Analisis Factorial

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Introduccion

Análisis factorial es una técnica estadística de reducción de datos usada para explicar las correlaciones entre las variables observadas en términos de un número menor de variables no observadas llamadas factores. Las variables observadas se modelan como combinaciones lineales de factores más expresiones de error. El análisis factorial se originó en psicometría, y se usa en las ciencias del comportamiento tales como ciencias sociales, marketing, gestión de productos, investigación operativa, y otras ciencias aplicadas que tratan con grandes cantidades de datos.

Matriz

1.- Lectura de la matriz de datos

```
x<-as.data.frame(state.x77)
```

2.- Quitar los espacios de los nombres

```
colnames(x)[4]="Life.Exp"
colnames(x)[6]= "HS.Grad"
```

3.- Separa n (estados) y p (variables)

```
n<-dim(x)[1]
p<-dim(x)[2]

dim(x)
```

```
## [1] 50  8
```

```
str(x)
```

```
## 'data.frame':    50 obs. of  8 variables:
##  $ Population: num  3615 365 2212 2110 21198 ...
##  $ Income    : num  3624 6315 4530 3378 5114 ...
##  $ Illiteracy: num  2.1 1.5 1.8 1.9 1.1 0.7 1.1 0.9 1.3 2 ...
##  $ Life.Exp  : num  69 69.3 70.5 70.7 71.7 ...
##  $ Murder    : num  15.1 11.3 7.8 10.1 10.3 6.8 3.1 6.2 10.7 13.9 ...
##  $ HS.Grad   : num  41.3 66.7 58.1 39.9 62.6 63.9 56 54.6 52.6 40.6 ...
##  $ Frost     : num  20 152 15 65 20 166 139 103 11 60 ...
##  $ Area      : num  50708 566432 113417 51945 156361 ...
```

```
colnames(x)
```

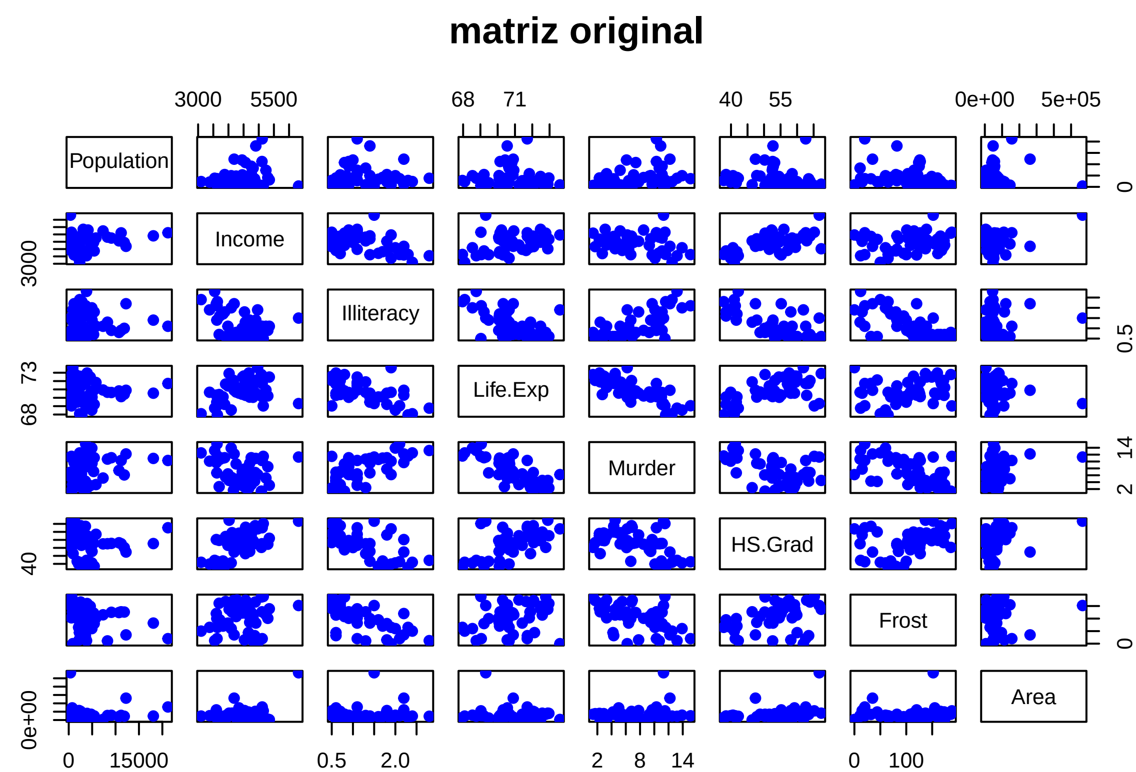
```
## [1] "Population" "Income"     "Illiteracy" "Life.Exp"   "Murder"
## [6] "HS.Grad"    "Frost"      "Area"
```

```
anyNA(x)
```

```
## [1] FALSE
```

4.- Generacion de un scater plot para la visualizacion de variables originales

```
pairs(x, col="blue", pch=19, main="matriz original")
```



Transformacion de alguna varibles

1.- Aplicamos logaritmo para las columnas 1,3 y 8

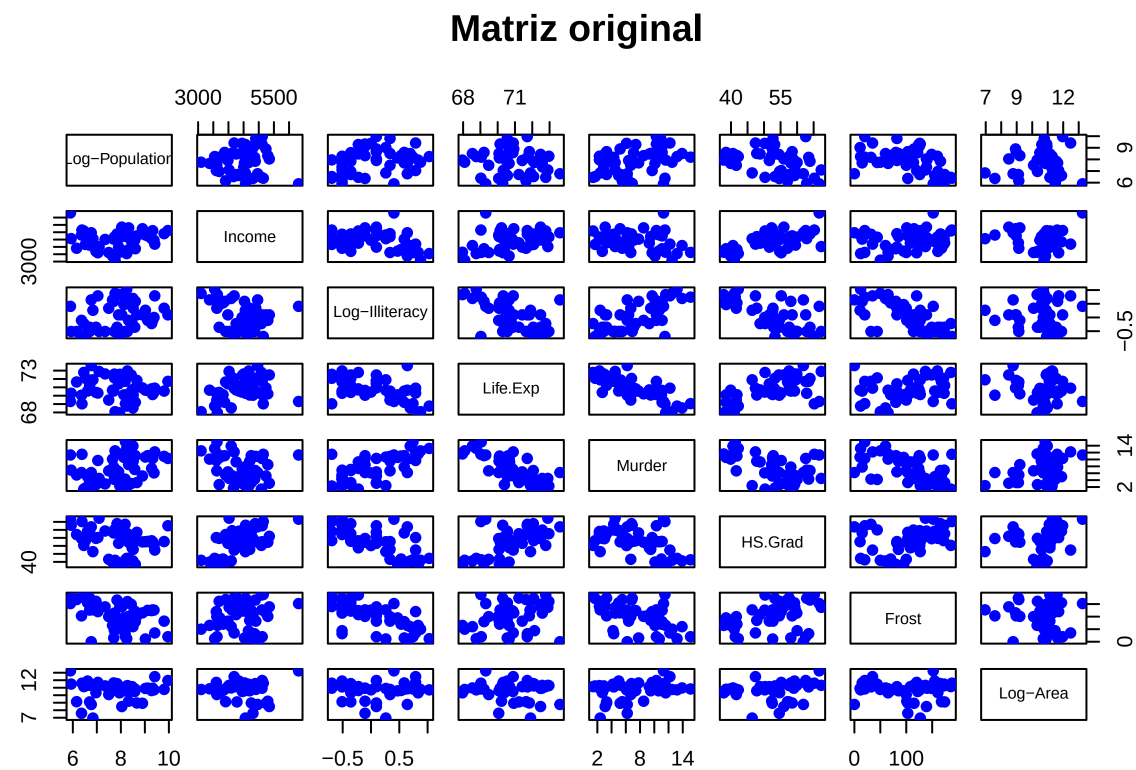
```
x[,1]<-log(x[,1])
colnames(x)[1]<-"Log-Population"

x[,3]<-log(x[,3])
colnames(x)[3]<-"Log-Illiteracy"
```

```
x[,8]<-log(x[,8])
colnames(x)[8]<-"Log-Area"
```

Creacion del grafico

Grafico scater para la visualizacion de la matriz original con 3 variables que se incluyeron.

```
pairs(x,col="blue", pch=19, main="Matriz original")
```



Nota: Como las variables tiene diferentes unidades de medida, se va a implementar la matriz de correlaciones para estimar la matriz de carga

Analisis Factorial de componentes principales (PCFA)

1.- Calcular la matriz de medias y de correlaciones Matriz de medias

```
mu<-colMeans(x)
mu
```

```
## Log-Population          Income Log-Illiteracy        Life.Exp          Murder
##    7.863443e+00    4.435800e+03    3.128251e-02    7.087860e+01    7.378000e+00
```

```
##       HS.Grad         Frost       Log-Area
##   5.310800e+01   1.044600e+02   1.066237e+01
```

Matriz de correlaciones

```r
R<-cor(x)
R
```

```
##                Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population     1.00000000  0.034963788     0.28371749 -0.1092630  0.3596542
## Income             0.03496379  1.000000000    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy     0.28371749 -0.351477726     1.00000000 -0.5699943  0.6947320
## Life.Exp          -0.10926301  0.340255339    -0.56999432  1.0000000 -0.7808458
## Murder             0.35965424 -0.230077610     0.69473198 -0.7808458  1.0000000
## HS.Grad           -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost             -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area           0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                   HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         1.0000000  0.36677970  0.196743429
## Frost           0.3667797  1.00000000 -0.021211992
## Log-Area        0.1967434 -0.02121199  1.000000000
```

2.- Reduccion de la dimensionalidad mediante Analisis factorial de componentes principales (PCFA).

1.- Calcular los valores y vectores propios.

```r
eR<-eigen(R)
```

2.- Valores propios

```r
eigen.val<-eR$values
eigen.val
```

```
## [1] 3.6796976 1.3201021 1.1357357 0.7517550 0.6168266 0.2578511 0.1366186
## [8] 0.1014132
```

3.- Vectores propios

```r
eigen.vec<-eR$vectors
eigen.vec
```

```
##             [,1]        [,2]        [,3]       [,4]        [,5]       [,6]
## [1,] -0.23393451 -0.41410075  0.50100922  0.2983839  0.58048485  0.0969034
## [2,]  0.27298977 -0.47608715  0.24689968 -0.6449631  0.09036625 -0.3002708
## [3,] -0.45555443  0.04116196  0.12258370 -0.1824471 -0.32684654 -0.6084112
## [4,]  0.39805075 -0.04655529  0.38842376  0.4191134 -0.26287696 -0.3565095
```

```
## [5,] -0.44229774 -0.27640285 -0.21639177 -0.2610739  0.02383706  0.1803894
## [6,]  0.41916283 -0.36311753 -0.06807465 -0.1363534 -0.34015424  0.3960855
## [7,]  0.36358674  0.21893783 -0.37542494 -0.1299519  0.59896253 -0.3507630
## [8,] -0.03545293 -0.58464797 -0.57421867  0.4270918 -0.06252285 -0.3012063
##            [,7]        [,8]
## [1,] -0.1777562 -0.23622413
## [2,]  0.3285840  0.12483849
## [3,] -0.3268997 -0.39825363
## [4,] -0.3013983  0.47519991
## [5,] -0.4562245  0.60970476
## [6,] -0.4808140 -0.40675672
## [7,] -0.4202943 -0.06001175
## [8,]  0.2162424 -0.05831177
```

4.- Calcular la proporcion de variabilidad

```
prop.var<-eigen.val/sum(eigen.val)
prop.var
```

```
## [1] 0.45996220 0.16501277 0.14196697 0.09396938 0.07710332 0.03223139 0.01707733
## [8] 0.01267665
```

5.- Calcular la proporcion de variabilidad acumulada

```
prop.var.acum<-cumsum(eigen.val)/sum(eigen.val)
prop.var.acum
```

```
## [1] 0.4599622 0.6249750 0.7669419 0.8609113 0.9380146 0.9702460 0.9873233
## [8] 1.0000000
```

Estimacion de la matriz de carga

Se estima la matriz de carga usando los autovalores y autovectores. Se aplica la rotacion varimax

Primera estimacion de Lamda mayuscula Se calcula multiplicando la matriz de los 3 primeros autovectores por la matriz diagonal formada por la raiz cuadrada de los primeros 3 autovalores.

```
L.est.1<-eigen.vec[,1:3] %*% diag(sqrt(eigen.val[1:3]))
L.est.1
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.44874575 -0.47578394  0.53393005
## [2,]  0.52366367 -0.54700365  0.26312322
## [3,] -0.87386900  0.04729332  0.13063856
## [4,]  0.76356236 -0.05349003  0.41394671
## [5,] -0.84843932 -0.31757498 -0.23061066
## [6,]  0.80406070 -0.41720642 -0.07254777
## [7,]  0.69745163  0.25155014 -0.40009375
## [8,] -0.06800771 -0.67173536 -0.61195003
```

Rotacion varimax

```r
L.est.1.var<-varimax(L.est.1)
L.est.1.var
```

```
## $loadings
##
## Loadings:
##      [,1]   [,2]   [,3]  
## [1,]                0.840
## [2,]  0.785 -0.106  0.121
## [3,] -0.665         0.583
## [4,]  0.763  0.384 -0.168
## [5,] -0.573 -0.528  0.517
## [6,]  0.825 -0.202 -0.323
## [7,]  0.281        -0.794
## [8,]        -0.906       
##
##                 [,1]  [,2]  [,3]
## SS loadings    2.744 1.300 2.091
## Proportion Var 0.343 0.163 0.261
## Cumulative Var 0.343 0.506 0.767
##
## $rotmat
##            [,1]      [,2]       [,3]
## [1,]  0.7824398 0.1724744 -0.5983649
## [2,] -0.5274231 0.6944049 -0.4895169
## [3,]  0.3310784 0.6986089  0.6342970
```

Estimacion de la matriz de los errores

1.- Estimacion de la matriz de perturbaciones

```r
Psi.est.1<-diag(diag(R-as.matrix(L.est.1.var$loadings)%*% t(as.matrix(L.est.1.var$loadings))))
Psi.est.1
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]      [,7]
## [1,] 0.2871756 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [2,] 0.0000000 0.3573295 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2170499 0.0000000 0.0000000 0.000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.2427595 0.0000000 0.000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1261156 0.000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.174162 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.2902087
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
```

```
## [7,] 0.0000000
## [8,] 0.1696637
```

2.- Se utiliza el metodo Analisis de factor principal (PFA) para estimacion de autovalores y autovectores

```r
RP<-R-Psi.est.1
RP
```

```
##                 Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population      0.71282441  0.034963788     0.28371749 -0.1092630  0.3596542
## Income              0.03496379  0.642670461    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy      0.28371749 -0.351477726     0.78295012 -0.5699943  0.6947320
## Life.Exp           -0.10926301  0.340255339    -0.56999432  0.7572405 -0.7808458
## Murder              0.35965424 -0.230077610     0.69473198 -0.7808458  0.8738844
## HS.Grad            -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost              -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area            0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                    HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         0.8258380  0.36677970  0.196743429
## Frost           0.3667797  0.70979126 -0.021211992
## Log-Area        0.1967434 -0.02121199  0.830336270
```

Calculo de la matriz de autovalores y autovectores

```r
eRP<-eigen(RP)
```

Autovalores

```r
eigen.val.RP<-eRP$values
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

Autovectores

```r
eigen.vec.RP<-eRP$vectors
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

Proporcion de variabilidad

```r
prop.var.RP<-eigen.val.RP/ sum(eigen.val.RP)
prop.var.RP
```

```
## [1]  0.564152306  0.180134556  0.143675179  0.079382934  0.057632455
## [6]  0.004585668 -0.011014811 -0.018548286
```

Proporcion de variabilidad acumulada

```r
prop.var.RP.acum<-cumsum(eigen.val.RP)/ sum(eigen.val.RP)
prop.var.RP.acum
```

```
## [1] 0.5641523 0.7442869 0.8879620 0.9673450 1.0249774 1.0295631 1.0185483
## [8] 1.0000000
```

Estimacion de la matriz de cargas con rotacion varimax

```r
L.est.2<-eigen.vec.RP[,1:3] %*% diag(sqrt(eigen.val.RP[1:3]))
L.est.2
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.42621819 -0.27609775  0.56228420
## [2,]  0.48528446 -0.36092954  0.32467098
## [3,] -0.84791581  0.08163995  0.10816670
## [4,]  0.73812189  0.02688907  0.36866093
## [5,] -0.84699944 -0.34227865 -0.12211117
## [6,]  0.78817342 -0.40399024  0.04935203
## [7,]  0.66112453  0.12457105 -0.40191996
## [8,] -0.06868291 -0.77165602 -0.36531090
```

Rotacion varimax

```r
L.est.2.var<-varimax(L.est.2)
```

Estimacion de la matriz de covarianzas de los errores

```r
Psi.est.2<-diag(diag(R-as.matrix(L.est.2.var$loadings)%*% t(as.matrix(L.est.2.var$loadings))))
Psi.est.2
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4259446 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [2,] 0.0000000 0.5288176 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2626737 0.0000000 0.0000000 0.0000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.3185422 0.0000000 0.0000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1505261 0.0000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.2131389 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.3858568
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
```

```
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
## [7,] 0.0000000
## [8,] 0.2663776
```

Obtencion de los scores de ambos metodos PCFA

```
FS.est.1<-scale(x)%*% as.matrix(L.est.1.var$loadings)
FS.est.1
```

```
##                       [,1]          [,2]       [,3]
## Alabama        -5.84072356 -1.3993671511  4.0008109
## Alaska          2.12443806 -3.6163397014 -1.3435941
## Arizona        -0.77245459 -1.1030150088  1.7864181
## Arkansas       -4.26961555 -0.1287634469  1.8680205
## California      1.57843978 -1.6386262821  3.0959757
## Colorado        3.35619481 -0.5747409714 -1.9955520
## Connecticut     2.96609993  2.5265114588 -1.0120520
## Delaware        0.15111765  2.2707877284 -1.3473631
## Florida        -0.91278118 -0.8518787165  3.2141818
## Georgia        -5.10406769 -1.5374188978  3.5972606
## Hawaii          1.68679592  2.0782245763  0.6972161
## Idaho           1.93931571  0.0374520725 -2.6403015
## Illinois        0.36572803 -0.9730363911  1.3246992
## Indiana         0.69870165  0.1740586327 -0.1660034
## Iowa            3.77325852  0.8634090197 -2.4308546
## Kansas          3.22079390  0.2206198504 -1.7333568
## Kentucky       -3.97957229 -0.1711842990  1.8581455
## Louisiana      -6.15095874 -1.1449716511  4.2193388
## Maine           0.38912287  0.9352663421 -2.8385772
## Maryland        0.54556931  0.6481615589  0.7313943
## Massachusetts   1.95531363  1.9508870989 -0.0699601
## Michigan        0.06109118 -0.8995742724  1.1610156
## Minnesota       3.83625590  0.7199310360 -2.2609012
## Mississippi    -6.73875213 -1.1336057288  3.0124928
## Missouri       -0.63621057 -0.5673516660  0.5606479
## Montana         1.70022911 -0.7530855537 -2.9827203
## Nebraska        3.31393569  0.5702899251 -2.6630094
## Nevada          1.83953234 -2.1624547546 -2.8632403
## New Hampshire   1.76672303  1.8835104424 -3.2522623
## New Jersey      1.23076573  1.5154423999  0.6483326
## New Mexico     -2.42369795 -1.2184859435  0.1095350
## New York       -0.55160991 -0.8431042602  2.9025469
## North Carolina -4.53932589 -0.7126552652  2.8168209
## North Dakota    3.26810535  1.0664889529 -3.5180166
## Ohio            0.67643704 -0.0394642439  0.5816740
## Oklahoma       -0.43628926  0.0293430043  0.2108486
## Oregon          2.64633236 -0.0126633017 -0.6563722
## Pennsylvania   -0.06313819  0.0425262164  0.8538298
```

```
## Rhode Island    0.25059508  4.0533333045 -1.3779994
## South Carolina -6.20030464 -0.7067780563  3.0142562
## South Dakota    2.51505516  0.8539599931 -3.9694575
## Tennessee      -3.75602365 -0.3764569265  2.4225536
## Texas          -2.74825842 -2.0176142597  4.0126966
## Utah            3.40911641  0.2638533973 -3.0642167
## Vermont         1.26368503  1.7670538099 -3.5748058
## Virginia       -1.45435214 -0.4332714574  1.8388594
## Washington      2.95298764  0.0002978623 -0.1436737
## West Virginia  -3.41599674  0.5649932020  0.5132111
## Wisconsin       2.58972274  0.8701285803 -1.5397225
## Wyoming         1.92267355 -0.8906222579 -3.6087703
```

PFA

```
FS.est.2<-scale(x)%*% as.matrix (L.est.2.var$loadings)
FS.est.2
```

```
##                       [,1]         [,2]       [,3]
## Alabama        -5.69766092 -1.133005866  3.9030908
## Alaska          1.77921500 -3.310049553 -1.2425530
## Arizona        -0.80948635 -1.007423566  1.6833688
## Arkansas       -4.04451164 -0.036340306  1.8899610
## California      1.28900772 -1.589528660  2.7938220
## Colorado        3.21256763 -0.645092519 -1.9103448
## Connecticut     2.85639977  2.291700954 -1.1152442
## Delaware        0.22491218  2.168332191 -1.3109174
## Florida        -1.04778981 -0.760012075  2.9630979
## Georgia        -5.04193484 -1.243399542  3.4848855
## Hawaii          1.64548810  1.848120424  0.5487863
## Idaho           1.99602286 -0.067186945 -2.4442739
## Illinois        0.17329771 -0.870927790  1.1838509
## Indiana         0.66348403  0.140717116 -0.1900850
## Iowa            3.70915552  0.657976435 -2.3698485
## Kansas          3.13617617  0.071725764 -1.6894853
## Kentucky       -3.82119443 -0.051170443  1.8492550
## Louisiana      -5.97309240 -0.880509145  4.1021292
## Maine           0.58567717  0.845398887 -2.6098620
## Maryland        0.40855637  0.650876372  0.5867974
## Massachusetts   1.91021424  1.761365924 -0.1964750
## Michigan       -0.07208772 -0.823049544  1.0671998
## Minnesota       3.74953682  0.518054623 -2.2104937
## Mississippi    -6.45121865 -0.852611917  3.0320154
## Missouri       -0.64446964 -0.519762510  0.5472506
## Montana         1.72574501 -0.752576236 -2.7507980
## Nebraska        3.28773039  0.392513546 -2.5439122
## Nevada          1.69672312 -1.994626548 -2.6292009
## New Hampshire   1.87991014  1.704867403 -3.0632652
## New Jersey      1.10782292  1.425042094  0.4638907
## New Mexico     -2.26112419 -1.086582245  0.2653217
## New York       -0.72255151 -0.744949928  2.6624378
```

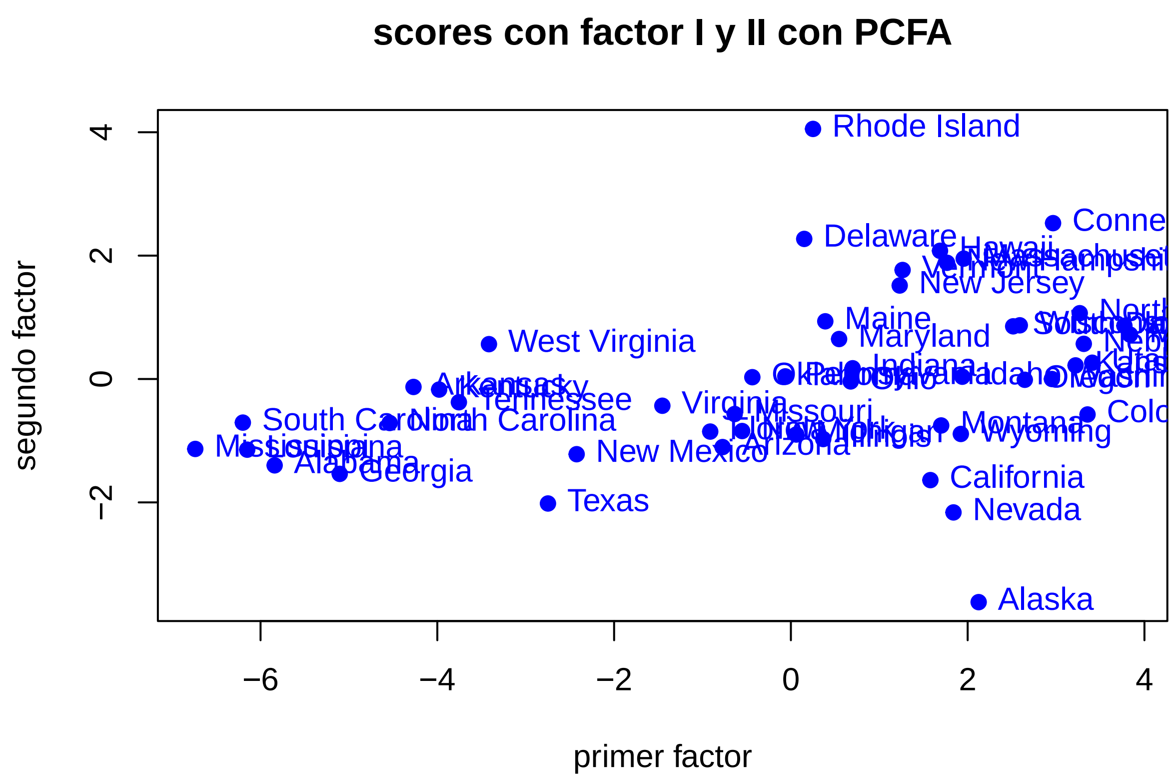
```
## North Carolina -4.42441540 -0.513264749  2.7372284
## North Dakota    3.22068093  0.897031063 -3.3556310
## Ohio            0.59453054 -0.051780182  0.4905274
## Oklahoma       -0.36512462  0.000708499  0.2244101
## Oregon          2.56050584 -0.129810062 -0.6934180
## Pennsylvania   -0.10451900  0.054229408  0.7553645
## Rhode Island    0.40356926  3.785456289 -1.3760426
## South Carolina -5.98815271 -0.435831413  2.9745853
## South Dakota    2.60764548  0.683975660 -3.7117087
## Tennessee      -3.63769564 -0.249263663  2.3593673
## Texas          -2.80670233 -1.827474308  3.8156526
## Utah            3.44131011  0.069209103 -2.8669774
## Vermont         1.44160727  1.580578146 -3.3086066
## Virginia       -1.50774364 -0.328200587  1.7151967
## Washington      2.81601549 -0.109025242 -0.2503494
## West Virginia  -3.18525955  0.632647668  0.5745805
## Wisconsin       2.55487697  0.699000994 -1.5141208
## Wyoming         1.92835024 -0.866073018 -3.3204601
```

Graficamos ambos scores

```r
par(mfrow=c(2,1))
```

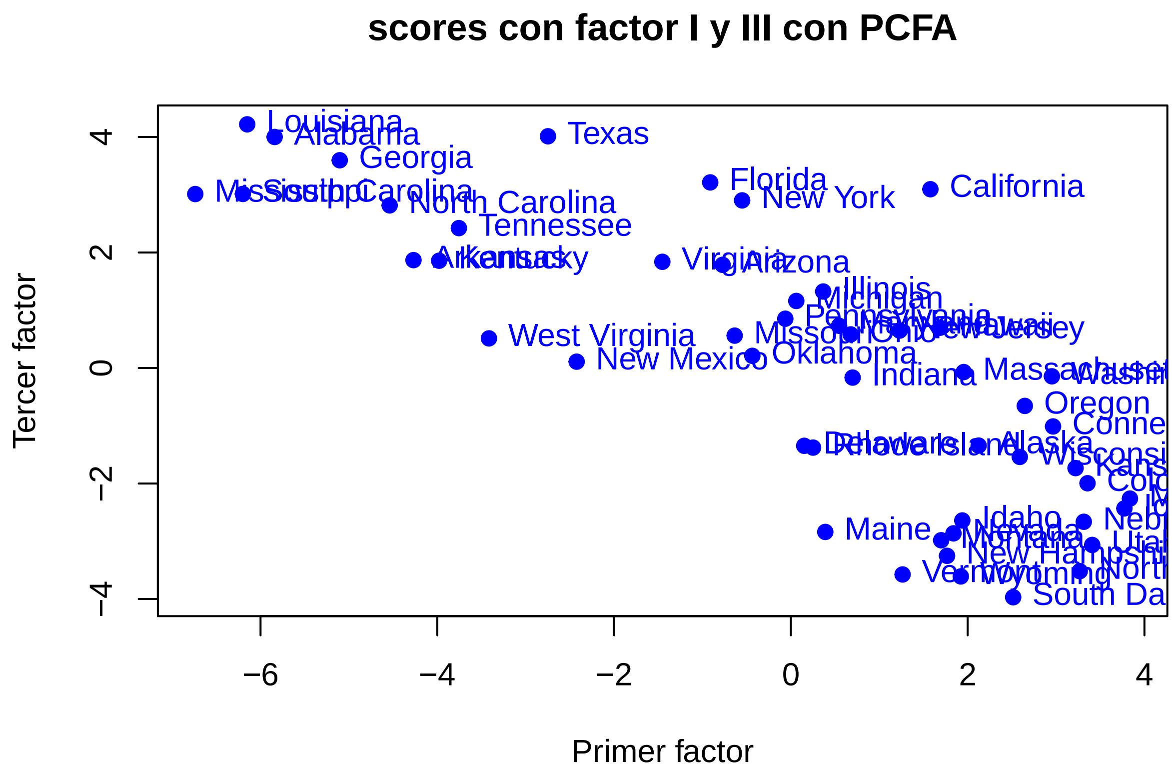
Factor I y II

```r
pl1<-plot(FS.est.1[,1], FS.est.1[,2], xlab="primer factor",
     ylab="segundo factor", main="scores con factor I y II con PCFA",
     pch=19, col="blue")
text(FS.est.1[,1], FS.est.1[,2], labels = rownames(x), pos=4, col="blue")
```



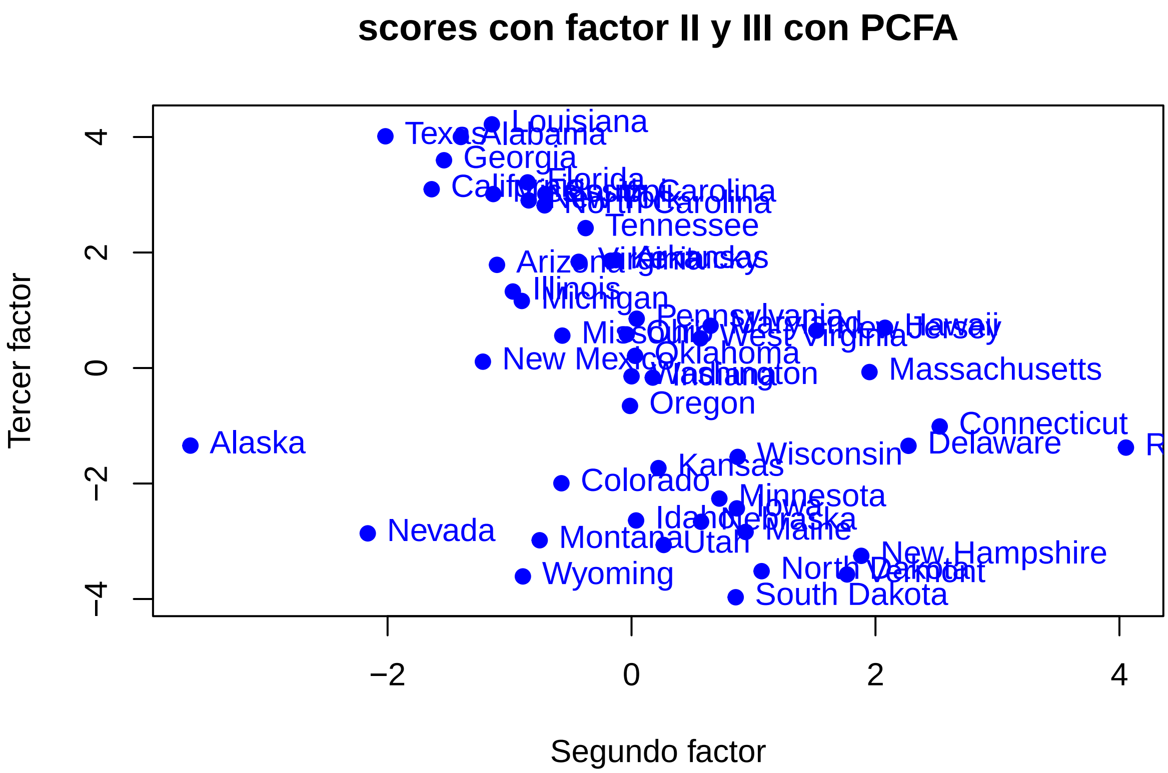
Factor I y III

```
pl2<-plot(FS.est.1[,1], FS.est.1[,3], xlab="Primer factor",
      ylab="Tercer factor", main="scores con factor I y III con PCFA",
      pch=19, col="blue")
text(FS.est.1[,1], FS.est.1[,3], labels = rownames(x), pos=4, col="blue")
```



Factor II y III

```
pl3<-plot(FS.est.1[,2], FS.est.1[,3], xlab="Segundo factor",
     ylab="Tercer factor", main="scores con factor II y III con PCFA",
     pch=19, col="blue")
text(FS.est.1[,2], FS.est.1[,3], labels = rownames(x), pos=4, col="blue")
```


scores con factor II y III con PCFA
Tercer factor
Segundo factor
4
2
0
−2
−4
−2
0
2
4
Louisiana
Texas
Alabama
Georgia
California
Florida
North Carolina
Tennessee
Arizona
Arkansas
Illinois
Michigan
Pennsylvania
Hawaii
Missouri
West Virginia
New Mexico
Oklahoma
Washington
Massachusetts
Oregon
Alaska
Connecticut
Delaware
Wisconsin
Kansas
Colorado
Minnesota
Iowa
Nevada
Idaho
Nebraska
Maine
Montana
Utah
New Hampshire
North Dakota
Vermont
Wyoming
South Dakota