Lecture 23 Bayesian Matting

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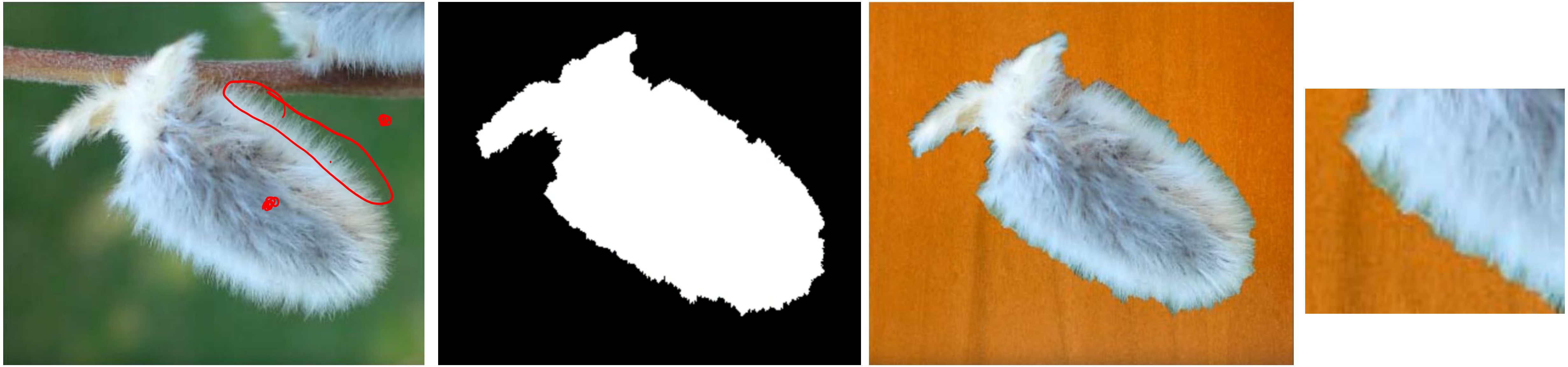
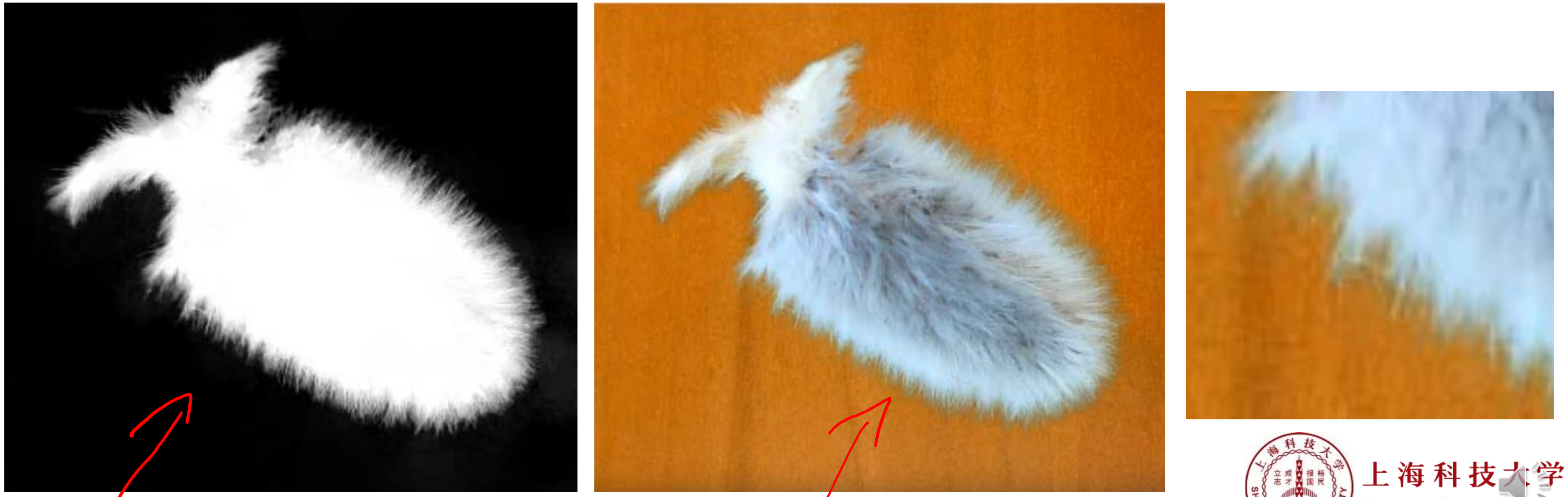
Motivation





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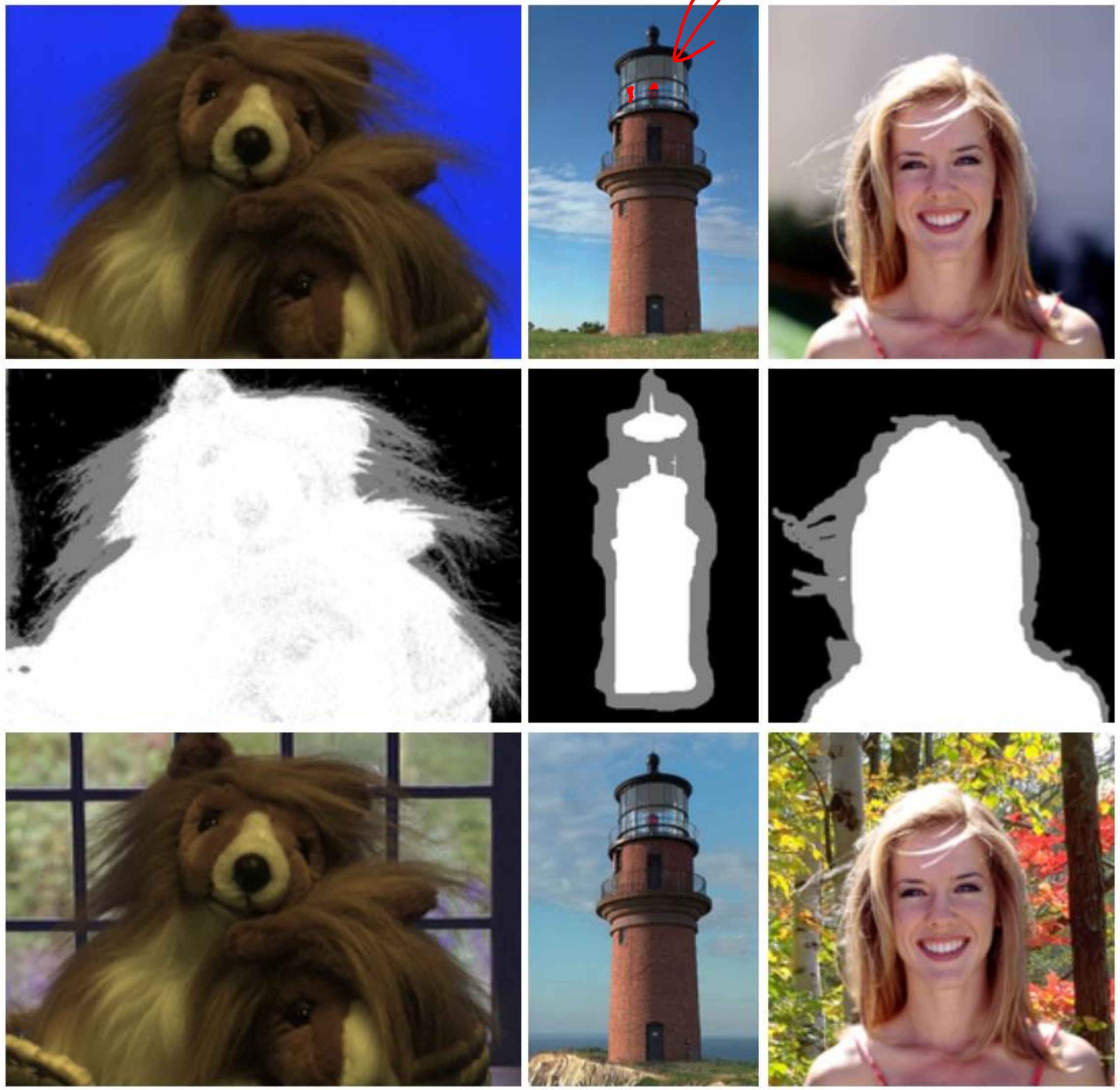


Introduction

Foreground

- Basic function of image matting

$$I(x,y) = \alpha(x,y)F(x,y) + [1-\alpha(x,y)]B(x,y)$$

Background

$I(x,y), F(x,y)$, $B(x,y)$: Full RGB image;

$\alpha(x,y)$: gray level image [0, 255] [0, 1] 0.6, 0.8

- What is the function of α's?

Finite pixel size

Finite shutter spread

Motion blur

Wispiness/ fuzziness/ Translucency

Why is matting hard?

$$I(x,y) = \alpha(x,y)F(x,y) + [1 - \alpha(x,y)]B(x,y)$$

know: $I(x,y) = \begin{bmatrix} R \\ G \\ B \end{bmatrix} \leftarrow$ greylevel image

unknow: $\alpha(x,y)$, $F(x,y) = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$, $B(x,y) = \begin{bmatrix} R \\ G \\ B \end{bmatrix}$

- 3 equations in 7 unknowns

Vlahos blue-screen matting

$$\alpha = 1 - a_1(I_b - a_2 I_g)$$

a1=1,a2=1

a1=2,a2=1

a1=2,a2=2

Vlahos blue-screen matting

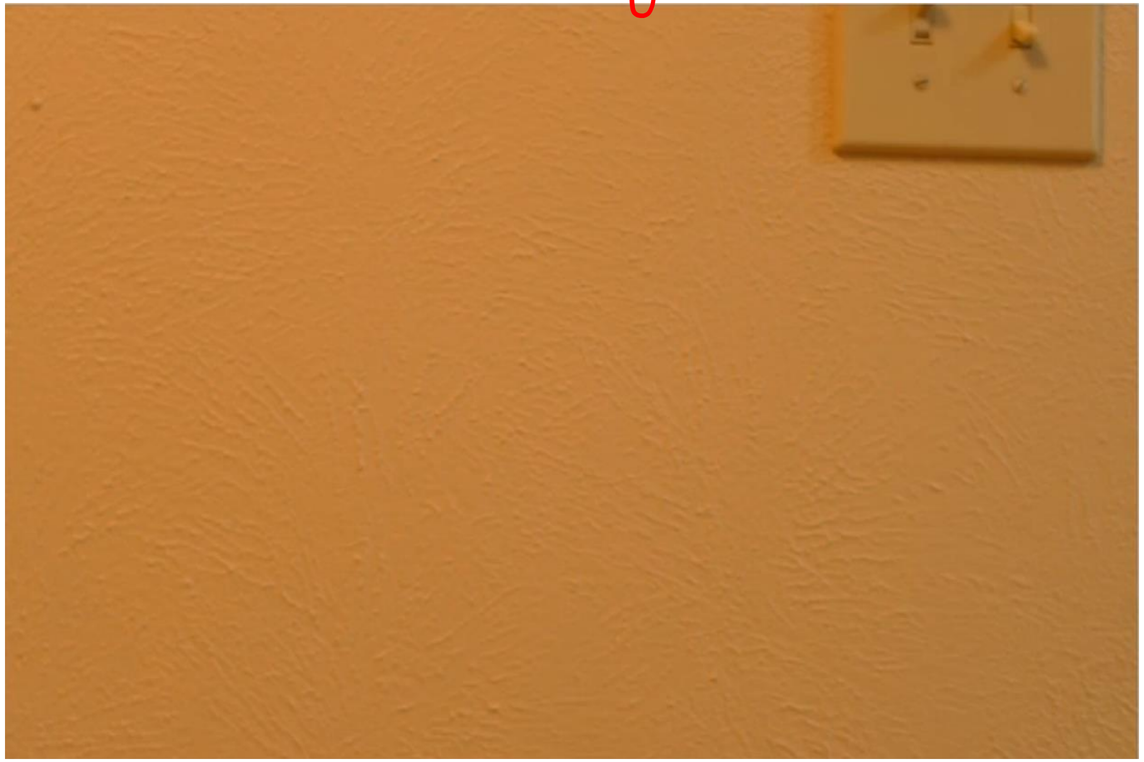
a1=1,a2=1

a1=2,a2=1

a1=2,a2=2

Getting ground-truth matting

$I(x,y)$ $B(x,y)$



Getting ground-truth matting

$$I = \underset{1}{\alpha} \cdot \underset{3}{F} + [1 - \alpha] \cdot B$$

When B is known, 3 equations in 4 unknowns.

Take 2 images with different known background.

$$\begin{cases} I_1 = \alpha \cdot F + [1 - \alpha] \cdot B_1 \\ I_2 = \alpha \cdot F + [1 - \alpha] \cdot B_2 \end{cases}$$

Then 6 equations in 4 unknowns.

$F \quad \alpha$

Bayesian image matting

- Known: I(x,y), full RGB
- Unknown: $\alpha(x,y)$ gray level image; $F(x,y), B(x,y)$: full RGB
- Optimization target:

$$\arg\max P(F,B,\alpha|I) = \max \frac{P(I|F,B,\alpha)P(F,B,\alpha)}{P(I)}$$

Bayesian rule: $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$

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Bayesian image matting

$\log P(I) = 1 = 0$

- Take logs:

$$\arg\max \log P(F,B,\alpha|I) \approx \log(P(I|F,B,\alpha)) + \log(P(F,B,\alpha)) - \log P(I)$$

- Then we assume $P(F,B,\alpha) = P(F)\,P(B)P(\alpha)$, and get:

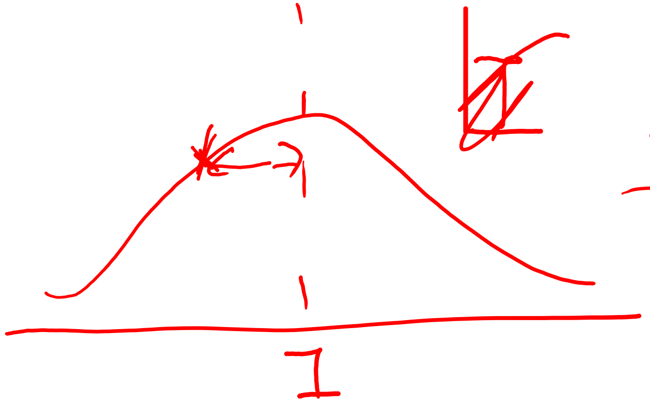
$$\arg\max \log P(F,B,\alpha|I) \approx \underbrace{\log(P(I|F,B,\alpha))}_{\text{Data term}} + \underbrace{\log(P(F)) + \log(P(B)) + \log(P(\alpha))}_{\text{Prior term}}$$

Bayesian image matting

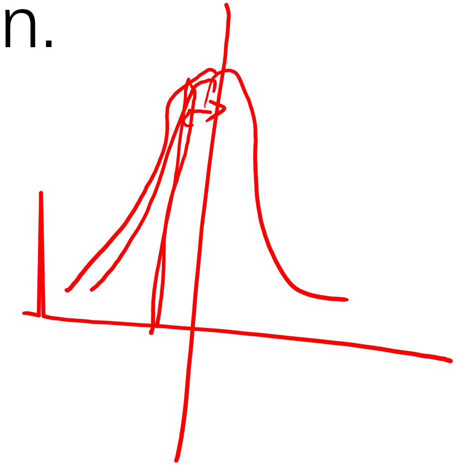
- Data term:

$$\arg\max \log P(F,B,\alpha|I) = \log[e^{-\frac{1}{2\sigma^2}\|I-(\alpha\cdot F+[1-\alpha]\cdot B)\|^2}]$$

- I, F, B, α should be consistent with the matting equation.
- σ is tunable equation.



$$I = \alpha\cdot F + (1-\alpha)\cdot B$$



Trimap

$\alpha \in (0,1)$

Bayesian image matting

- Prior term:

$P(F), P(B), P(\alpha)$ comes from trimap/ scribbles.

- Gaussian assumption for $P(F)$ and $P(B)$.
- Constant assumption for $P(\alpha)$.
- Fit Gaussian PDFs to color labeled in the trimap:

log

$$\mathrm{P(B)} = \frac{1}{2\pi\sigma} e^{-\frac{1}{2\sigma^2}(B-\mu_B)^2 \Sigma_B^{-1} (B-\mu_B)}$$

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Bayesian image matting

$$\begin{bmatrix} 6\times 6 \end{bmatrix} \begin{bmatrix} F \\ B \end{bmatrix} = \begin{bmatrix} 6\times 1 \end{bmatrix}$$

- Taking partial derivatives for F and B then equals to 0 and we get estimation of F and B.

$$-\frac{1}{\sigma^2}\|I-(\alpha F+(1-\alpha)B\|^2$$

$$\arg\max logP(F,B,\alpha|I) \approx \underbrace{\log(P(I|F,B,\alpha)}_{\text{Data term}}\ ) + \log(P(F))+$$

$$\log(P(B))+ \cancel{\log(P(\alpha))} \quad C$$

$$-\frac{1}{2\sqrt{2\pi\sigma^2}}(B-\mu_B)^T\Sigma_B^{-1}(B-\mu_B)$$

$$-\frac{1}{2\sqrt{2\pi\sigma^2}}(F-\mu_F)^T\Sigma_F^{-1}(F-\mu_F)$$

- Then α can be calculated with:

$$I = \alpha F + (1-\alpha)B \Longrightarrow \alpha = \frac{(I-B)(F-B)}{(F-B)(F-B)}$$

https://grail.cs.washington.edu/projects/digital-matting/papers/cvpr2001.pdf

Results of Bayesian image matting

Take home message

- Blue screen matting
- Ground-truth matting
- Bayesian image matting
- Closed form matting