# Asignment 2 Devin Maya

Sec 1.1.4) a) $A = \{ \frac{1}{n} | n$ is a pos. int.$\}$ as a subset of the real line $R^1$.

By observing $\frac{1}{n}$ we know $n$ is a positive int. as $n$ approaches infinity the sequence $\frac{1}{n}$ tends to 0. Thus the limit points are the set A itself and $\{0\}$. Thus the set is neither open or closed since it doesn't contain an open interval around each of it's points and it doesn't contain its limit point 0.

b) $B = \{ (a,b) | 1 < a^2 + b^2 \leq 2 \}$ is a subset of the plane $R^2$

Here the limit points are when $1 = a^2 + b^2$ & $a^2 + b^2 = 2$ plus the set itself. Thus the circles.

This set isn't open b/c for the points in the set there isn't an $\varepsilon$ ball that can be contained on the outer boundary of $a^2+b^2=2$

We know it is not closed since the limit points are not contained since the boundary points are not included. Therefore neither open nor closed.

c) $C=\{(a,b) \mid a \neq 0 \text{ \& } b \neq 0\}$ as a subset of the plane $\mathbb{R}^2$

Here the limits are determined to be the X-axis and the y-axis

The set is not open b/c for considering the points near the x axis & y axis there is no $\varepsilon$ ball that is contained within the set. Also it is not closed b/c it doesn't contain points on the x axis and y axis which are the limit points. Thus it is neither open or closed.

d) $D=\{(x,\sin\frac{1}{x}) \mid x \text{ is a pos. real } \#\}$ as a subset of $\mathbb{R}^2$

Here the limit point to consider is the y axis which is observable it will range from $[-1,1]$ for any $\varepsilon>0$ we can find $\frac{1}{n}$ within the $\varepsilon$

distance from $0$ such that $\sin(\frac{1}{n})$ is close to $y$, since there is oscillation from the sin function.

Thus the set itself and $(0, y)$ where $y$ is in the interval $[-1, 1]$.

For points close to the $y$ axis no open $\varepsilon$ ball is contained entirely in the set. It is not closed b/c it does not consider the $y$ axis interval ranging from $[-1, 1]$. Thus it is neither.

e) The empty set $\phi$

There are no limit points

The empty set trivially satisfies the def. of an open set which is an empty union of open sets. Also since its complement is open the empty set is closed. Thus it is both open and closed.

1.1.5 Which points of the plane are limit points of X?

We know that the set X consists of all points in the plane $R^2$ where both coordinates are rational numbers thus it is the set of all points with rational coordinates. Now to consider limit points every point in the plane $R^2$ is a limit point of X. For any point in $R^2$ consider any $\varepsilon > 0$. Then there is always a rational number $P/Q$ such that $|a - P/Q| < \varepsilon$ and another rational number $G/H$ such that $|b - G/H| < \varepsilon$ Thus the two points are in X and the distances are less than $\varepsilon$ where $\varepsilon > 0$. Thus every open set with such given point intersects X in a point other than itself. Therefore every point in $R^2$ is a limit point of X.

1.1.7 prove the second half of prop. 1.1.6

We will begin by assuming that $x$ is a limit point of the set $A$. Thus by applying that every $\varepsilon$ ball around $x$ for $\varepsilon > 0$ contains a point of $A$ that is not equal to $x$, this is from the definition of a limit point. Now by taking the sequence of $x_i$ in $A$ for every $i$ in $\mathbb{N}$ $x_i \in B_{(1/i)}x$ & $x_i \neq x$. Now for each $x_i$ we know that the open ball $B_{(1/i)}x$ contains a point in $A$ but not equal to $x$ since $x$ is a limit point of $A$. Thus a sequence can be constructed and we want to show it converges to $x$. Let $\varepsilon > 0$ & choose $N$ such that $1/N < \varepsilon$ which can be done since the sequence converges to $0$ as $n$ goes to infinity. Then for all $i \geq N$, $x_i \in B_{(1/i)}x \subseteq B_{(1/N)}x \subseteq B_{\varepsilon}(x)$ b/c $1/i \leq 1/N < \varepsilon$. Thus the sequence has a part that lies in the $\varepsilon$ ball around $x$ thus the

Sequence converges to $X$. Therefore $X$ is a limit point of $A$ there exists a sequence $X_i$ in $A$ such that $X_i \neq X$ for all $i$ & the limit of $X_i$ as $i$ approaches infinity is $X$.

1.1.8. We will let $U_i$ be a collection of open sets in $\mathbb{R}^n$. The objective is to show that the union of any collection of open sets in $\mathbb{R}^n$ is again an open set. By definition of an open set each $U_i$ for every point $x$ in $U_i$ there exists an open ball $B_{\epsilon}$ within $U_i$. Now consider a point in $U$. Such a point we will call $p$ is in $U$ if and only if it is in at least one of the sets $U_i$ since $U$ is the union of all $U_i$.

Now for every point in $U$ we can find an open ball centered at $p$ which is entirely contained in $U$ thus $U$ is an open set. Therefore the union of any collection of open sets in $\mathbb{R}^n$ is indeed an open set.

1.1.9 a) Let $x$ be a point in the intersection of the sets $U_1 \dots U_{15}$. We know that the point $x$ is in each set and the sets are open thus there is an open ball around such point $x$ in each $U_i$. From the possible balls it is optimal to choose the smallest possible one. We will denote such a ball by $B_r(x)$ which is in each $U_i$ which is in the intersection of all the $U_i$. Therefore for every point in the intersection of the $U_i$ there is an open ball around that point contained in the intersection. Thus the intersection of the $U_i$ is an open set.

b) An example of an infinite family of open sets in $\mathbb{R}^1$ whose intersection where the intersection is not open. This collection of open intervals $(-1/n, 1/n)$ for $n \in \mathbb{N}$. Here the intersection is $\{0\}$ where it is not open set in $\mathbb{R}^1$. Thus the intersection of an infinite collection of open sets may not be open.

1.1.12 Suppose $C$ is closed in $A$. By definition the complement of $C$ in $A$ is open in $A$. Thus it can be written as $A \cap U$ for some open set $U$ in $\mathbb{R}^n$. Since $U$ is open in $\mathbb{R}^n$ its complement $D$ in $\mathbb{R}^n$ is closed. Then $C = A - (A \cap U)$ so $A \cap (\mathbb{R}^n - U) = A \cap D$. $C$ is closed in $A$ it can be written as the intersection of $A$ with a closed set $D$ in $\mathbb{R}^n$. Now we must show the other side, $C = A \cup D$ for some closed set $D$ in $\mathbb{R}^n$ thus the complement of $C$ in $\mathbb{R}^n$ is the union of the complement of $A$ in $\mathbb{R}^n$ & the complement of $D$ in $\mathbb{R}^n$. Both sets are open since $A$ & $D$ are open & closed in $\mathbb{R}^n$.

$C$ is closed in $\mathbb{R}^n$ thus their union is open.

Now we will consider a point $X$ in $A$ that is not in $C$. This is to show $C$ is closed in $A$ not just in $\mathbb{R}^n$. Then there is an open set in $\mathbb{R}^n$ containing $X$ that doesn't intersect $D$ and also not intersect $C$, the open set intersected with $A$ is an open set in $A$ containing $X$ that does not intersect $C$ thus the complement of $C$ in $A$ is open in $A$. Thus $C$ is closed $A$. Therefore $C$ is closed in $A$ if & only if $C = A \cap D$ for a closed set $D$ in $\mathbb{R}^n$.

1.1.13 a) Take a point in $U$. Now considering a ball around the point which is the smallest. In this case it is possible to find points that are not in $U$. It is possible to find points where $y \neq 0$. Thus the point is not in $U$ & $U$ is not open $\mathbb{R}^2$.

b) $U$ is a subset of $\mathbb{R}^2$ since it is not a subset of $\mathbb{R}^1$ to determine if it is open in $\mathbb{R}^1$ the interval that applies to the given is $(-1,1)$.

c) A set is open if it can be written as the intersection of an open set where the open set for some $\varepsilon > 0$. $U = D^2 \cap U$ so $U$ is open in the subspace of the Disk.

1.1.14 Consider the subset of the plane to be a set with only a single point, thus this singleton point set is open in the subspace topology. where here the open set in the subspace topology is that a set is open iff it is the intersection of the subspace with an open set in the larger space. Now considering an open ball centered at $x$ where the intersection of such ball with the point $\{x\}$ so $\{x\}$ is open.

1.1.18 a) the set of rationals $\mathbb{Q}$ as a subset of the real line $\mathbb{R}^1$

Here the limit points of $\mathbb{Q}$ in $\mathbb{R}$ are all the numbers in $\mathbb{R}$ so the real numbers. Since you can consider the smallest possible interval at an arbitrary point there will always be a rational number that lies within that interval. Thus every real number is a limit point of $\mathbb{Q}$. Now we know $\mathbb{Q}$ in $\mathbb{R}^1$ has all reals as its limit points thus it is neither open or closed. Since its complement in $\mathbb{R}^1$ the set of all irrationals is not open. Now considering around any irrational number with the smallest possible epsilon ball or interval possible in this case will always have a rational number within said interval.

b) The topologist's comb

For each $X = 1/n$ it approaches any $Y$ in the interval $[0,1]$ where the limit points in the set itself also considering the line itself where $X=0$.
Now for points close to the y-axis there is no open ball around these points is contained entirely in the set.

c) Hawaiian Earring

The origin here is the limit point due to the ball around the origin always intersecting with B.

The set is not open because an open ball at the origin is not fully contained in B.
Also it is not closed because the origin is its limit point that is not contained.

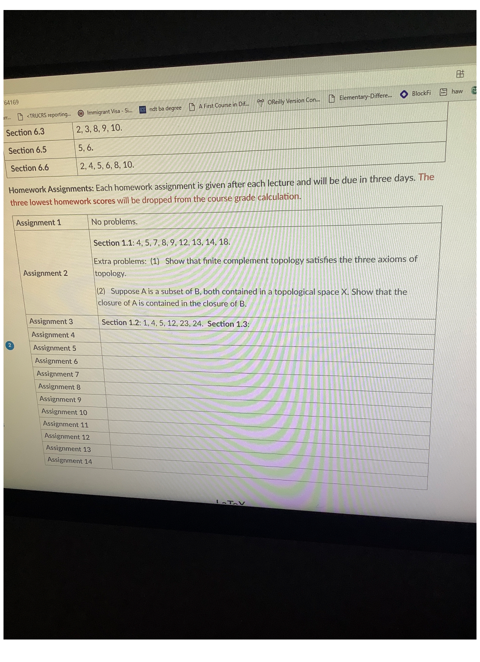
Extra Prob 1.

A subset $U$ of $X$ is open if and only if $X - U$ is finite

The empty set is open because its given vacuosly satisfied by definition. The complement of the whole set $X$ in $X$ is empty so $X$ is open due to it being finite.

Now the intersection of any finite collection of open sets is open. Let $U_i$ be a finite collection of open sets if and only if its complement is finite. Observing the complement of the union is the intersection of the complements and the collection of finite sets is finite therefore the intersection is open. Now the union of any collection of open sets is open. Here let $U_i$ be any collection of open sets then each $X - U_i$ is finite. Since the union's complement is the intersection of the complements and the intersection of a collection of finite sets is finite therefore the union is open of any collection.

2) The closure of a set P in a topological space X is the union of P and its limit points. A is a subset of B so each member of A is in B thus the limit points are present in both. Therefore the closure of A is a subset of the closure of B. where the closure of A has the points of A with the limit points of A and the same applies to B. Thus the closure of A is contained in the closure of B.



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| Section 6.3 | 2, 3, 8, 9, 10. |
| Section 6.5 | 5, 6. |
| Section 6.6 | 2, 4, 5, 6, 8, 10. |

**Homework Assignments:** Each homework assignment is given after each lecture and will be due in three days. The three lowest homework scores will be dropped from the course grade calculation.

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| Assignment 1 | No problems. |
| Assignment 2 | **Section 1.1**: 4, 5, 7, 8, 9, 12, 13, 14, 18.<br>Extra problems: (1) Show that finite complement topology satisfies the three axioms of topology.<br>(2) Suppose A is a subset of B, both contained in a topological space X. Show that the closure of A is contained in the closure of B. |
| Assignment 3 | **Section 1.2**: 1, 4, 5, 12, 23, 24. **Section 1.3**: |
| Assignment 4 | |
| Assignment 5 | |
| Assignment 6 | |
| Assignment 7 | |
| Assignment 8 | |
| Assignment 9 | |
| Assignment 10 | |
| Assignment 11 | |
| Assignment 12 | |
| Assignment 13 | |
| Assignment 14 | |

Followed Assignment 2 problems,