Lecture 8-1 Parametric Transform and Scattered Data Interpolation

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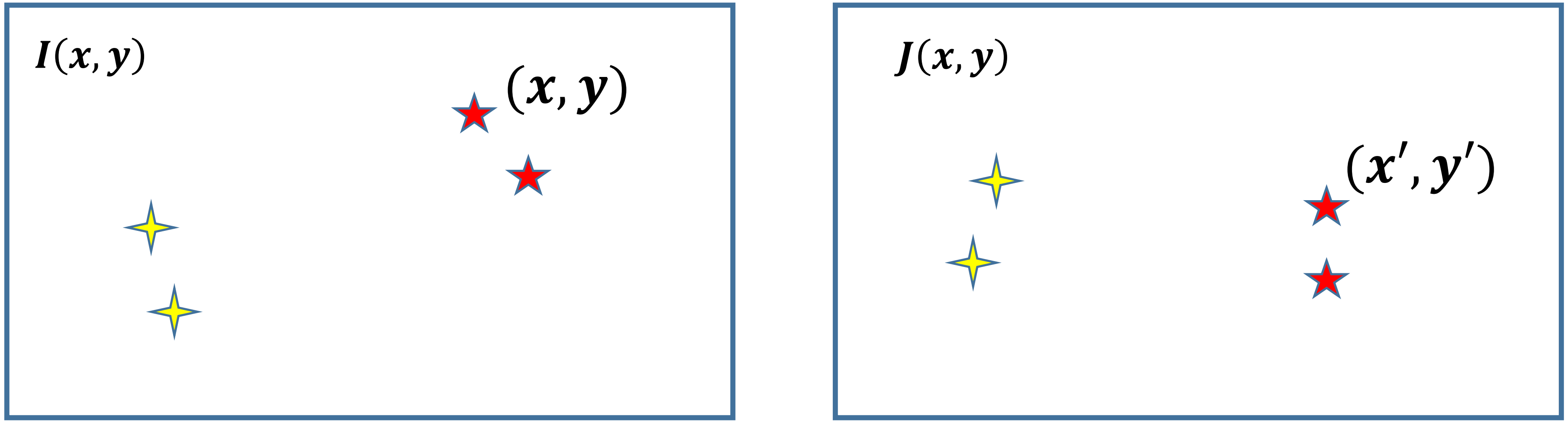
SIST Building 2 302-F

Outline

- Dense correspondence
    - -Simplest Image Registration
- Scattered data interpolation
    - -Thin Plate Spline Interpolation

Dense correspondence

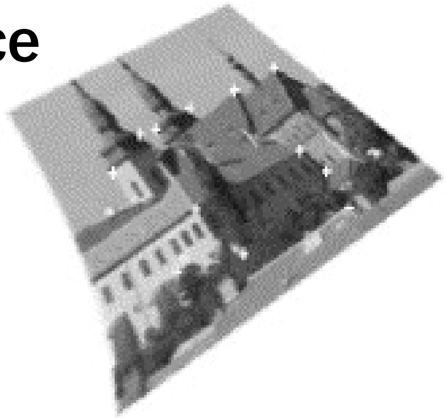


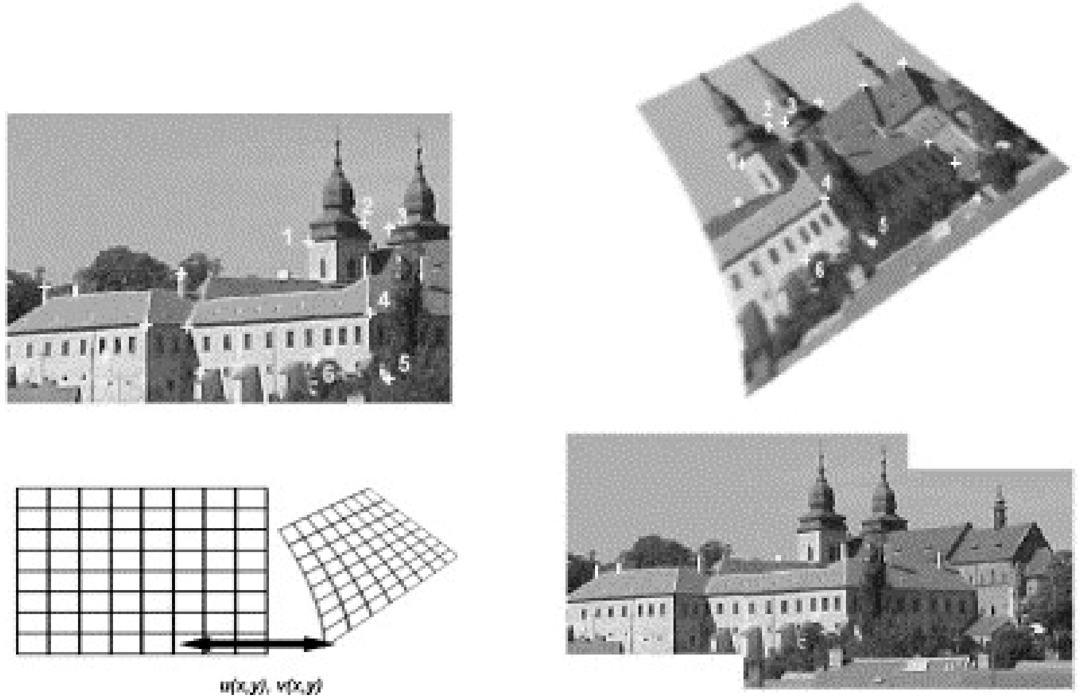
- For every pixel (x, y) we want to find a motion vector (u, v) so that $I(x,y) = J(x+v, y+u) = J(x', y')$.

- Where u and v are easy functions of (x, y).

Application examples

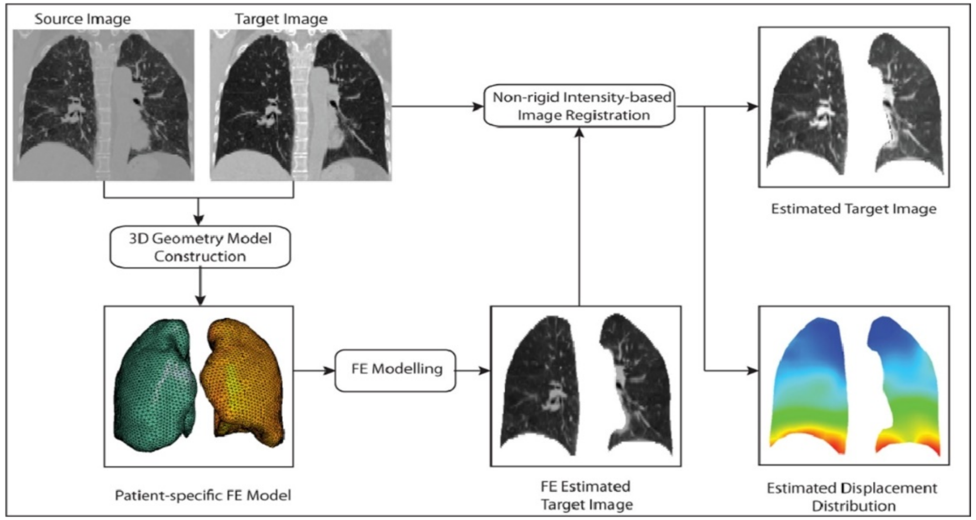
High-resolution scence





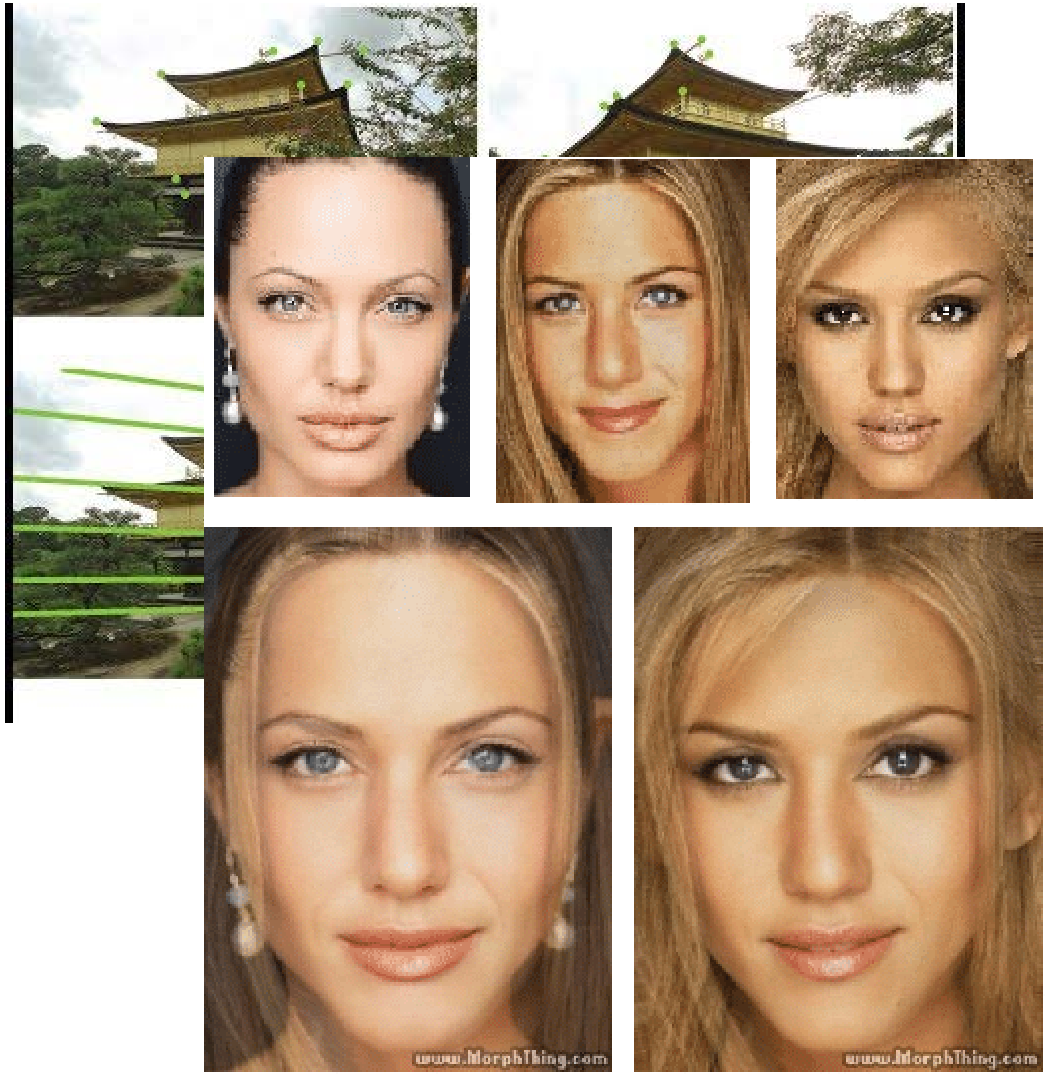
Remote sensing image

Medical image



Other extensions

Epipolar geometry



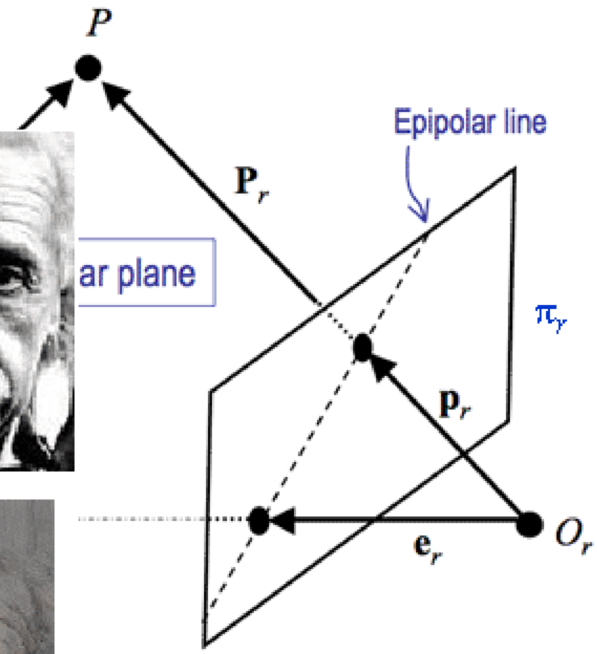


Photo morphing

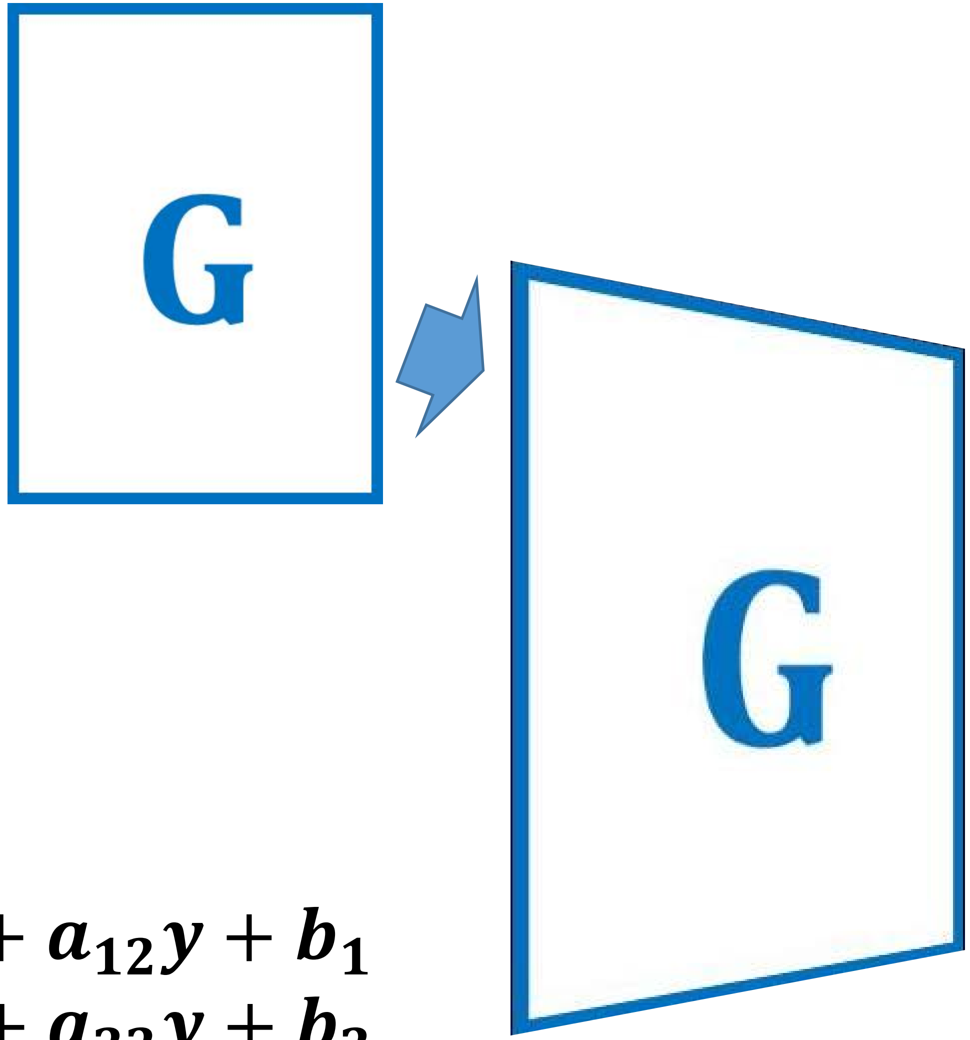
Parametric transformations

- $$\begin{bmatrix}\widetilde{x}'\\\widetilde{y}'\\\widetilde{z}'\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & b_1\\a_{21} & a_{22} & b_2\\c_1 & c_2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$$

- $$\begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}\frac{a_{11}x+a_{12}y+b_1}{c_1x+c_2y+1}\\\frac{a_{21}x+a_{22}y+b_2}{c_1x+c_2y+1}\end{bmatrix} = \begin{bmatrix}\frac{\widetilde{x}'}{\widetilde{z}'}\\\frac{\widetilde{y}'}{\widetilde{z}'}\end{bmatrix}$$

- Then we have:

$$(c_1x + c_2y + 1)x' = a_{11}x + a_{12}y + b_1$$
$$(c_1x + c_2y + 1)y' = a_{21}x + a_{22}y + b_2$$

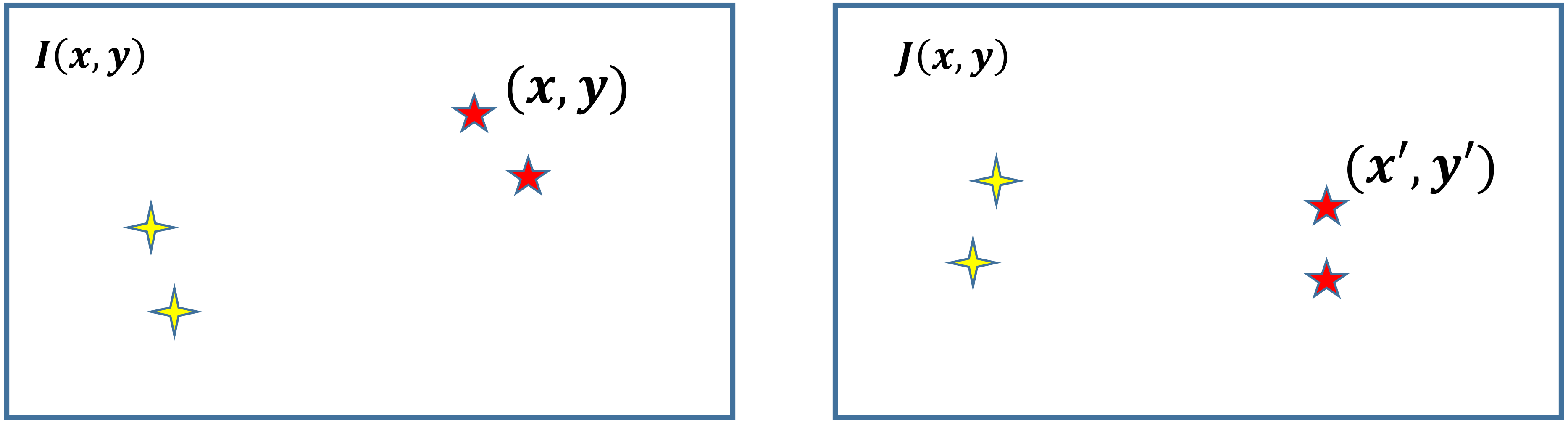


Linear equations

$$(c_1 x + c_2 y + 1)x' = a_{11}x + a_{12}y + b_1$$

$$(c_1 x + c_2 y + 1)y' = a_{21}x + a_{22}y + b_2$$

- We already know: x', y', x, y
- We don't know: $a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2, c_1, c_2$



Linear equations

$$(c_1 x_i + c_2 y_i + 1) x_i{}' = a_{11} x_i + a_{12} y_i + b_1$$

$$(c_1 x_i + c_2 y_i + 1) y_i{}' = a_{21} x_i + a_{22} y_i + b_2$$

$$a_{11} x_i + a_{12} y_i + b_1 \text{-} c_1 x_i x_i{}' - c_2 y_i x_i{}' = x_i{}'$$

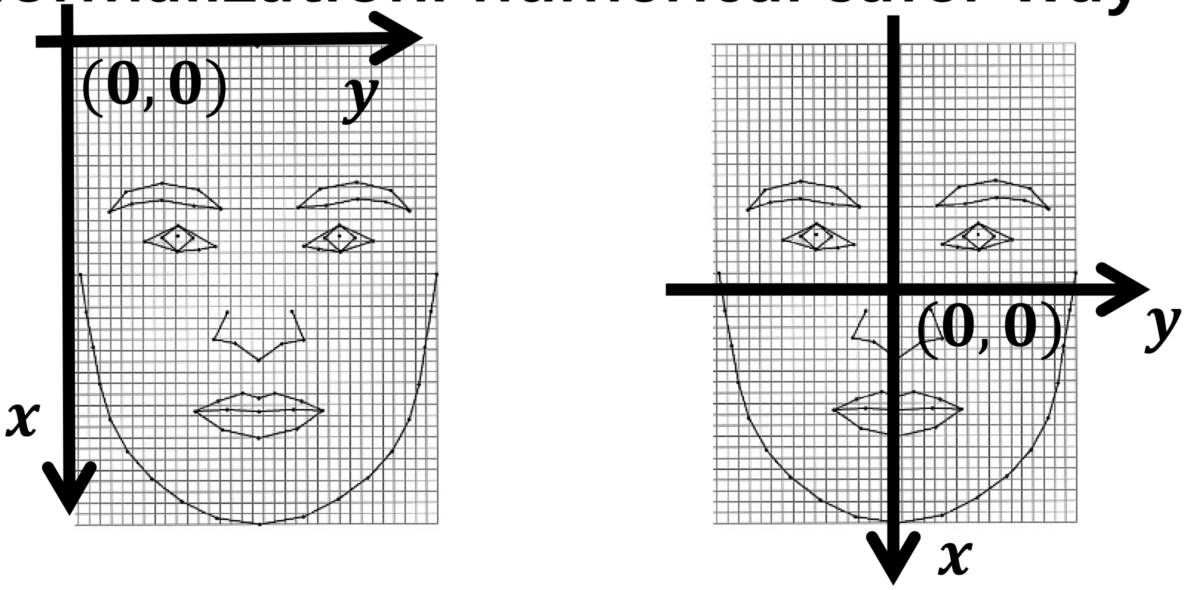
$$a_{21} x_i + a_{22} y_i + b_2 \text{-} c_1 x_i y_i{}' - c_2 y_i y_i{}' = y_i{}'$$

$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 & -x_i\, x'_i & -y_i\, y'_i \\ 0 & 0 & x_i & y_i & 0 & 1 & -x_i\, y'_i & -y_i\, y'_i \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} x'_i \\ y'_i \end{bmatrix}$$

Every 4 pair of landmarks will determine 8 parameters

Some tips

- Pre-normalization: numerical safer way



- Outlier rejection: ex. RANSAC

Outline

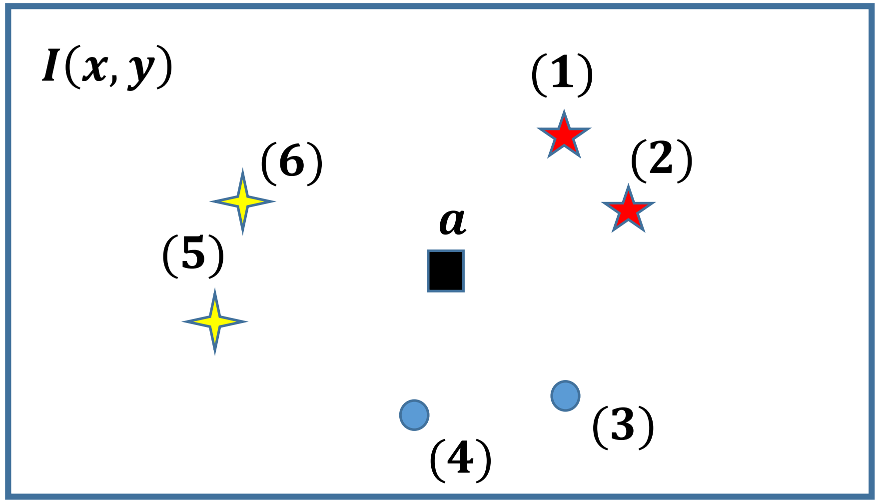
- Dense correspondence

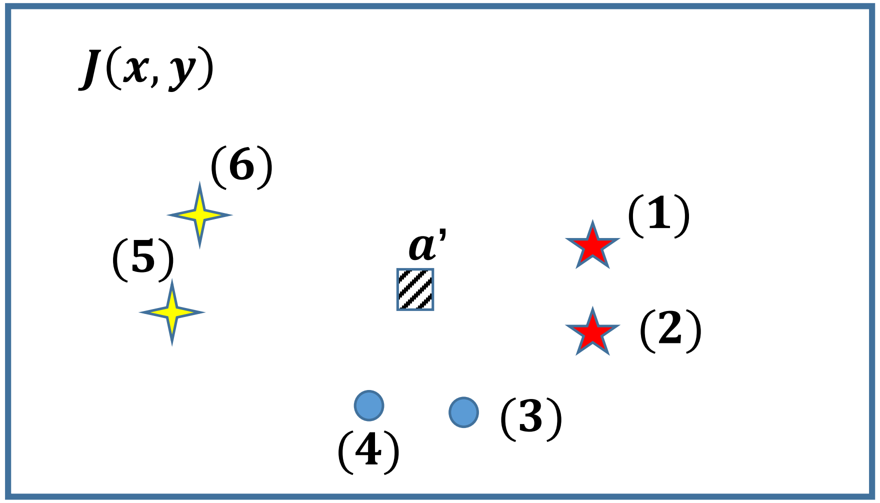
    -Simplest Image Registration

- Scattered data interpolation

    -Thin Plate Spline Interpolation

Scattered Data interpolation





- What we know: pixel (1) (2) (3) (4) (5) (6)
- What we don’t know: pixel a and b
- Key idea: image interpolation

Thin plate spline interpolation

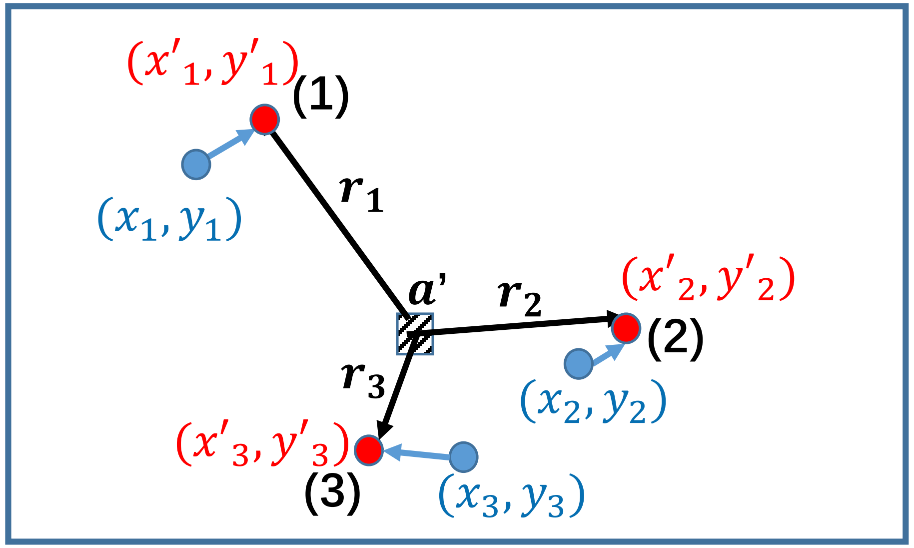
- Suggested for image registration by Ardi Goshtasby in 1988 [IEEE Trans. Geosci. and Remote Sensing, vol 26, no.1, 1988]
- Based on an analogy to the approximate shape of thin metal plates deflected by normal forces at discrete points.

Thin plate spline interpolation

- To describe the unknown pixel $\boldsymbol{a}'$
- Find its distance to each of the $\boldsymbol{N}$ localized points.

$$\boldsymbol{x}' = \sum_{\boldsymbol{i}=1}^{\boldsymbol{N}} \boldsymbol{\omega}_{1i}\, \varphi(r_i) + \boldsymbol{a}_{11}\boldsymbol{x} + \boldsymbol{a}_{12}\boldsymbol{y} + \boldsymbol{b}_1$$

$$\boldsymbol{y}' = \sum_{\boldsymbol{i}=1}^{\boldsymbol{N}} \boldsymbol{\omega}_{2i}\, \varphi(r_i) + \boldsymbol{a}_{12}\boldsymbol{x} + \boldsymbol{a}_{22}\boldsymbol{y} + \boldsymbol{b}_2$$



Radio-basis function

$$\varphi(r_i) = {r_i}^2 log r_i$$

$${r_i}^2 = \|(x, y) - (x_i - y_i)\|_2$$

Thin plate spline interpolation

- For N pairs of $(x, y) \longrightarrow (x', y')$
- Find the $6 + 2N$ coefficients a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2,ω_{1i}, ω_{2i} that satisfy

$$x' = \sum_{i=1}^{N} \omega_{1i}\, \varphi(r_i) + a_{11}x + a_{12}y + b_1$$

$$y' = \sum_{i=1}^{N} \omega_{2i}\, \varphi(r_i) + a_{12}x + a_{22}y + b_2$$

for all N pairs ($2N$ equations) and also satisfy $\cdots$

Thin plate spline interpolation

- … these 6 equations:

$$\sum_{i=1}^{N} \omega_{1i} = 0 \qquad \sum_{i=1}^{N} \omega_{2i} = 0$$

$$\sum_{i=1}^{N} \omega_{1i} x_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i} x_i = 0$$

$$\sum_{i=1}^{N} \omega_{1i} y_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i} y_i = 0$$

- Use all these $2N + 6$ equations to compute (x, y) for every point (x', y') in the image.

Linear equations

$$x' = \sum_{i=1}^{N} \omega_{1i}\,\varphi(r_i) + a_{11}x + a_{12}y + b_1$$

$$y' = \sum_{i=1}^{N} \omega_{2i}\,\varphi(r_i) + a_{12}x + a_{22}y + b_2$$

$$\sum_{i=1}^{N} \omega_{1i} = 0 \qquad \sum_{i=1}^{N} \omega_{2i} = 0$$

$$\sum_{i=1}^{N} \omega_{1i}x_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i}x_i = 0$$

$$\sum_{i=1}^{N} \omega_{1i}y_i = 0 \qquad \sum_{i=1}^{N} \omega_{2i}y_i = 0$$

$$\begin{bmatrix} \varphi(r_{11}) & \varphi(r_{12}) & \dots\dots & \varphi(r_{1n}) \\ & & \dots\dots & \\ & & \dots\dots & \\ \varphi(r_{11}) & \varphi(r_{12}) & \dots\dots & \varphi(r_{1n}) \\ x_1 & \dots\dots & & x_n \\ y_1 & \dots\dots & & y_n \\ 1 & \dots\dots & & 1 \end{bmatrix} \begin{bmatrix} \omega_{11} & \omega_{11} \\ \omega_{12} & \omega_{12} \\ \vdots & \vdots \\ \omega_{1n} & \omega_{2n} \\ a_{11} & a_{21} \\ a_{12} & a_{22} \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} x'_1 & y'_1 \\ x'_1 & y'_2 \\ \vdots & \vdots \\ x'_n & y'_n \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Why does TPS behave well?

- As (x, y) moves away from the N fiducial points, the terms in the sum

$$\sum_{i=1}^{N} \omega_{1i}\,\varphi(r_i) \qquad \sum_{i=1}^{N} \omega_{2i}\,\varphi(r_i)$$

begin to cancel out. When the sum $\longrightarrow$ 0.

$$x' \longrightarrow a_{11}x + a_{12}y + b_1$$
$$y' \longrightarrow a_{21}x + a_{22}y + b_2$$

Results

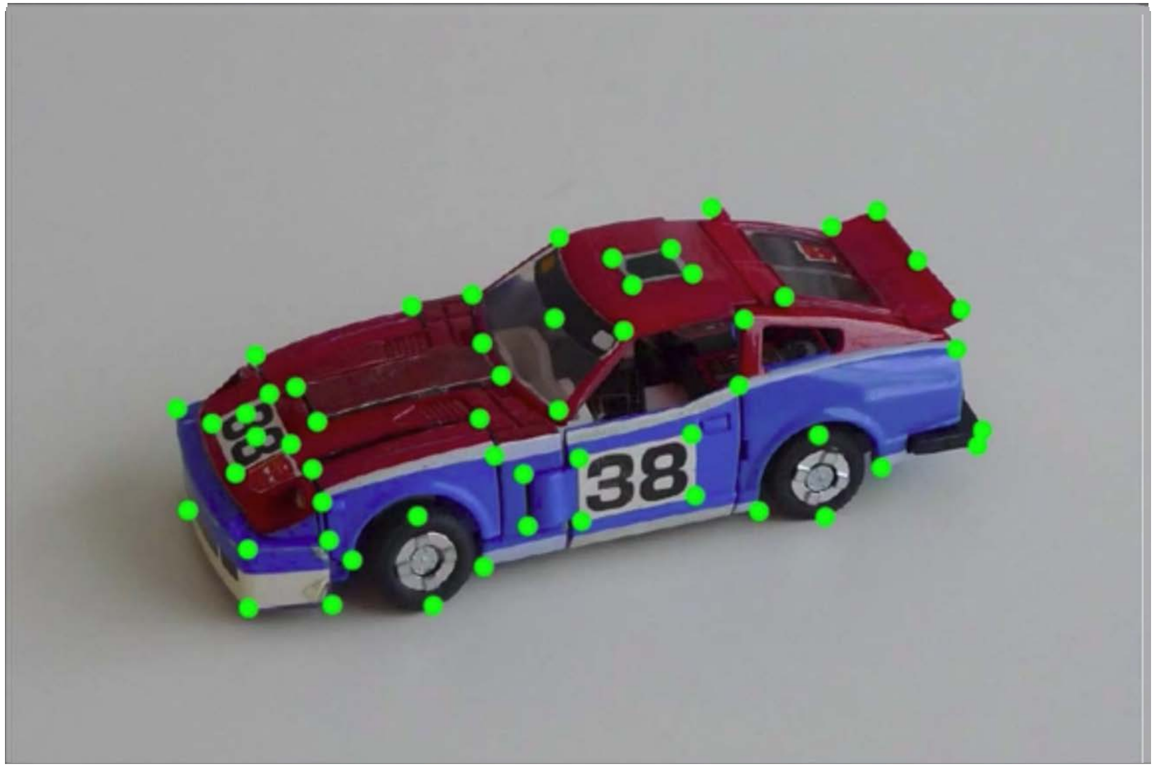
Image 1

Results

Image 2

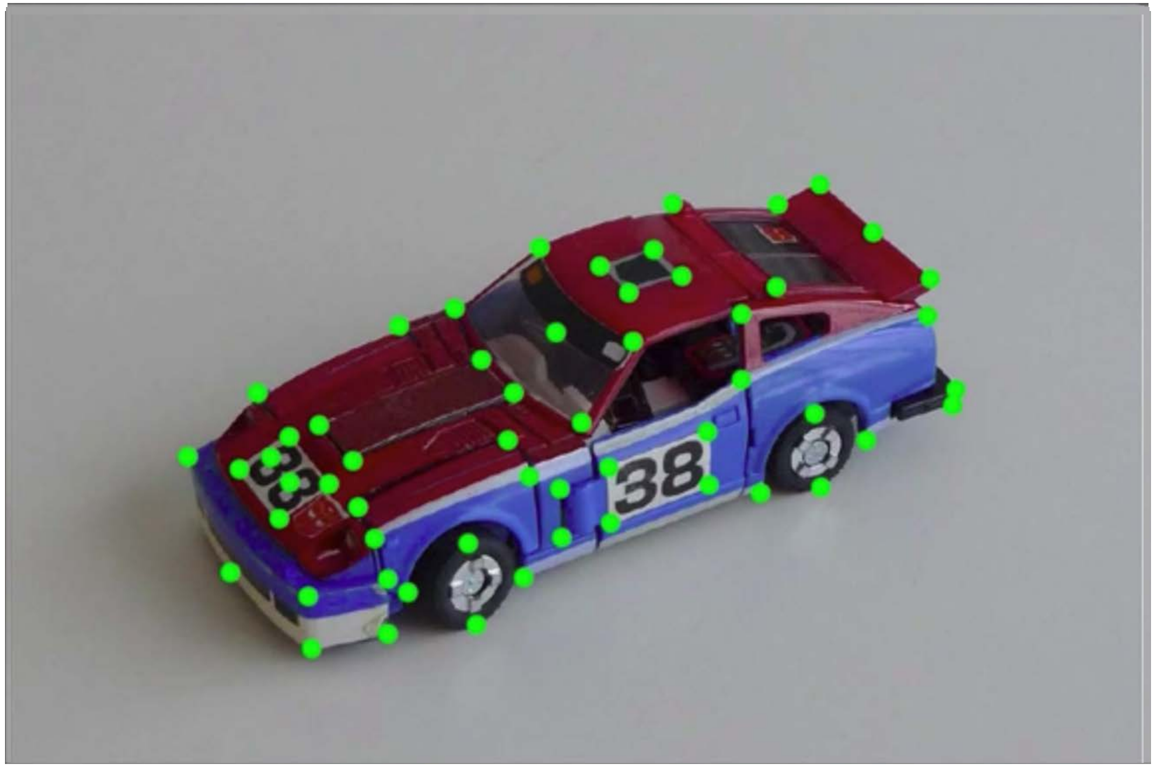
Results

Landmark selected on Image 2



Results

Landmark selected on Image 1

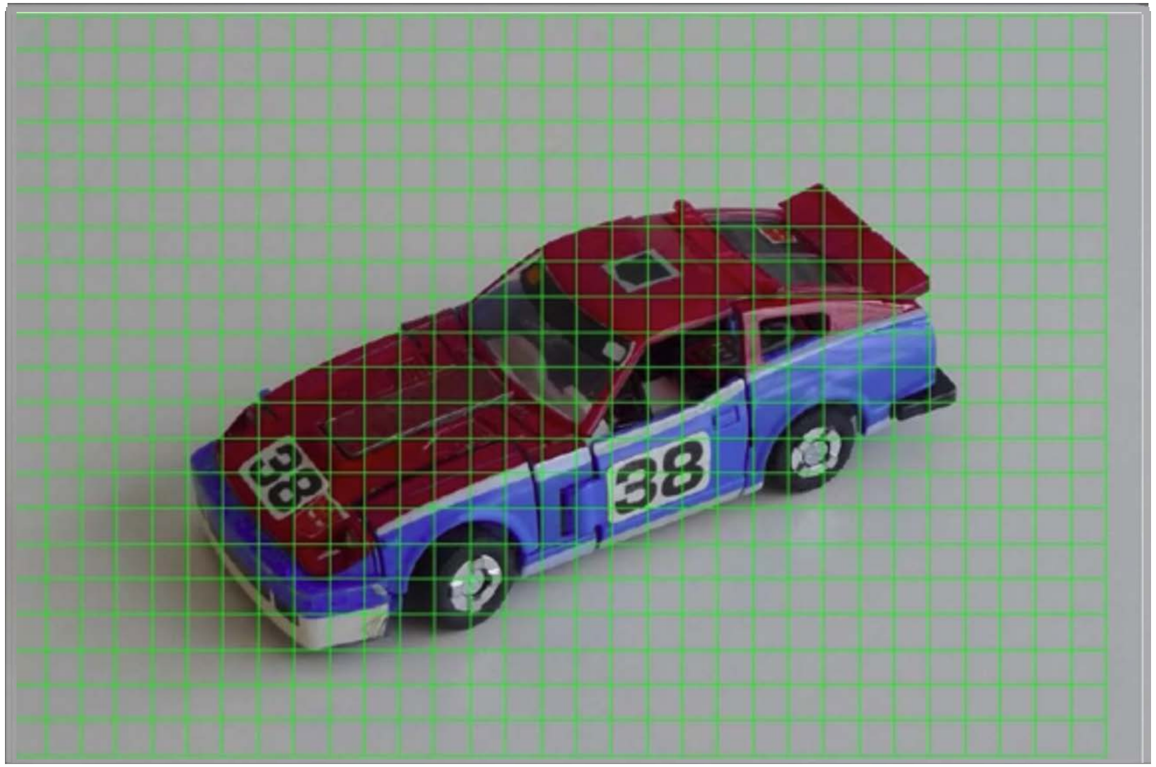


Results

Registration result from Image 1 to Image 2

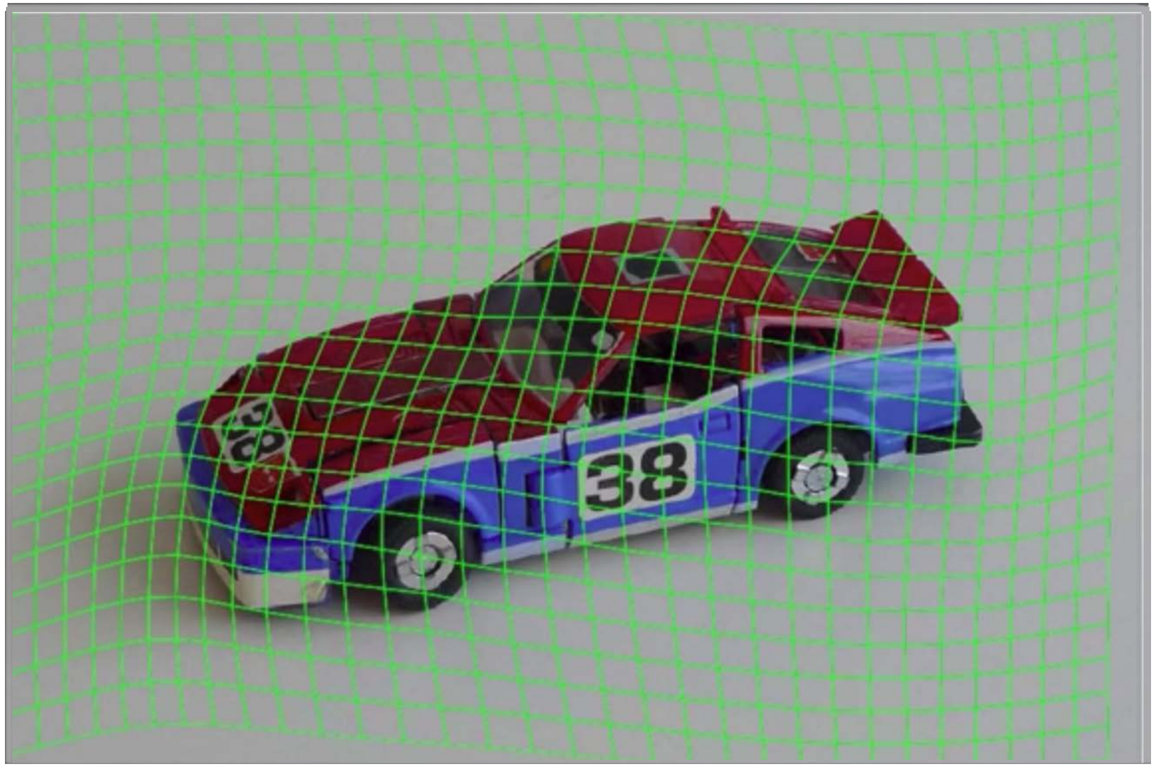
Results

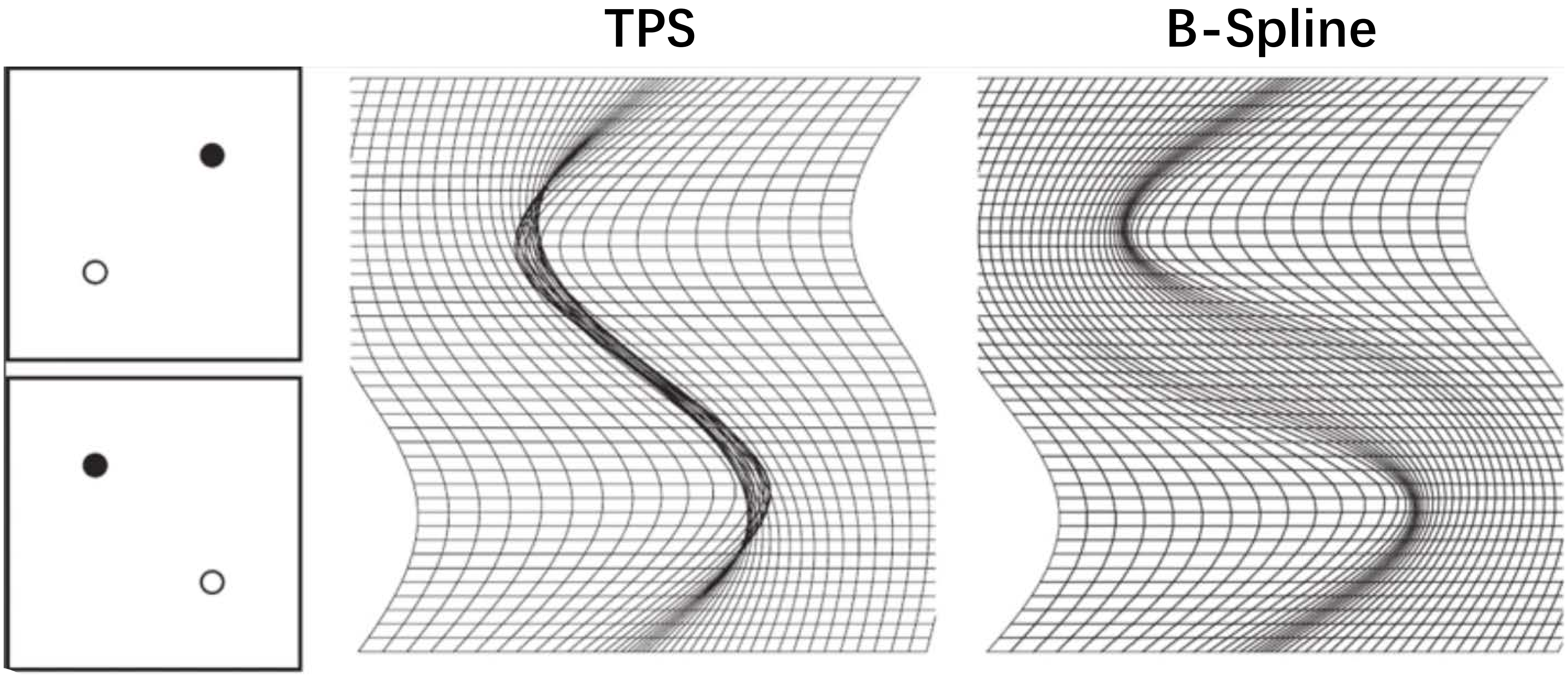
We put a grid to show the transform



Results

The deformation field from Image 1 to Image 2





(a) (b) (c)

Take home message

- Not invertible transform.

  -Enforce diffeomorphic transformations using PDE-based fluid flow method.

- Transformations depends on all the correspondences.

  - (B-spline).

- Straight lines generally don't stay straight
- Image intensities don't play a role!

  -Optical flow

- Active Shape Models … find landmarks