1) Continuing the argument in order to deduce that there are no 2×2 arrays satisfy the condition. Now instead of placing $(1,1)$ in the top left corner let us place $(1,2)$ in the top left corner. Thus if we put $(1,2)$ in the top left corner,

$$\begin{bmatrix} (1,2) & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

then based on the requirement, the top right corner, as well as the bottom left corner, have to be an officer of the second team and first rank $(2,1)$. Thus

$$\begin{bmatrix} (1,2) & (2,1) \\ (2,1) & \end{bmatrix}$$

However this cannot be a solution since there is only one possible officer $(2,1)$ where he cannot be repeated. Thus $(1,2)$ cannot be placed in the top left corner since it leads to a contradiction.

Now going down the remaining two options for the top left corner which are $(2,1)$ & $(2,2)$

① $\begin{bmatrix} (2,1) & \_ \\ \_ & \_ \end{bmatrix}$ & ② $\begin{bmatrix} (2,2) & \_ \\ \_ & \_ \end{bmatrix}$

For case ① where $(2,1)$ is in the top left corner the top right and bottom left must be $(1,2)$ which will again repeat the same officer. This contradiction for the needed condition to properly construct such a square will also occur for case ② when $(2,2)$ is in the top left corner without loss of generality. Since the only option is (1,1) for the top right & bottom left. Therefore there are no 2x2 arrays that satisfy the condition.

2) Now if selecting 9 military officers from 3 different teams & 3 different ranks to form a 3x3 array.

Using (i,j) to still indicate that the officer is of i team & j rank.

Thus the 9 total distinct officers are

(1,1) (1,2) (1,3) (2,1) (2,2) (2,3) (3,1) (3,2) (3,3)

By placing (1,1) in the first row & first column which is the first entry where such a 3x3 is numbered by entries as follows:

1	2	3
4	5	6
7	8	9

After placing (1,1) in the first entry

1	2	3
4	5	6
7	8	9

$\longrightarrow$

(1,1)	2	3
4	5	6
7	8	9

We know that entry 2 and entry 4 must be officers that are not in team 1 and not rank 1. Thus the possible options are (2,2) (2,3) (3,2) for entries 2 & 4.

By placing (2,2) for entry 2 and (2,3) for entry 4 the square becomes:

(1,1)	2	3
4	5	6
7	8	9

$\longrightarrow$

(1,1)	(2,2)	3
(2,3)	5	6
7	8	9

Now observing that in the 7^{th} entry the officer must be of team 3 and rank 2. Also observing for entry 3 the officer must be of team 3 and rank 3. Thus the square becomes:

(1,1)	(2,2)	3
(2,3)	5	6
7	8	9

$\longrightarrow$

(1,1)	(2,2)	(3,3)
(2,3)	5	6
(3,2)	8	9

observing that entry 5 must either be of team 1 or 3 but must be of rank 1. The 4 remaining officers are (1,2), (1,3), (2,1), (3,1). From these 4 officer (3,1) satisfies the required condition for entry 5. Thus after solving for entry 5 it is straightforward observing the solutions for entry 6 and entry 8. Where entry 6 must be of team 1 and rank 2 so entry 6 is (1,2)

Now for entry 8 it must be officer of team 1 and rank 3 so entry 8 is (1,3). The final entry 9 is the remaining officer (2,1) which also satisfies the condition.

(1,1)	(2,2)	(3,3)
(2,3)	(3,1)	6
(3,2)	8	9

$\rightarrow$

(1,1)	(2,2)	(3,3)
(2,3)	(3,1)	(1,2)
(3,2)	(1,3)	(2,1)

Therefore we have shown in a 3X3 case an arrangement that satisfies the given conditions. It is shown that each row and each column contain officers from different teams and different ranks.

3) was shown in discussion section to not be solvable since the construction in such a way will lead to a row or column where an officer is of the same team or rank thus leading to contradictions in the requirements.

Therefore it is not possible to construct an order 8 Euler square using any 2 different order 4 Latin squares. This is because the same issue observed in problem 1 will occur.

4) Beginning by constructing an order n Euler square when $n \geq 3$. For the case when $n = 3$ forming a latin square with left shift operations

is

	1	2	3
ℓ	2	3	1
ℓ^2	3	1	2

.

Where row 2 has operation ℓ & row 3 has operation ℓ^2 or ℓ^{n-1}.

Now a latin square using the right shift

is

	1	2	3
r	3	1	2
r^2	2	3	1

.

Where row 2 has operation r and row 3 has operation r^2 since r^{n-1}

Thus after overlapping the two results in

(l, r)
(l^2, r^2)

(1,1)	(2,2)	(3,3)
(2,3)	(3,1)	(1,2)
(3,2)	(1,3)	(2,1)

This is an Euler square of order 3.

Now for an order n Euler square where n is an odd number. If n is odd and the first latin square is after performing the left shift to the n rows up to l^{n-1}. Another latin square is obtained by performing the right shift for n rows up to r^{n-1}. When the two squares are combined, the two Latin squares will form an Euler square. This is because by the left shift and right shift operations it is guaranteed that each row and column will have distinct entries thus after combining the two Latin squares the entries remain unique across the entire square producing an Euler square.

However, observing for when n is even

after performing l, r shifts that is

for example $n = 4$

	1	2	3	4
l	2	3	4	1
l^2	3	4	1	2
l^3	4	1	2	3

	1	2	3	4
r	4	1	2	3
r^2	3	4	1	2
r^3	2	3	4	1

After combining the two Latin squares

	(1,1)	(2,2)	(3,3)	(4,4)
(λ, r)	(2,4)	(3,1)	(4,2)	(1,3)
(λ^2, r^2)	(3,3)	(4,4)	(1,1)	(2,2)
(λ^3, r^3)	(4,2)	(1,3)	(2,4)	(3,1)

This is clearly not an Euler square since many entries repeat more than once thus each entry is not distinct. This is true for when n is even since there will always be repetitions which is a violation of an Euler square therefore this method does not work for even numbers.