

# Numerical Relativity

## Exercise-1

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Generally: We are studying Sod Shock Tube problem and the Tolman-Oppenheimer-Volkoff (TOV) equation evolution using Einstein tool kit.

## Exercise 2-1

Sod problem (A method stimulates any Hydrodynamical system on a computer. This case represents a 1 Dimensional tube): We considered the non-relativistic case for an ideal fluid ( $P = (\gamma - 1)\rho\varepsilon$ ) applying the HLLE (Riemann solver). It has been found as the grid resolution increases, the stimulated rest mass density converges toward the exact solution (Figure 1). These distortions are related to the second and first order error  $[\Delta x^2, \Delta x]$  for the rarefaction and the shock wave respectively as explained mathematically in Godunov method. One can see for the rarefaction distortion diminish as we increase the resolution more than the shock.

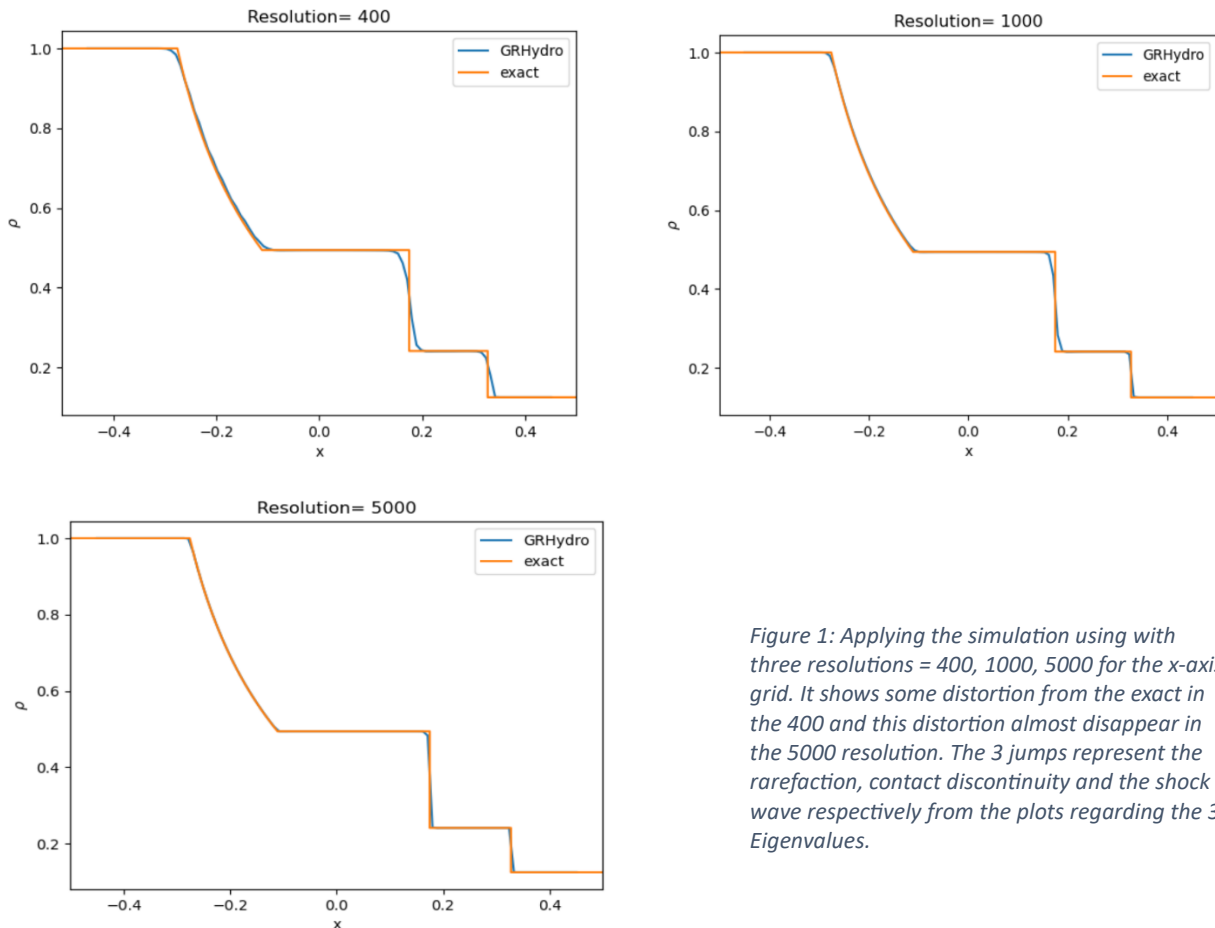


Figure 1: Applying the simulation using with three resolutions = 400, 1000, 5000 for the x-axis grid. It shows some distortion from the exact in the 400 and this distortion almost disappear in the 5000 resolution. The 3 jumps represent the rarefaction, contact discontinuity and the shock wave respectively from the plots regarding the 3 Eigenvalues.

## Exercise 2-2

TOV evolution of the maximum rest-mass density of a stable star: We are studying the evolution regarding TOV equations using Marquina (another Riemann solver) with the polytropic form of the TOV equation ( $P = K\rho^\gamma$ ). Where  $K=100$  and  $\gamma = 2$ . Data from Cactus simulation in Figure 2 shows oscillation in the density due to the low resolution as shown for the green line. For the exact relation, we are expecting to see straight line regarding the evolution with time. Increasing the resolution would give a more stable function as we see for  $R_1$  and  $R_0$ .

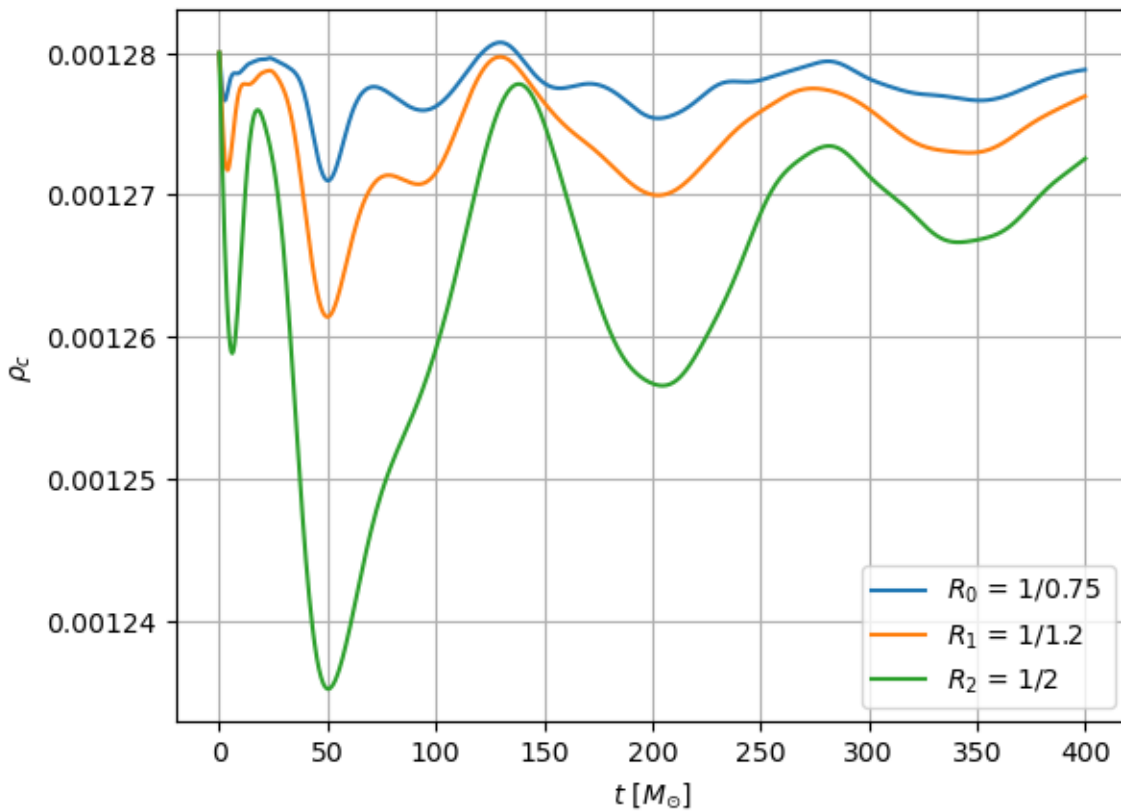


Figure 2: The maximum rest-mass density vs the evolution time in unit of solar mass.  $R$  stands for the Resolution used in the program for the 3 dimensions ( $x, y, z$ ). Increasing the resolution give more stable function.

We can see the effect due to the resolutions using the mesh representation in Figure 3. The higher resolution we have the clearer image. Compare  $R_0, R_1, R_2$  in Figures 2 & 3 for better understanding.

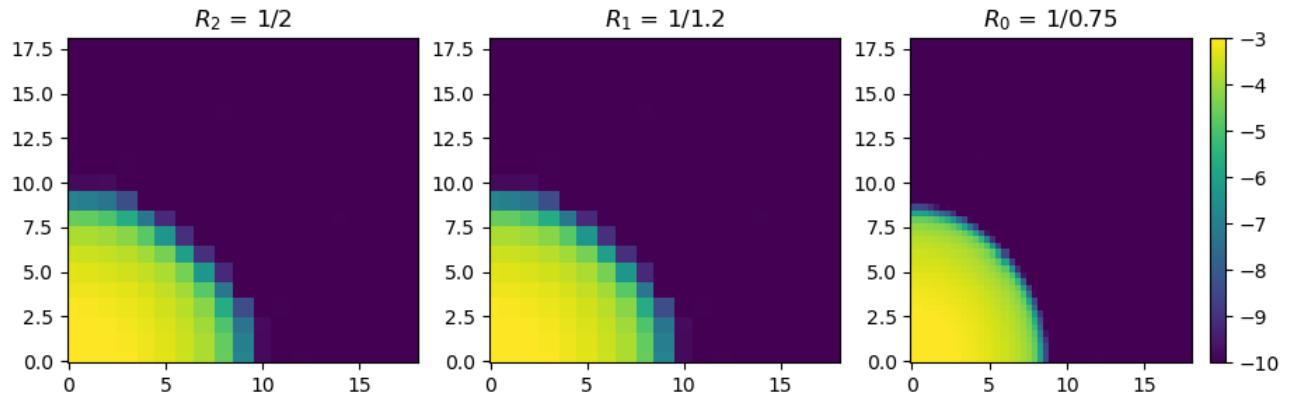


Figure 3: 2-Dimensional (x and y axes) representation of a stable star using. Zero on both axes shows the center of the star where the density is the highest and get lower as we move out.