Qiskit 3 - Bell

1. Theoretical introduction
2. Bell state 1 as a Quantum circuit
3. Bell state 1 visualization and analysis

Theoretical introduction

Mathematics

The 4 Bell states represent a 2-qubit maximum entangled system. Consider the 2-qubits A and B:
$\mathbf{A} =$ |
$\mathbf{B} =$ |

Consider furthermore the 4 Bell states notated as $|\psi_i\rangle$:

$$|\psi_{\mathbf{00}}\rangle = \frac{|0_a\rangle \otimes |0_b\rangle + |1_a\rangle \otimes |1_b\rangle}{\sqrt{2}}$$
$$|\psi_{\mathbf{01}}\rangle = \frac{|0_a\rangle \otimes |0_b\rangle - |1_a\rangle \otimes |1_b\rangle}{\sqrt{2}}$$
$$|\psi_{\mathbf{10}}\rangle = \frac{|0_a\rangle \otimes |1_a\rangle + |0_b\rangle \otimes |1_a\rangle}{\sqrt{2}}$$
$$|\psi_{\mathbf{11}}\rangle = \frac{|0_a\rangle \otimes |1_a\rangle - |0_b\rangle \otimes |1_a\rangle}{\sqrt{2}}$$

These states can also be represented in the more simplified form as follows:

$$|\psi_{\mathbf{00}}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$|\psi_{\mathbf{01}}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$
$$|\psi_{\mathbf{10}}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
$$|\psi_{\mathbf{11}}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Quantum circuits

We can assign the state $|\psi_{\mathbf{00}}\rangle$ for example to our 2 qubits **A** and **B** by the following procedure. Make sure that each qubit are in the states $|0_a\rangle$ and $|0_b\rangle$ respectively. Now apply a Hadamard operator to one of them, and then use that as a control qubit for a Cnot that targets the other. Thats it.

Note though that this procedure can be varied in interesting ways. Recall that **HX = ZH** and so forth.

Note furthermore that this can be generalized such that one can one can reach a bell state from any initial states of our qubits.

Questions

- Measurement - What is the hypothesis of the results we are going to get?
- Measurement - What do we measure? Two quantumregisters with respect to a classicalregister? We are measuring the probability of us finding the system in one of the four states (00, 01, 10, 11)? Is this 1 wavefunction or wavefunctions?
- Entanglement - These states are maximum entangled, verify this mathematically, and then visually.
- Visualization - Blochs etc etc. State plots etc.

Bell state 1 as a Quantum circuit

We want to achieve the following:
$|\psi_{00}\rangle = \mathbf{C_{not}}$

Where the superpositioned $\mathbf{H}|0_a\rangle$ is the control-unit and $|0_a\rangle$ is the target-unit.

In Matrix notation:

$$|\psi_{00}\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} (\begin{pmatrix} 1_a \\ 0_a \end{pmatrix} \otimes (\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1_b \\ 0_b \end{pmatrix}))$$

Hence our hypothesis when it comes to measuring such system, is that we will find the Bell state 1 in the state of 00 half of the times and in the state of 11 half of the times.

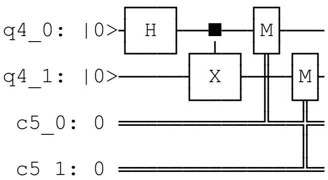
In [1]:

```python
from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit
from qiskit import Aer, execute
from qiskit.tools.visualization import plot_histogram, circuit_drawer
```

In [14]:

```python
sim = Aer.get_backend('qasm_simulator')
qubits = QuantumRegister(2)
classical = ClassicalRegister(2)
c1 = QuantumCircuit(qubits, classical)
c1.h(qubits[0])
c1.cx(qubits[0], qubits[1])
c1.measure(qubits, classical)
circuit_drawer(c1)
```

Out[14]:



In [13]:

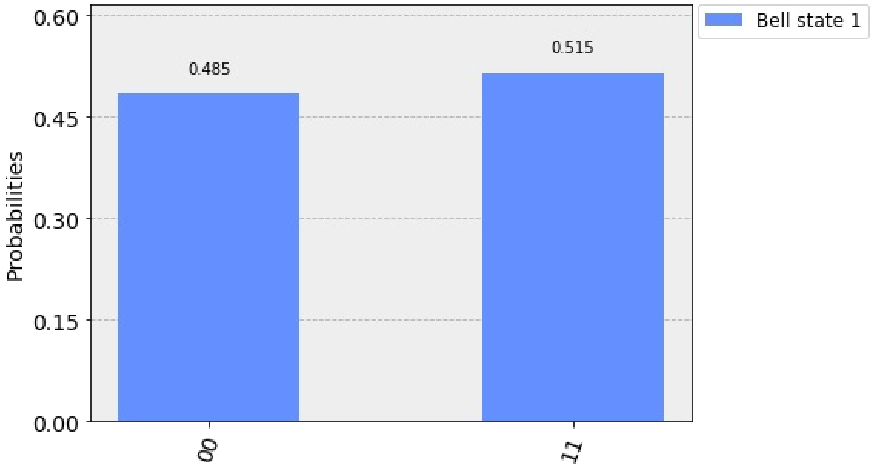
```python
print(c1.qasm())
```

```
OPENQASM 2.0;
include "qelib1.inc";
qreg q3[2];
creg c4[2];
h q3[0];
cx q3[0],q3[1];
measure q3[0] -> c4[0];
measure q3[1] -> c4[1];
```

In [15]:

```
job1 = execute(c1, sim, shots=1000)
result1 = job1.result()
count1 = result1.get_counts()
plot_histogram(count1, legend=['Bell state 1'])
```

Out[15]:

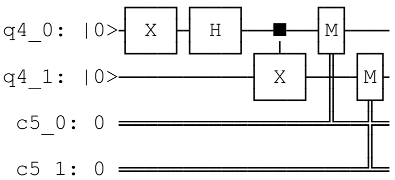


Bell state 2

In [28]:

```
c2 = QuantumCircuit(qubits, classical)
c2.x(qubits[0])
c2.h(qubits[0])
c2.cx(qubits[0], qubits[1])
c2.measure(qubits, classical)
circuit_drawer(c2)
```

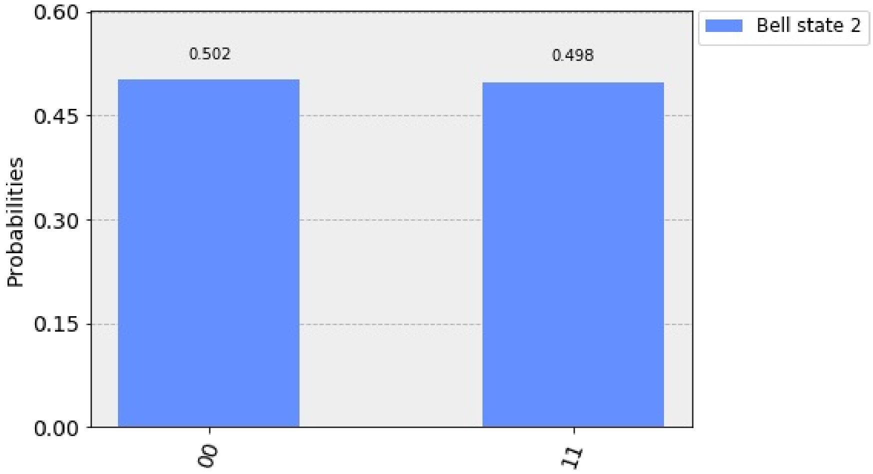
Out[28]:



In [29]:

```
job2 = execute(c2, sim, shots=1000)
result2 = job2.result()
count2 = result2.get_counts()
plot_histogram(count2, legend=['Bell state 2'])
```

Out[29]:

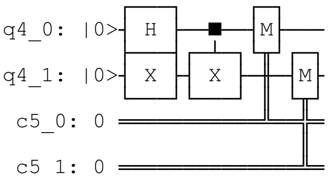


Bell state 3

In [23]:

```
c3 = QuantumCircuit(qubits, classical)
c3.x(qubits[1])
c3.h(qubits[0])
c3.cx(qubits[0], qubits[1])
c3.measure(qubits, classical)
circuit_drawer(c3)
```

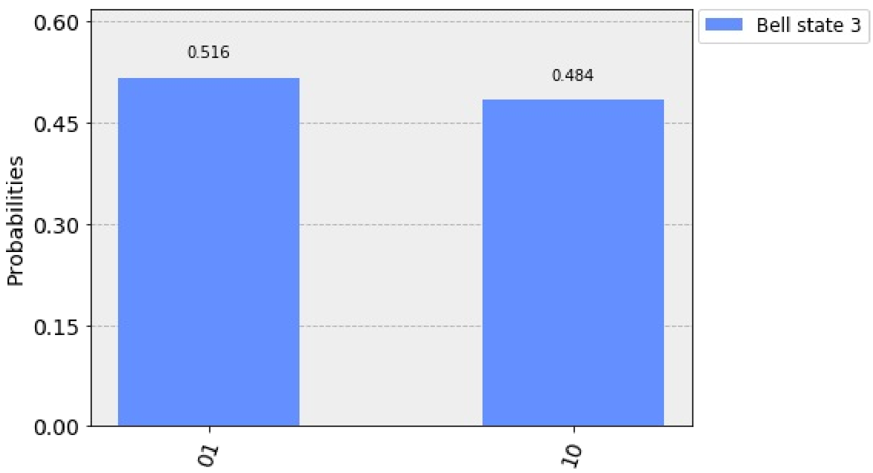
Out[23]:



In [24]:

```
job3 = execute(c3, sim, shots=1000)
result3 = job3.result()
count3 = result3.get_counts()
plot_histogram(count3, legend=['Bell state 3'])
```

Out[24]:

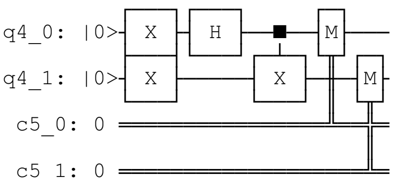


Bell state 4

In [25]:

```
c4 = QuantumCircuit(qubits, classical)
c4.x(qubits)
c4.h(qubits[0])
c4.cx(qubits[0], qubits[1])
c4.measure(qubits, classical)
circuit_drawer(c4)
```

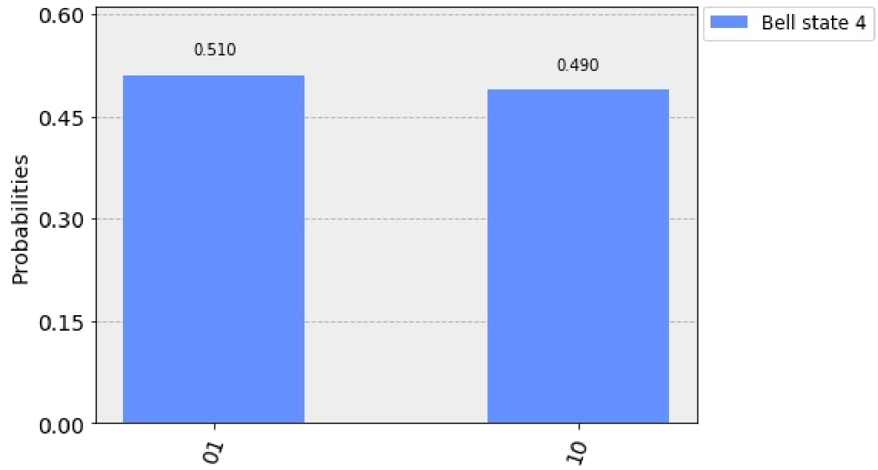
Out[25]:



In [26]:

```
job4 = execute(c4, sim, shots=1000)
result4 = job4.result()
count4 = result4.get_counts()
plot_histogram(count4, legend=['Bell state 4'])
```

Out[26]:

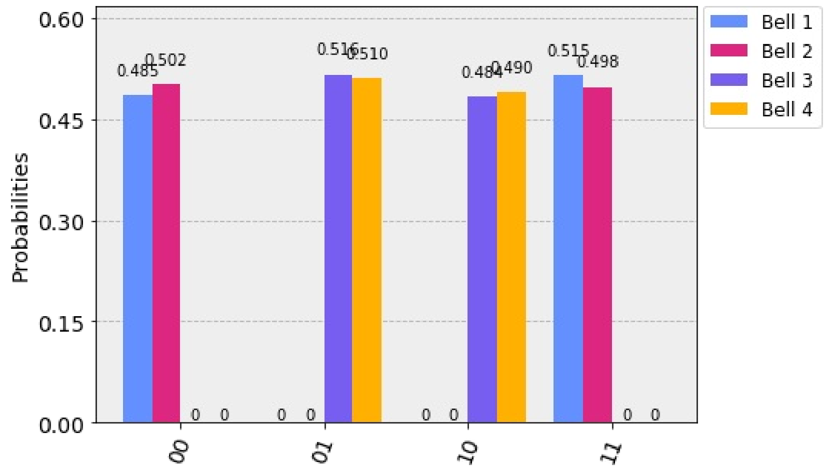


Visualization of results

In [30]:

```
legend = ['Bell 1', 'Bell 2', 'Bell 3', 'Bell 4']
plot_histogram([count1, count2, count3, count4], legend=legend)
```

Out[30]:



In [ ]:

```
```