non-zero covariance, beacon estimates are initialised using Equations 7 and 8, and all observation covariances are finite, then the state estimate of the AGV will remain unchanged if and only if*

$$\nabla\mathbf{h}_1^x - \nabla\mathbf{h}_1^p\nabla\mathbf{g}_1^x = \mathbf{0} \tag{9}$$

for all timesteps k.

Proof. Assume that the beacon is initialized at time step 0. We prove Equation 9 by considering the first two time steps.

The beacon estimate is initialized into the map at time step 0. Therefore, the state of the system with the initialized beacon is given by Equations 7 and 8. Because the vehicle is stationary and no process noise is injected, the predicted state of the vehicle at time step 1 is $\hat{\mathbf{x}}_1(1 \mid 0) = \hat{\mathbf{x}}_1(0 \mid 0)$ and $\mathbf{P}_1(1 \mid 0) = \mathbf{P}_1(0 \mid 0)$.

A necessary and sufficient condition to ensure that the vehicle estimate does not change is that the Kalman weight (gain matrix) which is applied to it should be $\mathbf{0}$. Therefore, the weight should be of the form

$$\mathbf{W}(k+1) = \begin{bmatrix} \mathbf{W}_v(k+1) \\ \mathbf{W}_p(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{W}_p(k+1) \end{bmatrix},$$

where the dimension of $\mathbf{0}$ is the same as $\hat{\mathbf{x}}_v(k \mid k)$ and $\mathbf{W}_p(k+1)$ is the weight applied to the beacon. Because the Kalman weight is given by

$$\mathbf{W}(k+1) = \mathbf{P}(k+1 \mid k)\nabla^T\mathbf{h}_1^x\mathbf{S}^{-1}(k+1),$$

the condition is equivalent to demanding that

$$\mathbf{P}(k+1 \mid k)\nabla^T\mathbf{h}_1^x = \begin{bmatrix} \mathbf{0} \\ \mathbf{W}_p\mathbf{S}^{-1}(k+1) \end{bmatrix}, \tag{10}$$

where the second term is the weight that is applied to the beacon and is not considered in the proceeding analysis. Therefore, we need only consider the *numerator* of the weight and can dispense with the calculation of $\mathbf{S}(k+1)$. Substituting, the numerator of the weight on the vehicle is

$$\begin{aligned}\mathbf{W}_v(1) &= \mathbf{P}_v(1 \mid 0)\,\nabla\mathbf{h}_1^{xT} - \mathbf{P}_v(1 \mid 0)\,\nabla\mathbf{g}_1^{xT}\,\nabla\mathbf{h}_1^{pT} \\ &= \mathbf{P}_v(1 \mid 0)\,(\nabla\mathbf{h}_1^{x} - \nabla\mathbf{h}_1^{p}\,\nabla\mathbf{g}_1^{x})^T.\end{aligned} \tag{11}$$

Because $\mathbf{P}_v(1 \mid 0)$ is nonsingular, a necessary and sufficient condition for this weight to be $\mathbf{0}$ is that

$$\mathbf{0} = \nabla\mathbf{h}_1^{x} - \nabla\mathbf{h}_1^{p}\,\nabla\mathbf{g}_1^{x}.$$

This result proves the theorem for a single timestep. In [7] we show that it can be readily extended to an arbitrary number of timesteps. □

The importance of this result is that the behavior of the vehicle and beacon depends critically on the Jacobian matrices used to initialize the beacon. In the special case that the observation model is linear and time invariant[1] ($\nabla\mathbf{h}_i^x = \mathbf{H}^x, \nabla\mathbf{h}_i^p = \mathbf{H}^p$ and $\nabla\mathbf{g}_i^x = \mathbf{G}^x$), the condition requires that

$$\mathbf{H}^p\mathbf{G}^x = \mathbf{H}^x.$$

However, in a general nonlinear system where the Jacobian matrices are functions of noisy observations and erroneous estimates, it is not clear that the condition in Equation 9 can be guaranteed to hold. Furthermore, because this condition is a *structural* relationship, normal tuning procedures (such as inflating the observation noise covariances) cannot circumvent the problem. If the vehicle estimate changes for one value of $\mathbf{R}(k)$, it will change for all (finite) values of $\mathbf{R}(k)$. In fact, as we now show, this condition cannot be guaranteed even for the simplest case of a position-orientation AGV model and a range-bearing sensor model.

4 A Concrete Example

Consider the following simple system. The vehicle state is described by its position (x_v, y_v) and orientation θ_v in some global coordinate system. The vehicle is equipped with a sensor that is able to return the range r and bearing ϕ of a target relative to the sensor platform. The sensor is a rotating scanner (such as a laser range finder) which completes one revolution per second.

[1] Even with nonlinear process and observation models, such a system structure arises in the relative map [6] and geometric projection filter [5] which decouple the problem of map building from vehicle localization. However, with these approaches the it appears that the map can only be used as an aid to beacon gating and it cannot be used to update the vehicle position estimate.

4.1 System Equations

The vehicle model is the standard equation for a steered bicycle [8]:

$$\mathbf{x}_v(k+1) = \begin{pmatrix} x_v(k) + V(k+1)\Delta T\cos(\delta(k+1) + \theta_v(k)) \\ y_v(k) + V(k+1)\Delta T\sin(\delta(k+1) + \theta_v(k)) \\ \theta_v(k) + \frac{V(k+1)\Delta T\sin(\delta(k+1))}{B} \end{pmatrix},$$

where the time step is ΔT, the control inputs are the wheel speed $V(k+1)$ and steer angle $\delta(k+1)$ and the vehicle wheel base is B. The process noises are additive disturbances which act on $V(k)$ and $\delta(k)$. The observation model is

$$\mathbf{h}_i[\mathbf{x}_v(k), \mathbf{p}_i(k), \mathbf{w}(k)] = \begin{bmatrix} \sqrt{(x_i - x_v)^2 + (y_i - y_v)^2} \\ \tan^{-1}\left(\frac{y_i - y_v}{x_i - x_v}\right) - \theta_v \end{bmatrix} \tag{12}$$

The Jacobian for this equation can be written as

$$\nabla\mathbf{h}_i^x = \begin{pmatrix}\mathbf{h}_i^{x_v y_v} & \mathbf{h}_i^{\phi} & \mathbf{h}_i^{x_i y_i}\end{pmatrix},$$

where

$$\mathbf{h}_i^{x_v y_v} = \begin{pmatrix} -(x_i - x_v)/r & -(y_i - y_v)/r \\ (y_i - y_v)/r^2 & -(x_i - x_v)/r^2 \end{pmatrix}, \tag{13}$$

$$\mathbf{h}_i^{\theta_v} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \tag{14}$$

$$\mathbf{h}_i^{x_i y_i} = -\mathbf{h}_i^{x_v y_v}. \tag{15}$$

The observation noise Jacobian is

$$\mathbf{h}_i^{w} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{16}$$

Let $\alpha(k) = \theta_v(k) + \phi(k)$. Inverting Equation 12, the beacon position is initialized as

$$\mathbf{g}_i[\mathbf{x}_v(k), \mathbf{w}(k)] = \begin{bmatrix} x_v(k) + r(k)\cos[\alpha(k)] \\ y_v(k) + r(k)\sin[\alpha(k)] \end{bmatrix}.$$

and the Jacobian is

$$\nabla\mathbf{g}_i^x = \begin{pmatrix}\mathbf{g}_i^{x_v y_v} & \mathbf{g}_i^{\theta}\end{pmatrix},$$

where for

$$\mathbf{g}_i^{x_v y_v} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{17}$$

$$\mathbf{g}_i^{\theta} = \begin{bmatrix} -r(k)\sin[\alpha(k)] \\ r(k)\cos[\theta_v(k) + \phi(k)] \end{bmatrix}, \tag{18}$$

$$\mathbf{g}_i^{w} = \begin{bmatrix} \cos[\alpha(k)] & -r(k)\sin[\alpha(k)] \\ \sin[\alpha(k)] & r(k)\cos[\alpha(k)] \end{bmatrix}. \tag{19}$$

We now consider two special cases for this system — a stationary vehicle with no process noise, and a vehicle which moves in a circle with nominally constant control inputs.

4.2 Behavior of a Stationary Vehicle

First consider the special case that the vehicle is stationary and *no process noise* acts on the system. In this situation the process model reduces to the identity matrix and

$$\mathbf{f}_v\left[\mathbf{x}_v\left(k\right),\mathbf{0},\mathbf{0}\right]=\mathbf{x}_v\left(k\right).$$

where the dimension of $\mathbf{0}$ is the same as $\hat{\mathbf{x}}_v\left(k\mid k\right)$. At timestep 0 the beacon is initialised into the map and the mean and covariance are set using Equations 7 and 8. This case can be analysised by Theorem 1. Assuming that Equation 10 has been obeyed up to timestep k, the numerator of $\mathbf{W}\left(k\right)$ is

$$\begin{pmatrix}\mathbf{P}_n\left(k\mid k\right) & \mathbf{P}_n\left(k\mid k\right)\boldsymbol{\nabla}^T\mathbf{g}_n^x\\ \boldsymbol{\nabla}\mathbf{g}_n^x\mathbf{P}_n\left(k\mid k\right) & \boldsymbol{\nabla}\mathbf{g}_n^x\mathbf{P}_n\left(k\mid k\right)\boldsymbol{\nabla}^T\mathbf{g}_n^x+\mathbf{A}\end{pmatrix}\begin{pmatrix}\begin{bmatrix}\mathbf{h}_i^{x_vy_vT}\\ \mathbf{h}_i^{\phi T}\end{bmatrix}\\ \mathbf{h}_i^{x_iy_iT}\end{pmatrix}$$

where $\mathbf{A}\left(k\right)$ is a positive semi-definite matrix corresponding to the partially filtered observation noise. Therefore, the weight on the vehicle $\mathbf{W}_v$ is

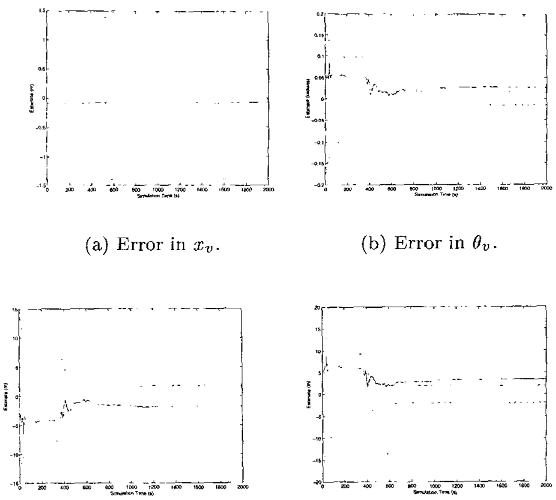
$$\mathbf{W}_v=\mathbf{P}_n\left(k\mid k\right)\left\{\begin{bmatrix}\mathbf{h}_i^{x_vy_vT}\\ \mathbf{h}_i^{\theta T}\end{bmatrix}+\begin{bmatrix}\mathbf{g}_i^{x_vy_vT}\\ \mathbf{g}_i^{\theta T}\end{bmatrix}\mathbf{h}_i^{x_iy_iT}\right\}.$$

Substituting from Equations 17 to 19 and using the property that $\mathbf{h}_i^{x_vy_vT}=-\mathbf{h}_i^{x_iy_iT}$,

$$\mathbf{W}_v=\mathbf{P}_n\left(k\mid k\right)\begin{bmatrix}\mathbf{0}\\ \mathbf{h}_i^{\theta T}-\mathbf{g}_i^{\theta T}\mathbf{h}_i^{x_vy_vT}\end{bmatrix}.$$

Therefore, the weight on the vehicle position states is always guaranteed to be $\mathbf{0}$. However, this is *not* the case for the vehicle orientation state. Substituting from Equations 14, 18 and 13, it can be readily shown that this weight is non-zero only if the angle between the vehicle and the beacon $(\theta_v+\phi)$ is the *same* as the value when the beacon was first initialized [7]. It should be emphasized that these analytical results reflect a fundamental failure in the structure of the cross correlation between the vehicle and beacon estimates. The errors occur irrespective of the magnitude of covariances and they are not the result of subtle numerical implementation errors.

This behavior can be clearly seen in a simulation study of this scenario. For the results in this paper we use the following conditions. The vehicle initially starts at the origin with the standard deviations in x_v and y_v of 0.7m and θ_v of 5°. It observes a beacon at (97.89, 70.1) with an accurate sensor whose observation noise covariance is $\mathbf{R}\left(k\right)=\mathrm{diag}(0.25\mathrm{m}^2,(1^\circ)^2)$. Figure 1 plots the time history of the estimates of x_v, θ_v, x_1 and y_1. As expected, the estimate of x_v does not change. However, the estimate of θ_v immediately starts to change and its covariance begins to decline. By the end of the run, the orientation covariance is less than 6% of its initial value. However, this update is entirely spurious — there is no additional information about the vehicle's orientation and so the orientation estimate becomes inconsistent. In turn, the beacon estimate becomes inconsistent as well. Extending the simulation further shows that, in the limit, the steady-state covariance of the beacon estimate does not become 0 but is a value greater than the initial vehicle position covariance.



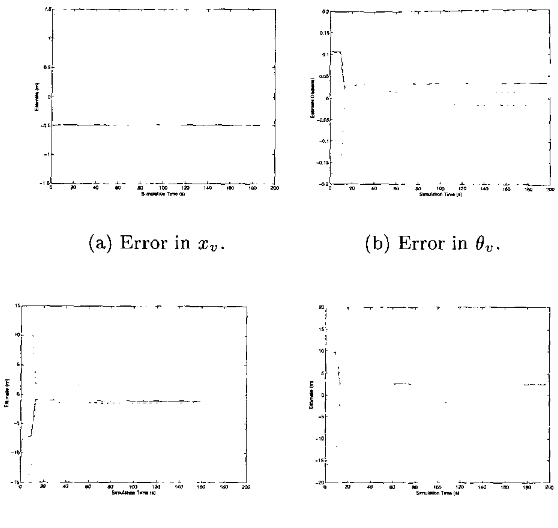
(a) Error in x_v. (b) Error in θ_v.

(c) Error in x_1. (d) Error in y_1.

Figure 1: Estimation errors and 2 standard deviation bounds. The standard deviation bounds are shown as dashed lines.

The value of $(\theta_v+\phi)$ also changes if the vehicle and beacon configuration changes. Figure 2 shows the result when the vehicle, at timestep 10, "teleports" to the position (50, 50). This jump occurs instantaneously and without uncertainty, i.e., at the same time the estimate changes, the true vehicle position changes by the same amount[2]. As can be seen the results are

[2]This is equivalent to the process model $x_v(k+1)=x_v(k)+\Delta x$, $y_v(k+1)=y_v(k)+\Delta y$ where Δx and Δy are known.



(a) Error in x_v. (b) Error in θ_v.

(c) Error in x_1. (d) Error in y_1.

Figure 2: Estimation errors and 2 standard deviation bounds. For a vehicle which "teleports" from position $(0,0)$ to $(50,50)$ at timestep 10.

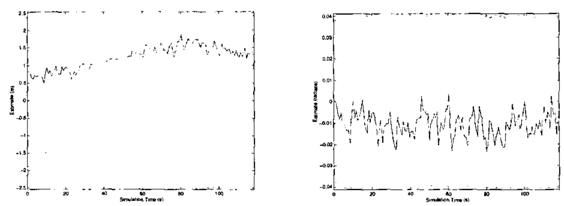
dramatic. Although the error in the estimate of x_v remains unchanged after the instantaneous translation, observations of the beacons lead to a large spurious reduction in both the vehicle orientation and the beacon position estimates. Again, these reductions correspond to inconsistent estimates.

It must be emphasized that the inconsistencies in both examples are not simply due to the fact that the observation Jacobian is calculated using the noise-corrupted sensor observations. Even if the *true* state of the beacon and the vehicle were always used to calculate the Jacobian of the observation equation, *any* motion by the vehicle that affects the observation Jacobian will lead to inconsistency analogous to a violation of the condition of Equation 9 in the stationary case described in Section 3. In fact, a perfectly known change in the AGV's orientation – *with no change in position* – will have the same effect. In the next section we generalize the example to include the accumulation of process noise by a moving vehicle.

4.3 The Behavior of a Moving Vehicle

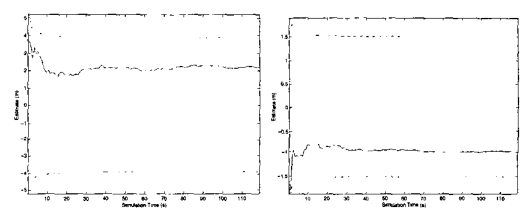
In this section we consider the case of a moving vehicle in a long-duration SLAM simulation. As in our example of a stationary vehicle with a range-bearing sensor, our goal is to keep the scenario as simple as possible to demonstrate the perniciousness of the inconsistency problem. To this end, we assume that the vehicle travels in a circle with constant control inputs $\mathbf{u}(k+1) = [1\ 2°]$ and process noise standard deviations $\mathbf{Q}(k) = \mathrm{diag}\{0.25, (0.3°)^2\}$. The vehicle observes an environment which contains 5 beacons.

The time history of the first 600 time steps of the vehicle and beacon estimates are shown in Figure 3 and Figure 4 respectively. The estimates appear to be white, zero mean and the covariances even appear to be slightly conservative. The errors in the beacons and the beacon estimates appear to have stabilized and it appears that this algorithm is performing in a satisfactory manner.



(a) Error in x_v. (b) Error in θ_v.

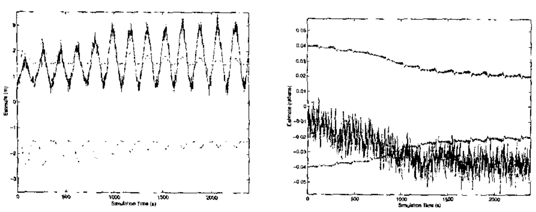
Figure 3: Results for the first 600 seconds (3000 timesteps) of a moving vehicle.



(a) Error in x_1. (b) Error in y_1.

Figure 4: Results for the first 600 seconds (3000 timesteps) of beacton 1.

Figures 5 and 6 show the time histories when the experiment is allowed to run for 16,000 time steps. These results clearly show that the apparent consistency is only a short-term phenomena: within less than five thousand beacon updates the map has become inconsistent. We speculate that this behavior has not been recognized in the literature for two main reasons. First, most systems reported in the literature (such as [9] or [6]) only present results for the first few hundred time steps. Over such short durations the signs of divergence are not very prominent. Second, the few long-duration studies (such as [3]) have all used compasses to measures the absolute orientation of the vehicle. A compass causes the orientation errors to be filtered out, and significantly reduces the rate of divergence [7].

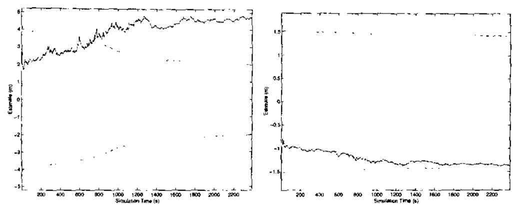


(a) Error in x_v. (b) Error in θ_v.

Figure 5: Results for the first 3200 seconds (16000 timesteps).

5 Conclusions

This paper has analyzed the properties of the full-covariance stochastic mapping approach to SLAM. We have shown that a simple but realistic scenario is guaranteed to produce inconsistent vehicle and beacon estimates. Furthermore, we have shown that the problem typically occurs after several hundred time steps, which is longer than the duration of many experimental runs. The full implications of this result (for example, whether one build a consistent map without adding process noise to the beacon estimates) is presently under investigation [7]. However, two conclusions can be immediately drawn from these results. First, it is questionable whether the Kalman filter framework developed in [1] provides a general, robust and rigorous solution to the stochastic SLAM problem. Second, the consistency of *any* map building algorithm cannot be fully assessed from experimental runs which are of short duration. As demonstrated in Subsection 4.3, an algorithm can appear to be consistent for many hundreds of time steps but, in fact, prove to be inconsistent.



(a) Error in x_1. (b) Error in y_1.

Figure 6: Estimation errors and 2 standard deviation bounds for beacon 1 for 3200 seconds (16000 timesteps). The standard deviation bounds are the pairs of dashed lines.

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