Qiskit 7 - Shores algorithm

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In [109]:

```
import numpy as np
from math import gcd

from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit, Aer, execute, IBMQ
from qiskit.tools.visualization import circuit_drawer, plot_histogram
```

1 - Introduction

Problem

The problem of factoring integers can be expressed in such way, that based on some integer N, one want to find the integers N_1 and N_2, such that: $N_1 N_2 = N$, meanwhile: $1 < N_1, N_2 < N$

Shores algorithm approaches this problem by reducing it into finding the period of a certain function.

Algorithm

step 1 Pick an integer N and use a classical algorithm to determine if it is prime or a power of prime. If so, exit.

step 2 Randomly choose an integer a such that $1 < a < N$. Perform Euclid's algorithm to determine if the $GCD(a, N)$ is 1. If not, exit.

step 3 Use the quantum circuit represented by the unitary operator $U_{fa,N}$ to find a period r.

step 4 If a is odd, or if $a^{\frac{r}{2}} \equiv -1$ Mod N, then return to step 2 and choose another a.

step 5 Use Euclid's algorithm to calculate $GCD((a^{\frac{r}{2}} + 1), N)$ and $GCD((a^{\frac{r}{2}} - 1), N)$. Return at least one of the nontrivial solutions.

2 - Choose N

So we will work with a non-prime, non power of prime positive integer such that it is manageable on a handful couple of qubits. Allow me to put together the following list of candidates:

- List of possible integers = [4, 6, 8, 10, 12, 14, 15, 18]

In [86]:

```
N = 4
N_bit = format(N, 'b')
print('N =', N)
print('N bit =', N_bit)
```

```
N = 4
N bit = 100
```

3 - Get a

If $GCD(a, N) = 1$ a is a co-prime of N and that is what we can use in following steps. Hence we ought to pull random a and check its common factors against N, if it is a 1 we keep it.

In [87]:

```
for i in range(N):
    a = np.random.randint(2, N)
    GCD = gcd(a, N)
    if GCD is 1:
        print('gcd is 1 for a:', a)
        break
```

```
gcd is 1 for a: 3
```

4 - Phase estimation

The modular function

First up is to find the powers of $a \; Mod \; N$, that is:

$$a^0 \, Mod N, a^1 \, Mod N, a^3 \, Mod N, \ldots$$

In other words we are to find the values x of the function:

$$f_{a,N}(x) = a^x \, Mod N$$

However we are rather interested in the period r of this function such that:

$$f_{a,N}(r) = a^r \, Mod N$$

It is known from a number theory theorem that for any co-prime $a \leq N$, the function $f_{a,N}(r)$ will output a 1 for some $r \leq N$. After it hits 1, the sequence of numbers will simply repeat itself. *(isn't this a mechanism we could use for reformulated optimization problems?).*

State dynamics

Next step is to implement the function $f_{a,N}$ on a quantum circuit, in order to do that we are to proceed according to the following procedure:

1: Define 2 quantunRegisters. The first one $|x\rangle_m$ and the second one $|y\rangle_n$. Where m = the binary length needed to hold N, and where n = the binary length needed to hold the periodic base of N.

| φ

2: Place the $|x\rangle_m$ qubits in an equally weighted superposition:

| φ

3: Evaluate the function $f_{a,N}(x)$ for all the superpositioned possibilities:

| φ

4: By measuring the bottom qubits $|y\rangle_n$ we obtain an estimate $a^{\bar{x}} \, Mod N$ for some $\bar{x}$. By the periodicity of $f_{a,N}$ we also have that:

$$a^{\bar{x}} \equiv a^{\bar{x}+r} \, Mod N$$

and,

$$a^{\bar{x}} \equiv a^{\bar{x}+2r} \, Mod N$$

such that for any $s \in \mathbb{Z}$ we have:

$$a^{\bar{x}} \equiv a^{\bar{x}+sr} \, Mod N$$

Furthermore we have that out of the 2^m superpositions in $|x\rangle$ in state $|\varphi_2\rangle$, there are $\frac{2^m}{r}$ of them that has the solution $\bar{x}$. This finally gives us:

| φ

Where t_0 is the first time the measured the value of $a^{t_0} \equiv a^{\bar{x}} \, Mod N$

To boil this down, we want to estimate $\bar{x}$ since that gives us a value to plug into the function $f_{a,N}$ which gives us the period base we are looking for. Hence this stage starts with preparing the states and end with taking the measurements of $|y\rangle$ such that a satisfying value can be detected. *(Do we use that value and re instantiate the x qubits? Or is the superpoitioned x state to be 'automatically manipulated' by our measurements of y such that it simply takes on our prefered value?)*

Quantum circuit

So how do we construct the black box unitary? Allow us to establish a relationship between the period of the function $f_{a,N}$ and the phase value of the eigenvalue. this way solving the phase helps us find the period.

Phase estimation can described such that if we know U and $|\psi\rangle$, we can estimate ϕ:

U |

Proceeding with the following specifications:

$$N = 4 = 100_{bin}$$
$$a = 3$$
$$n = 3$$
$$r = 2 = 01_{bin}$$
$$freq = (1, 3)$$
$$m = 6$$

Define registers

In [95]:

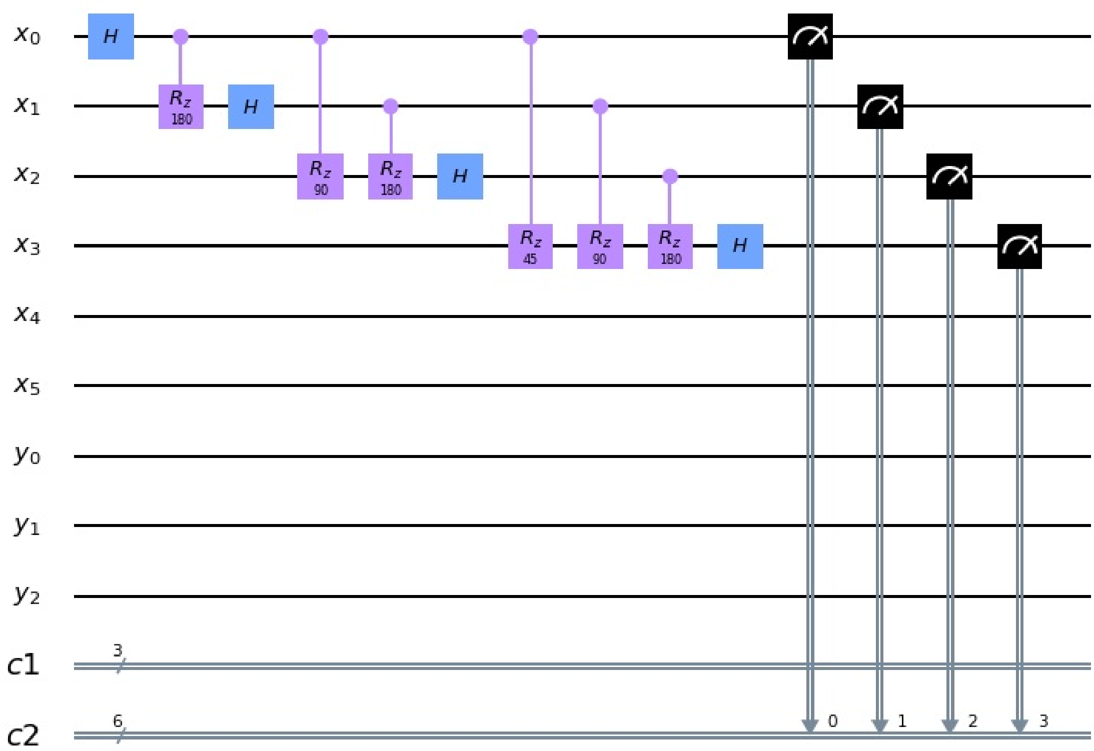
```
x = QuantumRegister(6, 'x')
y = QuantumRegister(3, 'y')
c1 = ClassicalRegister(3, 'c1')
c2 = ClassicalRegister(6, 'c2')
```

QFT - Quantum Fourier Transform

In [121]:

```
q1 = QuantumCircuit(x, y, c1, c2)
q1.barrier()
q1.h(x[0])
q1.crz(180, x[0], x[1])
q1.h(x[1])
q1.crz(90, x[0], x[2])
q1.crz(180, x[1], x[2])
q1.h(x[2])
q1.barrier()
q1.crz(45, x[0], x[3])
q1.crz(90, x[1], x[3])
q1.crz(180, x[2], x[3])
q1.h(x[3])
q1.barrier()
q1.measure([x[0], x[1], x[2], x[3]], [c2[0], c2[1], c2[2], c2[3]])
circuit_drawer(q1, output='mpl', plot_barriers=False)
```

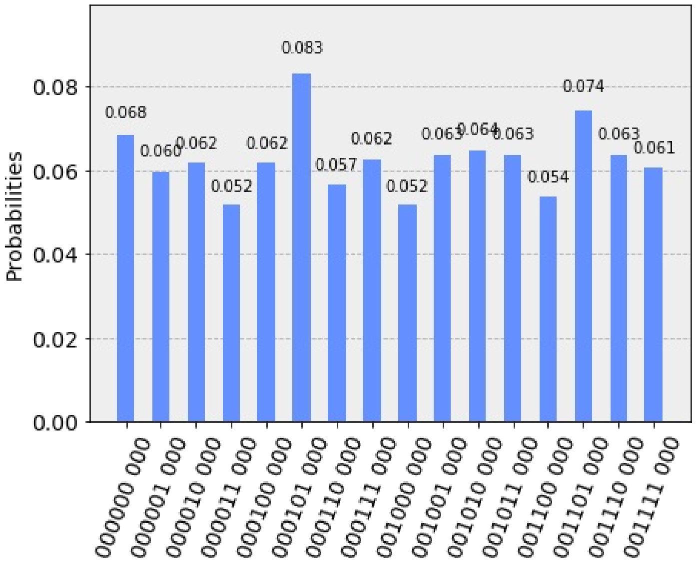
Out[121]:



In [125]:

```
sim = Aer.get_backend('qasm_simulator')
count = execute(q1, sim).result().get_counts()
plot_histogram(count)
```

Out[125]:



To be continued...

In [ ]: