Lecture 2-2 Basic relationship between pixels (chapter 2.5)

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Outline

- **Neighbors of Pixel**
- **Relationship between Pixels**
  - Adjacency
  - Connectivity
  - Regions
  - Boundaries
- **Distance measures**
  - Euclidean distance
  - City-block distance
  - Chessboard distance

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Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**

- **$N_D(p)$**

- **$N_8(p)$**

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$**

- **$N_8(p)$**

	q_1	
q_2	p	q_3
	q_4	

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$**

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

- **$N_8(p)$**

r_1		r_2
	p	
r_3		r_4

Neighbors of Pixel

If a pixel p at coordinate (x, y)

- **$N_4(p)$**

  (x+1, y), (x-1, y), (x, y+1), (x, y-1)

- **$N_D(p)$**

  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)

- **$N_8(p)$: $N_4(p) \cup N_D(p)$**

r_1	q_1	r_2
q_2	p	q_3
r_3	q_4	r_4

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Adjacency

To define adjacency of pixels, we need identify

- **Type of Neighbor**

  $N_4(p)$, $N_D(p)$, $N_8(p)$

- **The set of intensity values V**
  - Binary image: $V = \{1\}$
  - Gray-scale image: $V = [L_{min}, L_{max}]$

q_1	p	q_2

0	0	0
0	1	1
0	0	0

Adjacency in a binary image

	q_1	
q_2	p	q_3

0	39	0
11	13	16
0	0	0

Adjacency in a gray-scale image

Adjacency

Types of Adjacency:

- **4-adjacency**
- **8-adjacency**
- **M-adjacency (mixed adjacency)**

Adjacency

Types of Adjacency:

- **4-adjacency**
  - p, q $\in$ V
  - q $\in N_4(p)$
- 8-adjacency
- M-adjacency (mixed adjacency)

$$\begin{matrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{matrix}$$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Adjacency

Types of Adjacency:

- 4-adjacency
- 8-adjacency
  - $p, q \in V$
  - $q \in N_8(p)$
- M-adjacency (mixed adjacency)

$$\begin{array}{ccc} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{array}$$

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

Adjacency

Types of Adjacency:

- 4-adjacency
- 8-adjacency
- **M-adjacency (mixed adjacency)**
  - $p, q \in V$
  - $q \in N_4(p)$ or
    $q \in N_D(p)$ and $N_4(p) \cap N_4(q) \notin V$

$$\begin{matrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{matrix}$$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

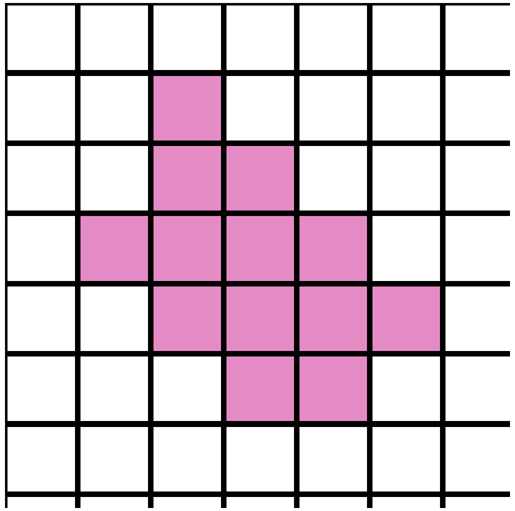
Connectivity

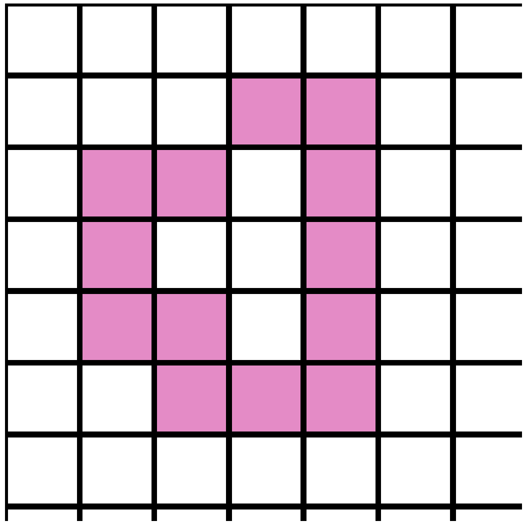
Important concept used in establishing boundaries of objects and components of regions in an image

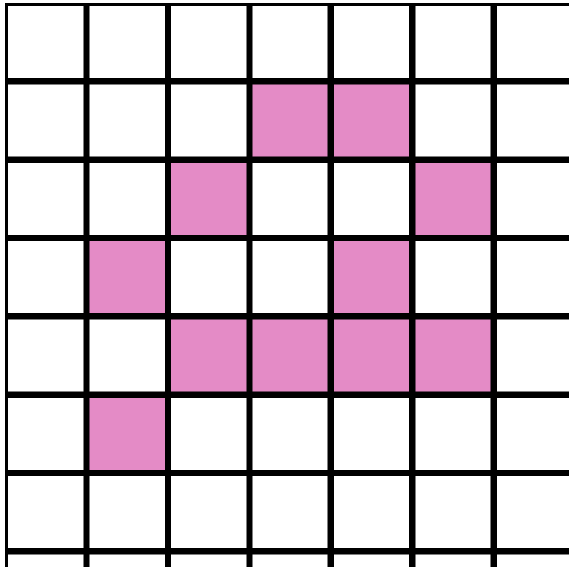
- *Path*
- *Connected*
- *Connected component*
- *Connected set*

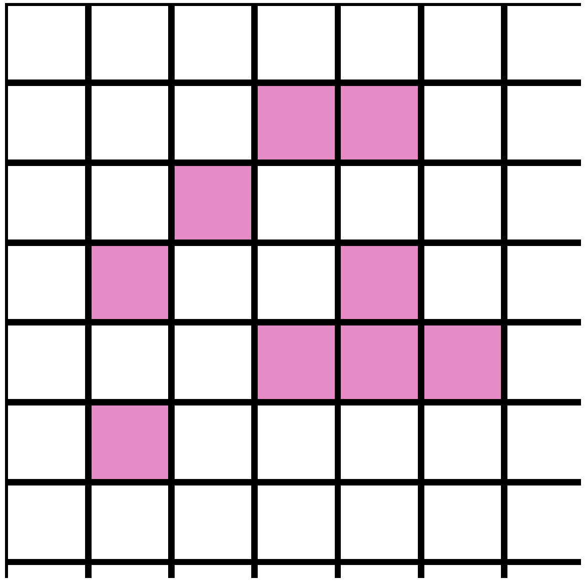
$$\begin{array}{ccc} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{array}$$

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$









Region

R: a subset of an image which is also a connected set

- *Adjacent region*
- *Disjoint region*

$$\begin{array}{l}\left.\begin{matrix}1&1&1\\1&0&1\\0&1&0\end{matrix}\right\} R_i\\ \left.\begin{matrix}0&0&1\\1&1&1\\1&1&1\end{matrix}\right\} R_j\end{array}$$

Pixels

- Neighbors of Pixel
- Relationship between Pixels
  - Adjacency
  - Connectivity
  - Regions
  - Boundaries
- **Distance measures**
  - Euclidean distance
  - City-block distance
  - Chessboard distance

Distance Measures

For pixels p, q and z, with coordinates (x, y), (s, t) and (v, w), D is a **distance function or metric** if

- $D(p, q) \geqq 0$ ($D(p, q) = 0$ only if $p = q$)
- $D(p, q) = D(q, p)$
- $D(p, z) \leq D(p, q) + D(q, z)$

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Distance Measures

- **Euclidean distance：**

$$D_e(p, q) = [\,(x-s)^2+(y-t)^2]^{1/2}$$

- City-block distance:
- Chessboard distance

		2		
	1.4	1	1.4	
2	1	0	1	2
	1.4	1	1.4	
		2		

Distance Measures

- Euclidean distance：

  $$D_e(p, q) = [\,(x-s)^2+(y-t)^2]^{1/2}$$

- **City-block distance:**

  $$D_4(p, q) = |x-s|+|y-t|$$

- Chessboard distance

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Distance Measures

- Euclidean distance:

$$D_e(p, q) = [(x-s)^2+(y-t)^2]^{½}$$

- City-block distance:

$$D_4 (p, q) = |x-s|+|y-t|$$

- **Chessboard distance**

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Distance Measures

D_m distance is defined as the shortest m-path between the point

	r_{12}	1
r_{21}	r_{22}	
1		

$$D_m = ?$$

Distance Measures

No m-path between the point

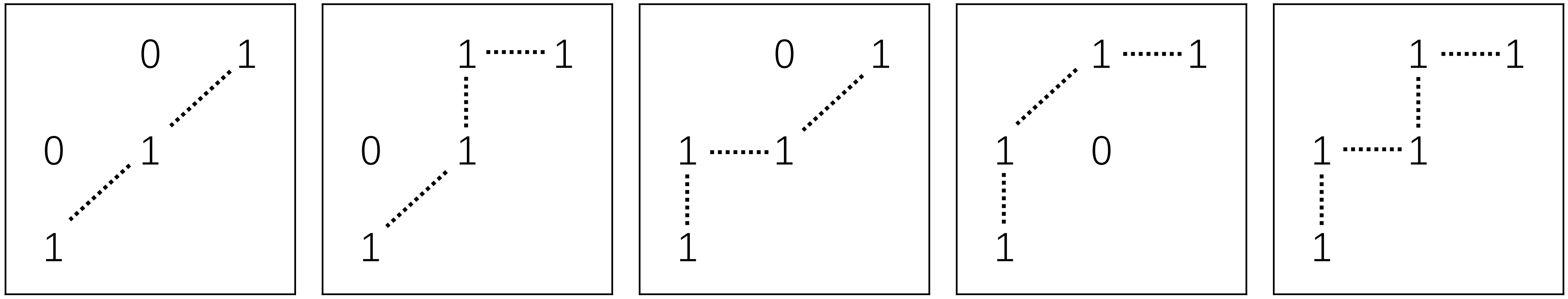
	0	1
0	0	
1		

	1	1
0	0	
1		

	0	1
1	0	
1		

Distance Measures

D_m **distance** is different by the values of r_{12}, r_{21} and r_{22}



$D_m = 2$ $D_m = 3$ $D_m = 3$ $D_m = 3$ $D_m = 4$

Take home message

- 1. Relationship between pixels.
- 2. Core problem: types of adjacency.
- 3. Connection and distance measurements.