作法

題目所給的點共有 45 個，為符合 best choice 中 m 需介於 1~n-1 的條件，因此我將 m 的最大值設為 44。又因 least square 可看作為 Ax=B 的式子，故我先將 45*45 的 A 矩陣與 1*45 的 B 矩陣建出來。並討論當 m 在 2~44 時所得到的方程式，帶入題目所給的 x 值後，計算出相對應 y 值，再利用 best choice case 2，來計算每一個 m 的 case 中最小的 error 發生在哪裡，即該 m 為 best choice。
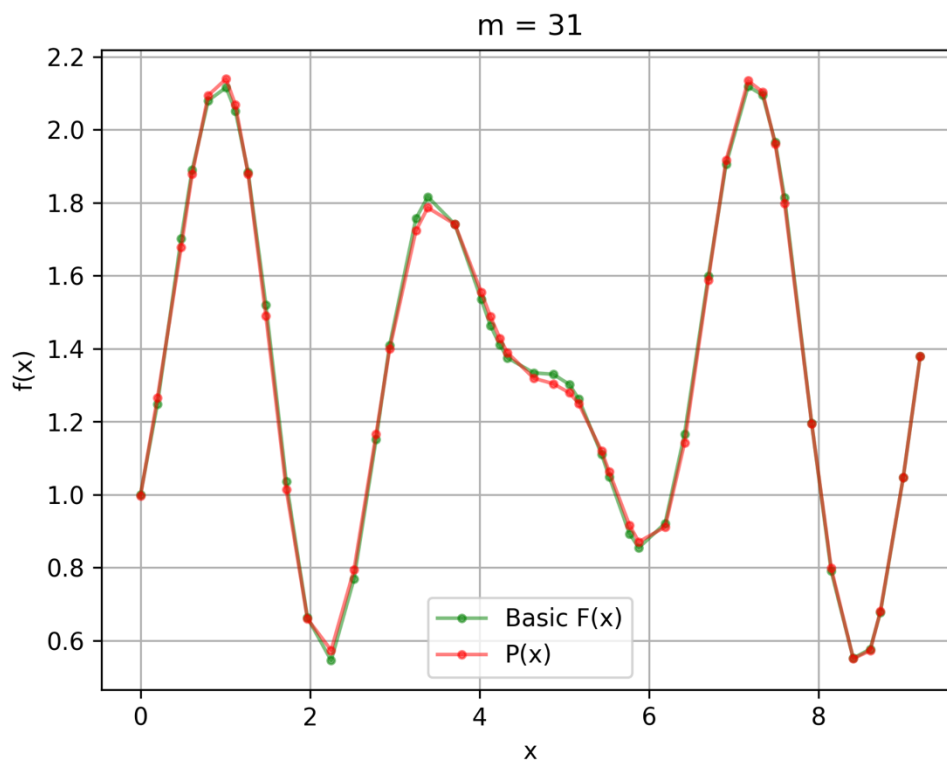
圖示說明

綠色：題目給定的 x 與 F(x) 的值
紅色：題目給定的 x 與 P(x) 的值

結論

1. 分析，n 次多項式的最佳選擇為何？
   我依照 best choice，case 2 的方式求得在 m = 31 時 error 值最小，因此 best choice 為 $P_{31}(X)$。

```
m = 2, err is : 0.4609520916322449
m = 3, err is : 0.4570805448239851
m = 4, err is : 0.4519448797581908
m = 5, err is : 0.4486753670235202
m = 6, err is : 0.4534999596178266
m = 7, err is : 0.2721268811768857
m = 8, err is : 0.255823539936228
m = 9, err is : 0.1838363828323003
m = 10, err is : 0.1860842986095324
m = 11, err is : 0.15380248496581564
m = 12, err is : 0.147949991084034
m = 13, err is : 0.1700178244300646
m = 14, err is : 0.14991784075760625
m = 15, err is : 0.09780149029940013
m = 16, err is : 0.11312949010217399
m = 17, err is : 0.10304248702301441
m = 18, err is : 0.12769404819673955
m = 19, err is : 0.24809928639881393
m = 20, err is : 0.1417533132272511
m = 21, err is : 0.24227010590447498
m = 22, err is : 0.04677453169821739
m = 23, err is : 0.07815402858981454
m = 24, err is : 0.08575699431195469
m = 25, err is : 0.10438711215202236
m = 26, err is : 0.03658367582811908
m = 27, err is : 0.04109885146713271
m = 28, err is : 0.0489842549204442
m = 29, err is : 0.060368828951328064
m = 30, err is : 0.11467365027500094
m = 31, err is : 0.030266200478694907
m = 32, err is : 0.07795054917725285
m = 33, err is : 0.20504657048675112
m = 34, err is : 0.062328328840389614
m = 35, err is : 0.06262146228566372
m = 36, err is : 0.12068098632193618
m = 37, err is : 0.03485307795241913
m = 38, err is : 0.1205538021339268
m = 39, err is : 0.05899830351407501
m = 40, err is : 1.2623230527043567
m = 41, err is : 0.10422827331690848
m = 42, err is : 0.1732520013763246
m = 43, err is : 0.23377442022622294
m = 44, err is : 0.09751189006343769
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2. 根據輸出的圖片與 error.txt 來看，大致來說會發現 m 值越大會有越趨近於原 F(X)的趨勢，error 也會減少。

m = 10

m = 19

m = 25

m = 44
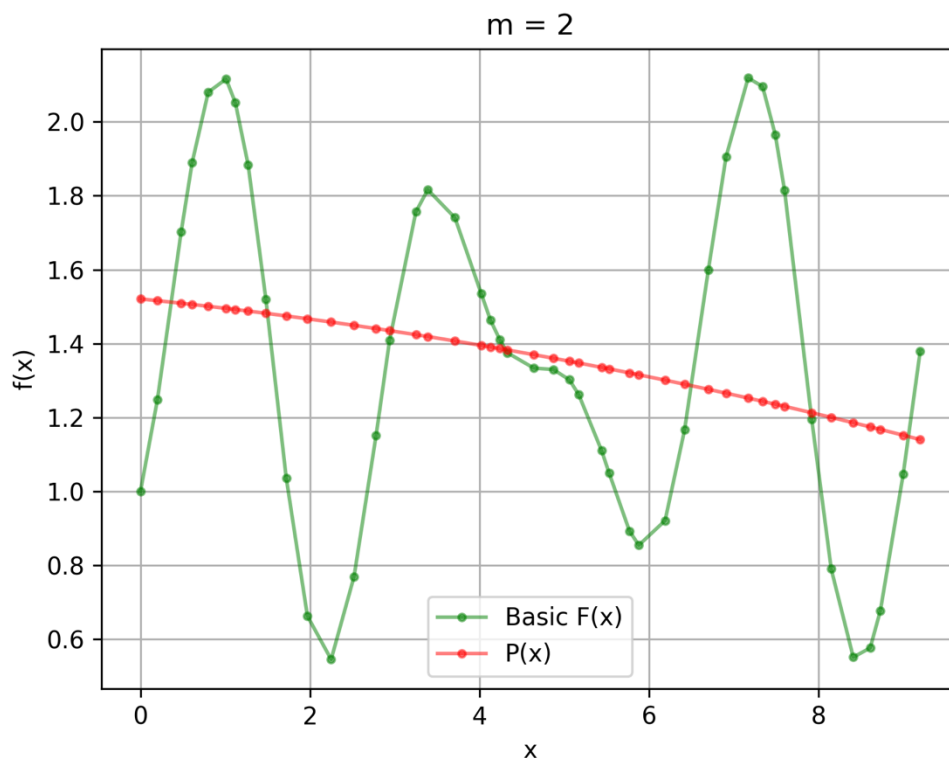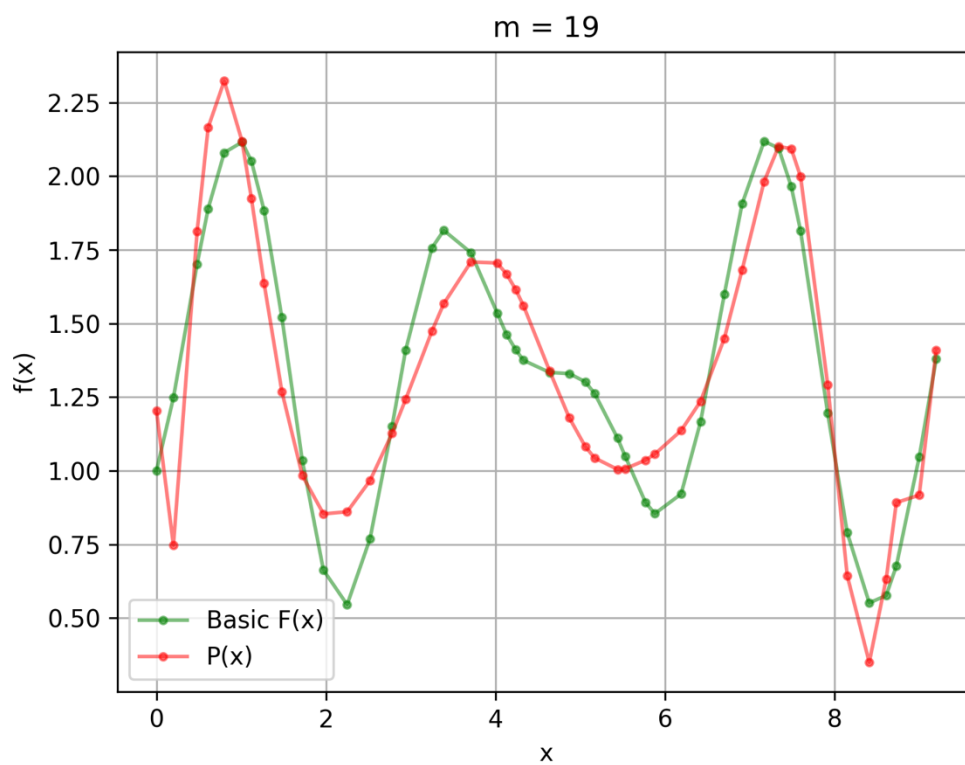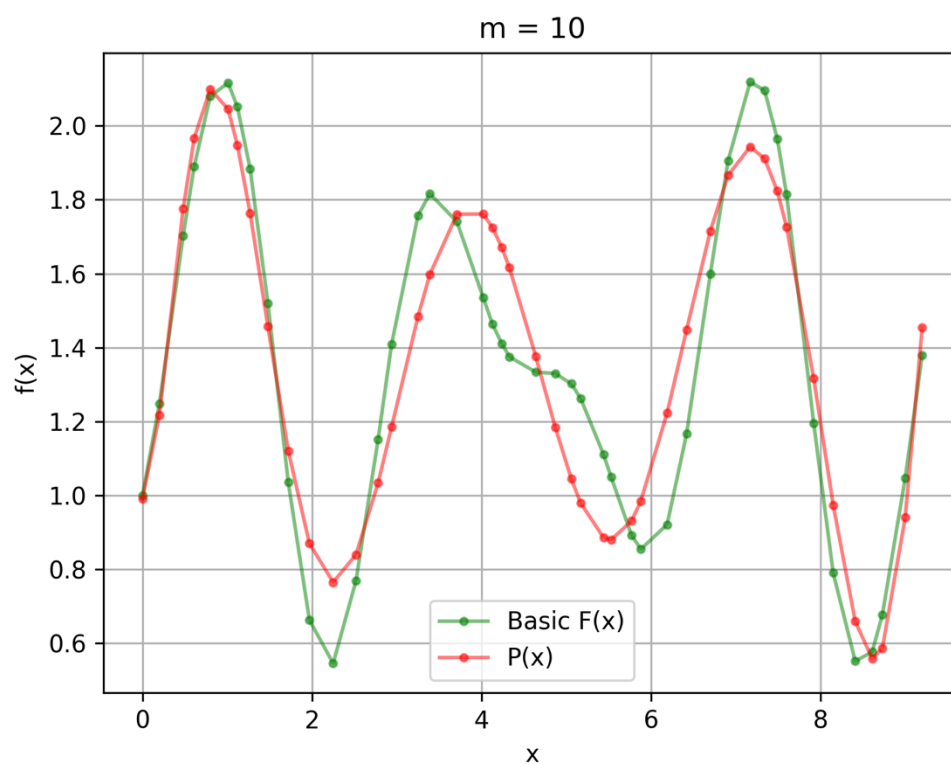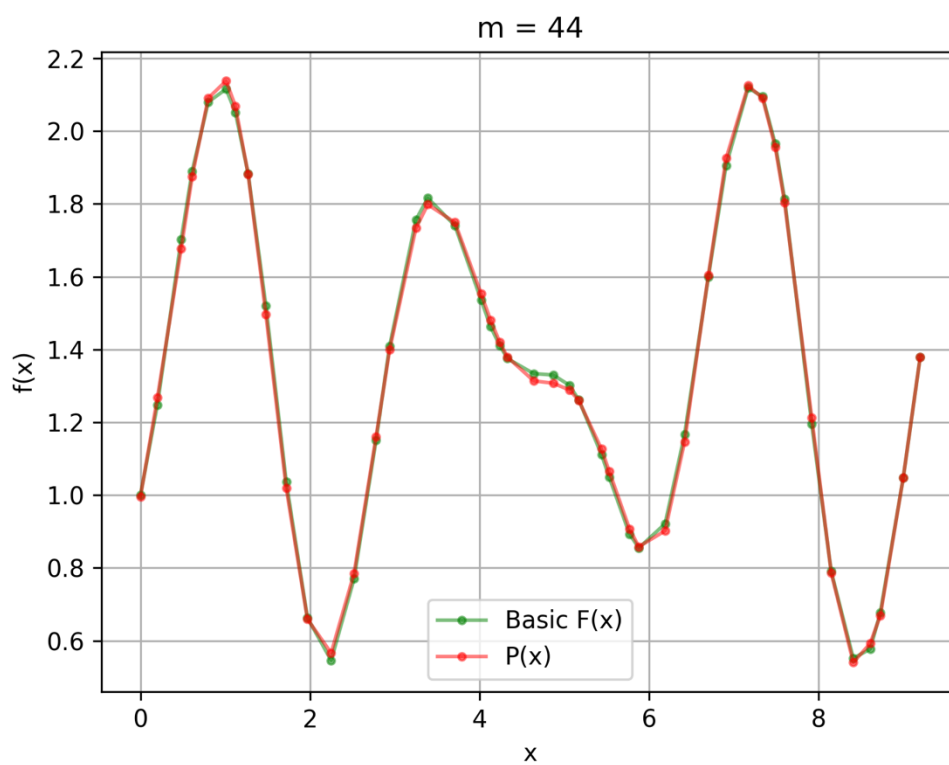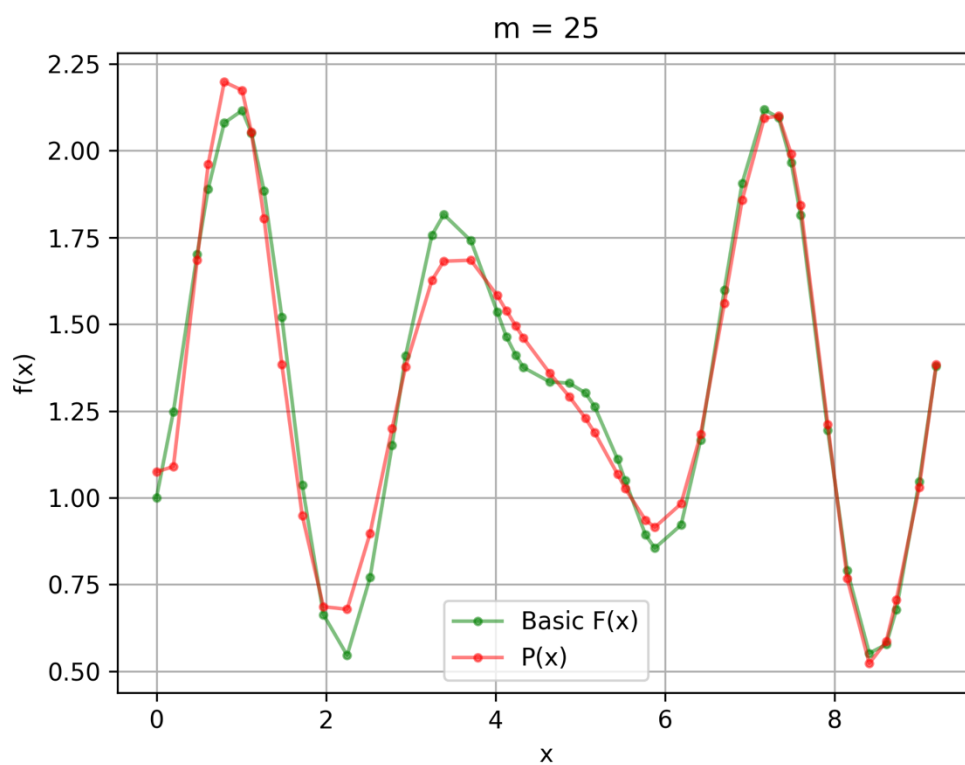
3. 但當在 P(x)趨近於 F(x)一定的程度後，m 值越大，不一定 error 就會更小，而是有可能會浮動導致 error 的值更大。依照我跑出來的數據來說，在 m = 40 的狀況下會突然浮動的很嚴重，與 F(x)相差很多，但其前後 m = 39 和 m = 41 的狀況下卻是非常趨近於 F(x)的。

m = 40

m = 41