Lecture 4-1 Intensity Transform （chapter 3.1-3.2 ）

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Outline

- **Histogram （直方图）**
  - Definition
  - Property
- **Intensity Transformation（灰度变换）**
  - Linear transform
  - Non-linear transform
- **Histogram Processing**
  - Histogram Equalization
  - Histogram Matching

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Definition

$$h(r_k) = n_k$$

Where r_k : the *k*th intensity value in the level range of [0, L-1]

n_k : the number of pixels in the image with intensity r_k

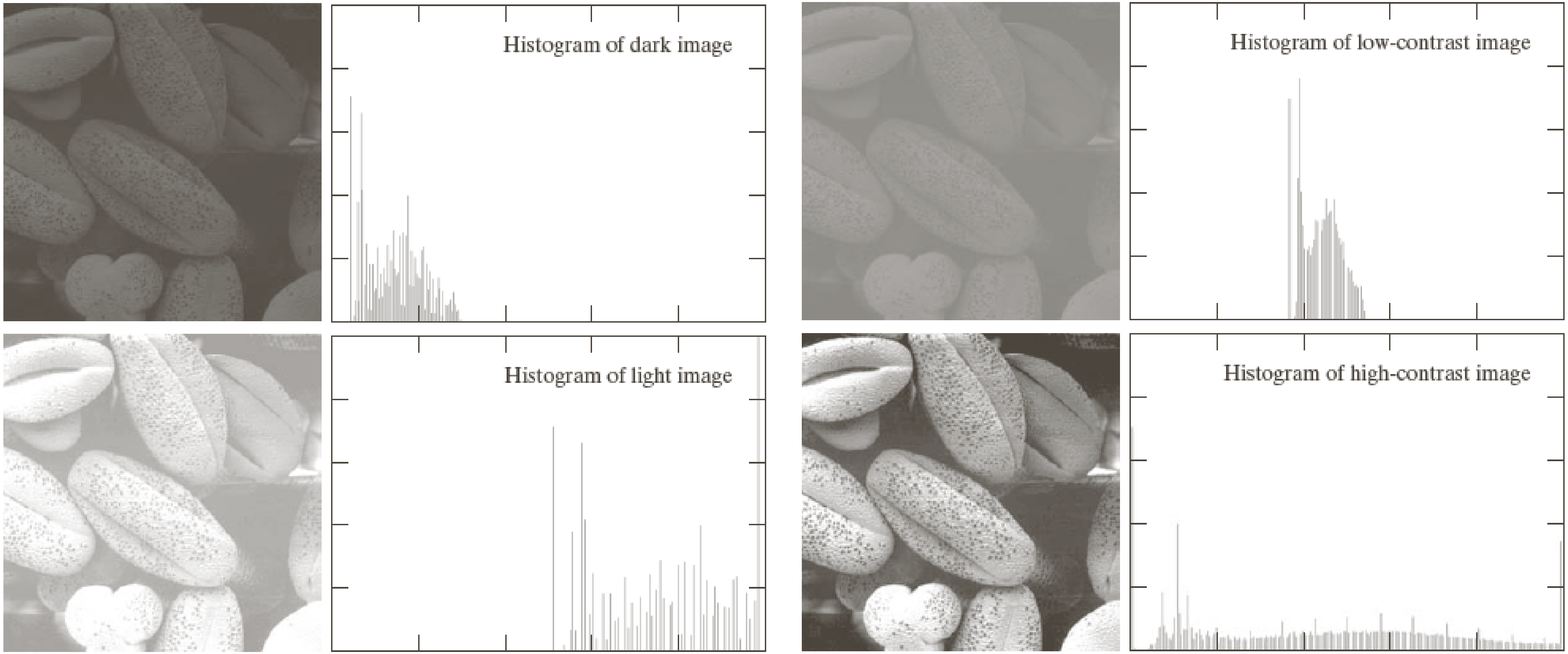
Normalized Histogram（归一化直方图）

$$p(r_k) = \frac{n_k}{MN}$$

Where $p(r_k)$: the probability of occurrence of intensity r_k in an image

M, N: the row and column dimensions of the image

Basic Image Type



Properties

The histogram of an image

- describe the number or probability of intensity, NO location (spatial) information
- can be same as other images
- $\sum_0^{L-1} n_k = M \cdot N$ or $\sum_0^{1} p(r_k) = 1$
- If Region C=A∪B, A and B are disjoint, $H_C = H_A + H_B$

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- **Histogram （直方图）**
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- **Intensity Transformation（灰度变换）**
  - Operate on single pixels of an image – point processing
  - Contrast manipulation and image thresholding（对比度和阈值处理）
- **Histogram Processing**
  - Histogram Equalization
  - Histogram Matching

Intensity Transformation

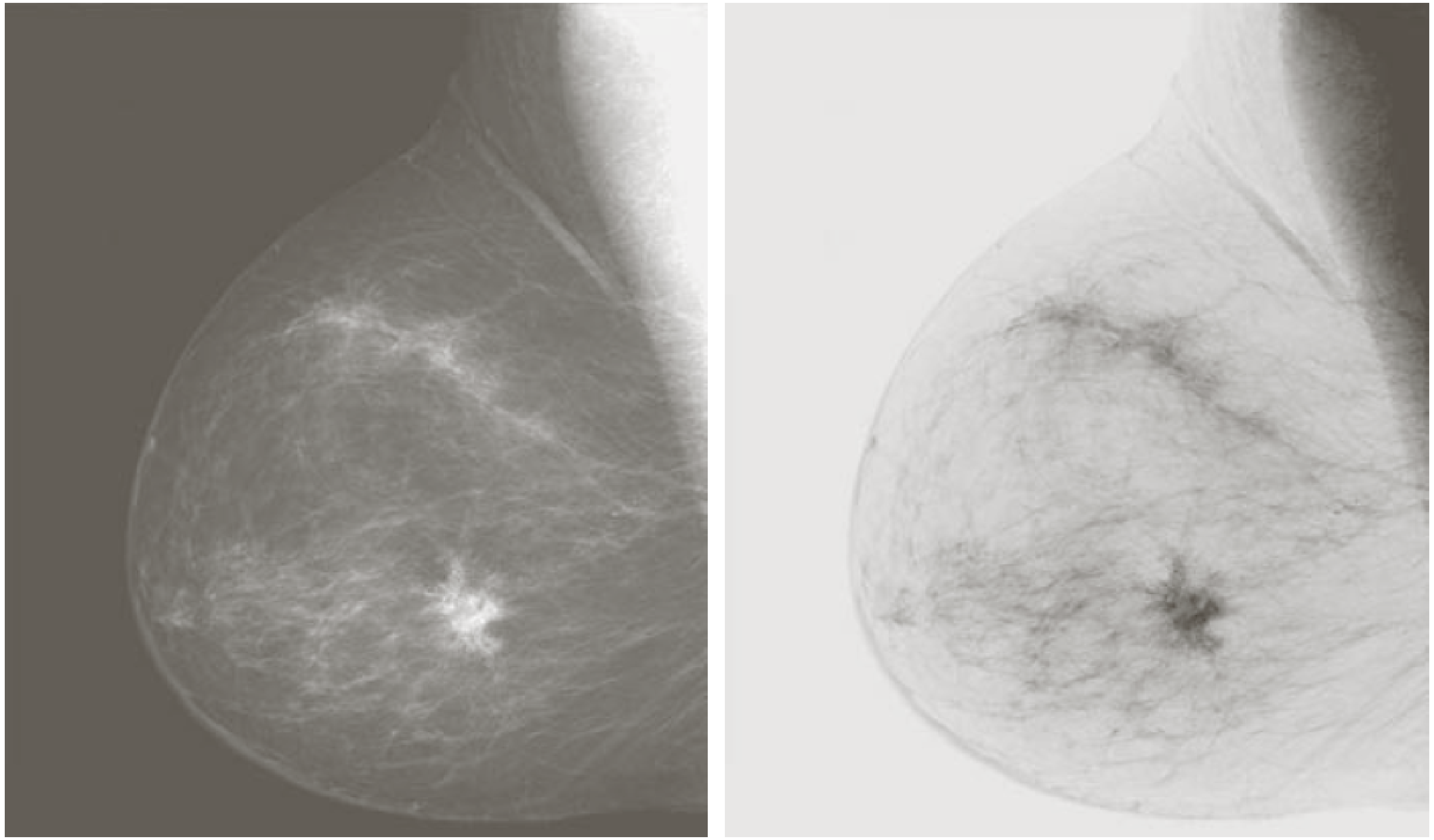
- **Simplest image processing techniques**

$$s = T(r)$$

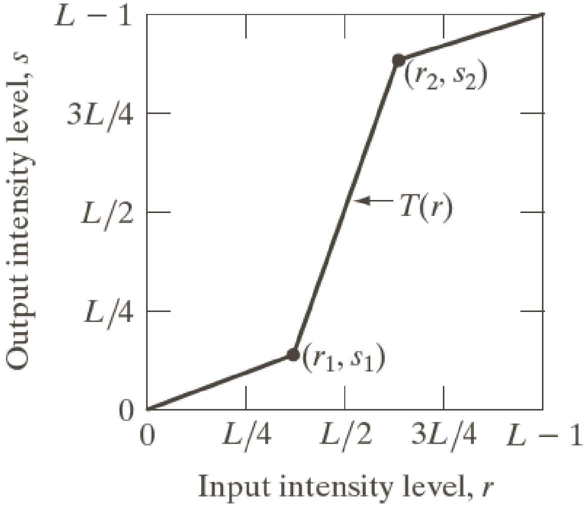
- **Types of Intensity Transformation**
  - Image Negatives（图像反转）
  - Log Transformation（对数变换）
  - Power-law (gamma) Transformation（幂律/伽马变换）
  - Piecewise-Linear Transformation（分段线性变换）

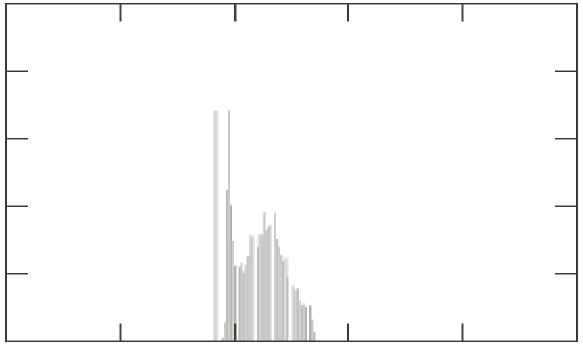
Image Negatives

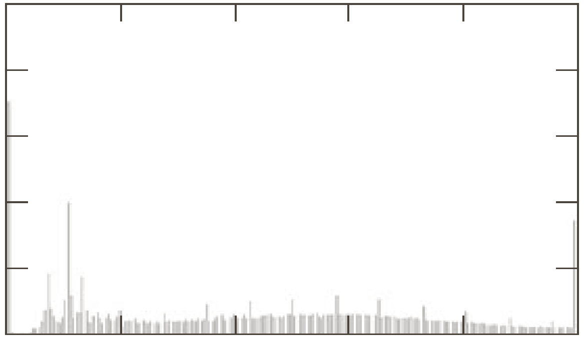
$$s = \mathrm{T}(r) = L - 1 - r$$



Contrast Stretching



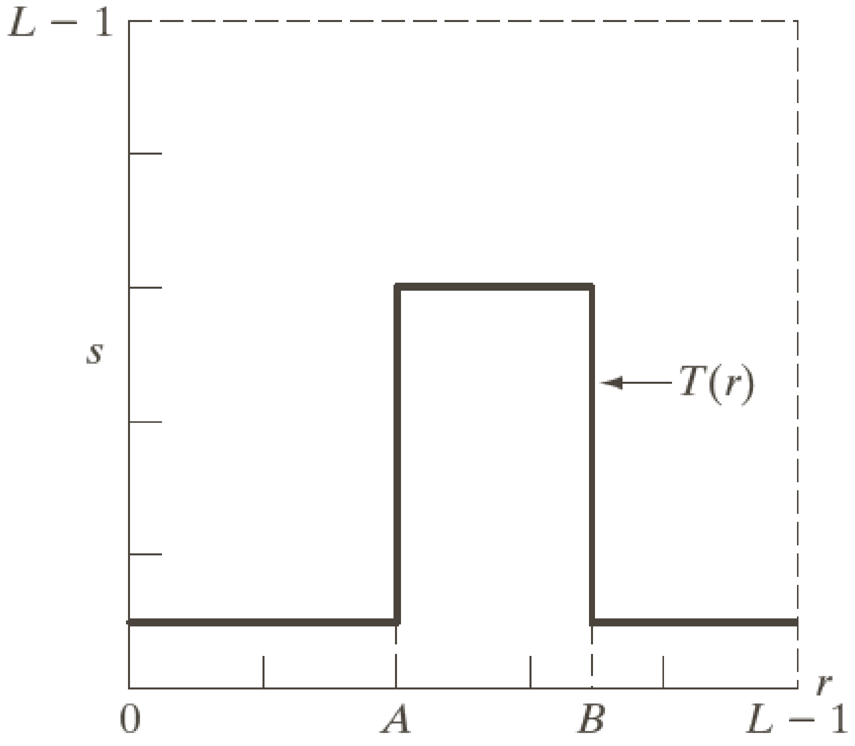


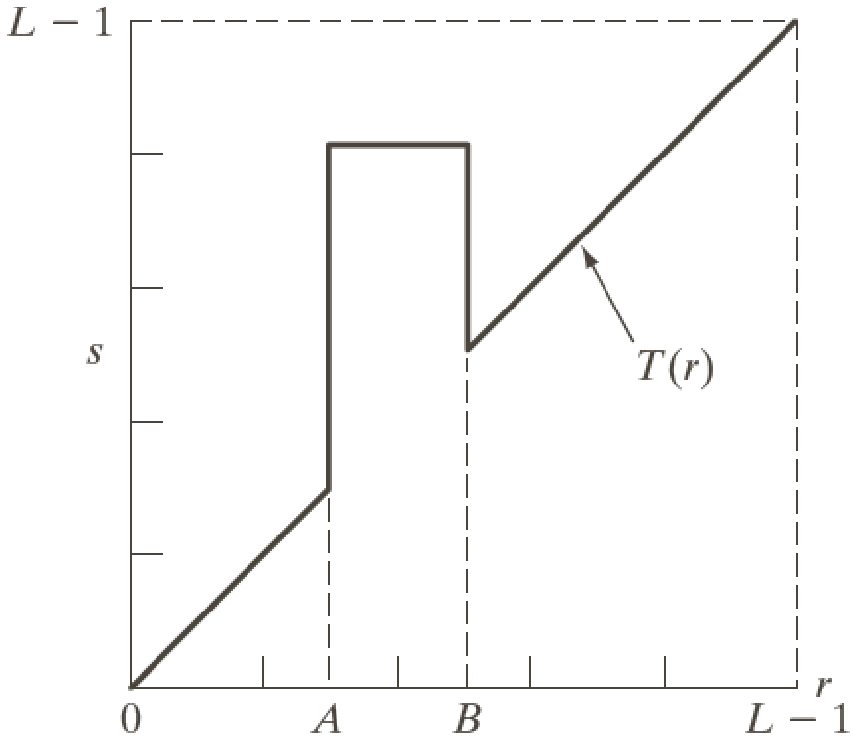


$r_1 = r_2$, $s_1 = 0$, $s_2 = L\text{-}1$

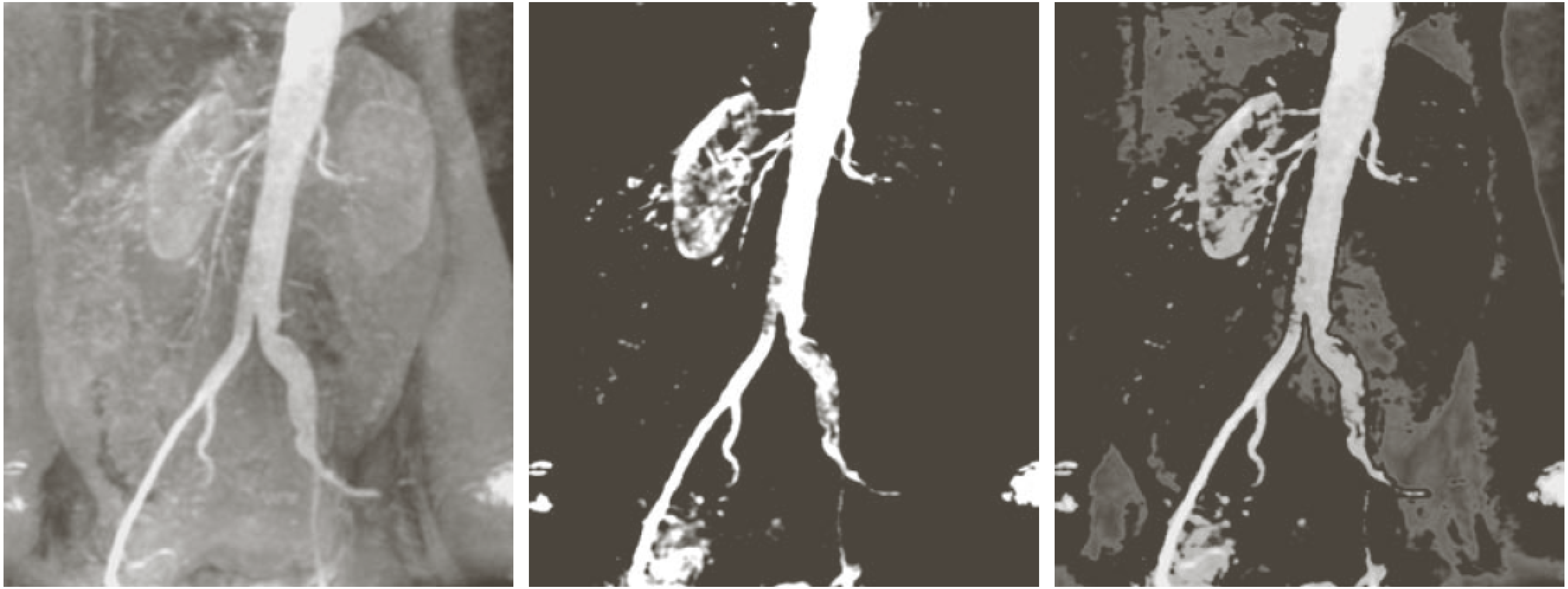
Intensity-level slicing

- What's the function of the transform $s = T(r)$ in the figures below?





Intensity-level slicing



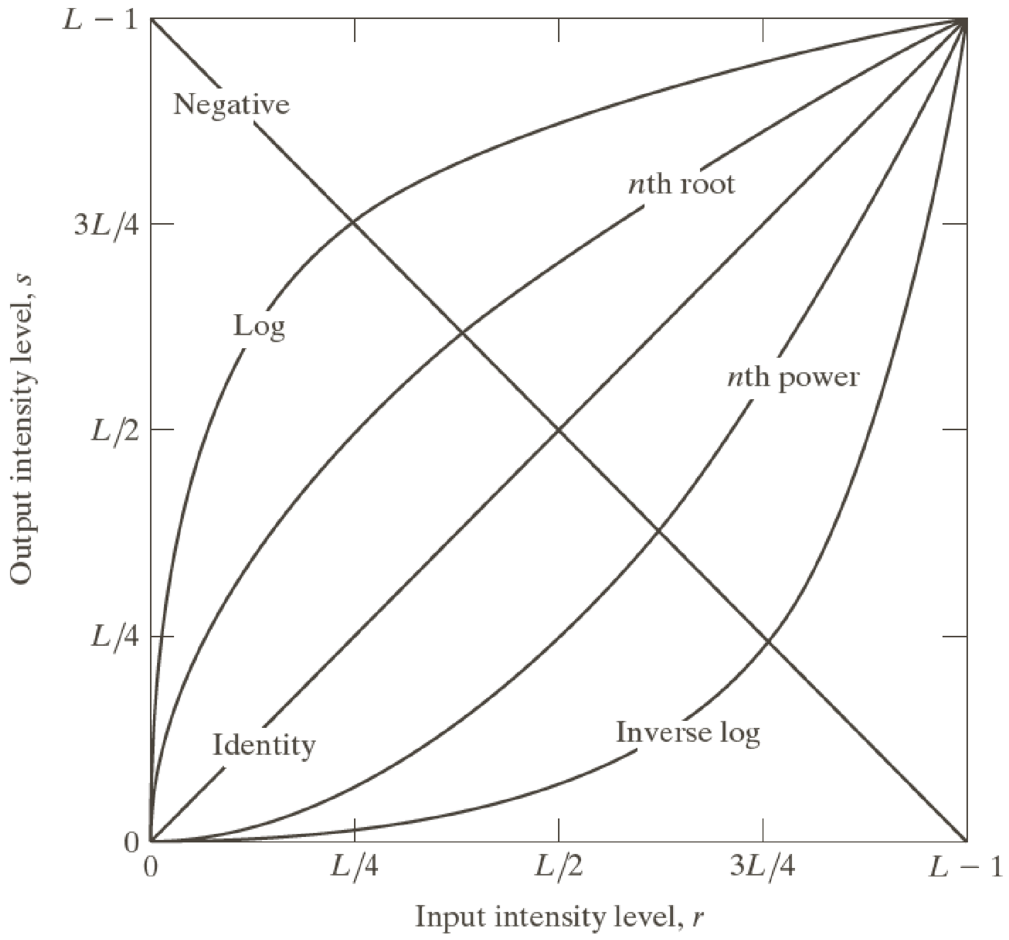
Log Transformation

- **Log Transformation (对数变换)**

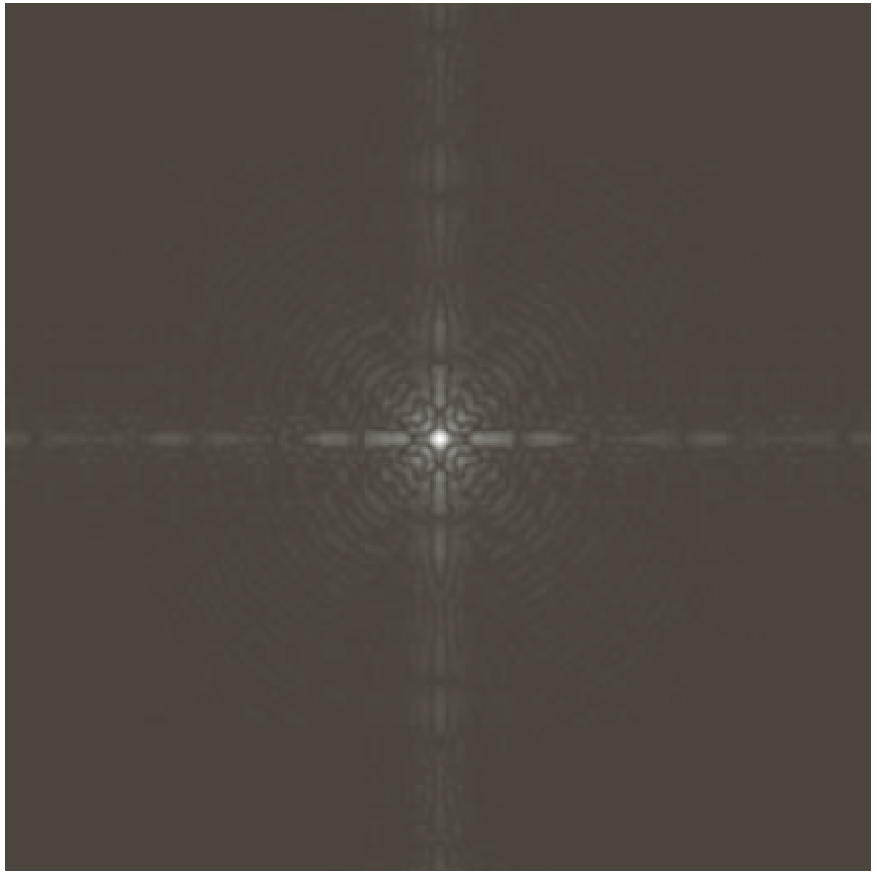
$$s = c \log(1 + r)$$

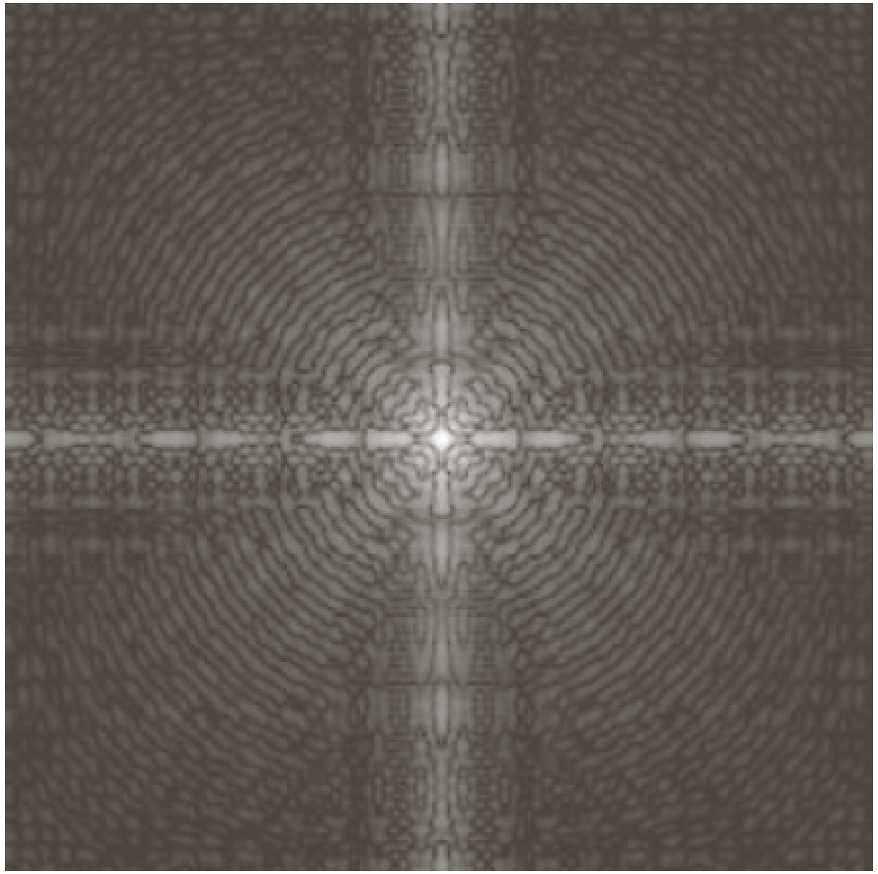
- **Inverse Log Transformation (反对数变换)**

$$s = c \cdot 2^r - 1$$



Fourier Spectrum





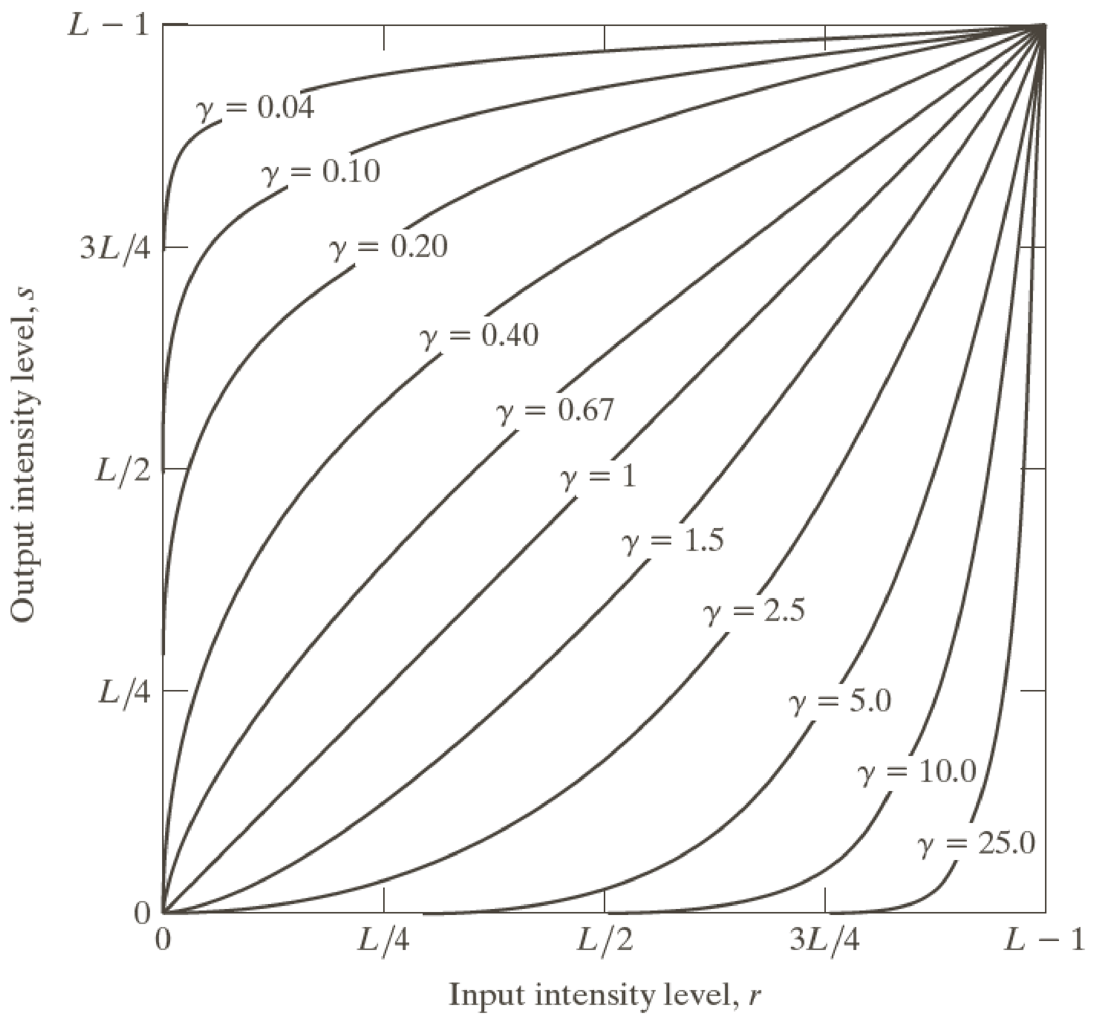
Gamma Transformation

- **Gamma Transformation (伽马变换)**

$$s = c \cdot r^{\gamma}$$

or

$$s = c \cdot (r + \varepsilon)^{\gamma}$$



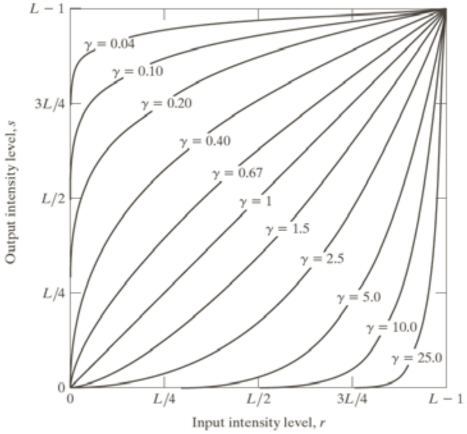
Gamma Transformation

Fractured spine

γ = 0.6

γ = 0.4

γ = 0.3



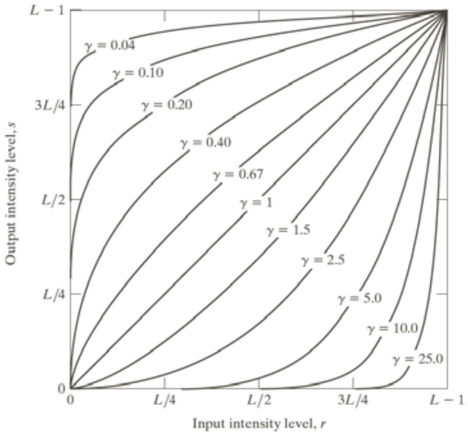
Gamma Transformation

Aerial image

γ = 3.0

γ = 4.0

γ = 5.0



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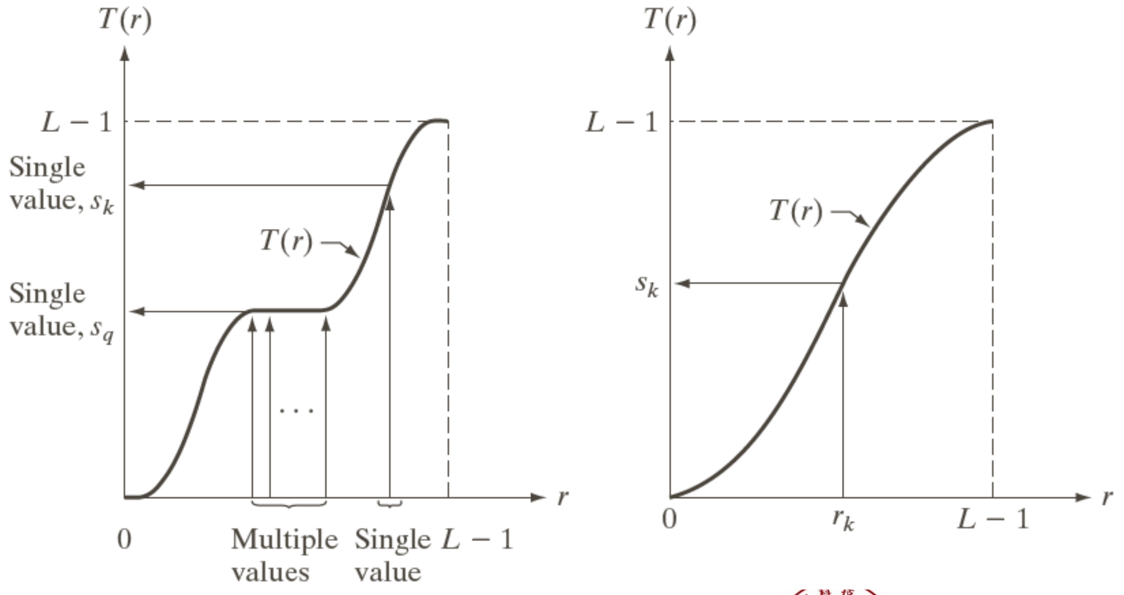
Basis of Histogram Processing

- Given intensity transformation $s = T(r)$, where $T(r)$
  - $T(r)$ is strictly monotonically increasing function （严格单调递增函数， $T(r_2) > T(r_1)$ if $r_2 > r_1$） in the interval $0 \leq r \leq L-1$
  - $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$
- The inverse transform $r = T^{-1}(s)$



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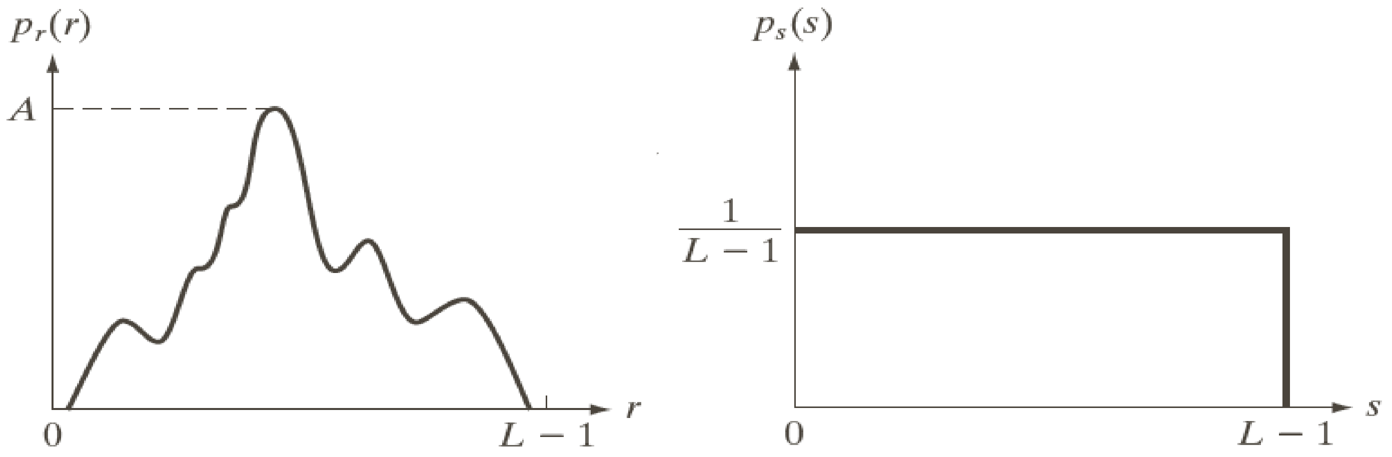
Histogram Equalization

- **Uniform Probability density function :** $p_s(s) = \frac{1}{L-1}$
- **The probability density function (PDF) of s is**

$$p_s(s) = p_r(r) \cdot \frac{dr}{ds} \Longrightarrow p_r(r) \cdot \frac{dr}{ds} = \frac{1}{L-1} \Longrightarrow (L-1)p_r(r) \cdot dr = ds$$

- **Transformation function :** $s = T(r) = (L-1)\int_0^r p_r(w)dw$



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Complementary prove

$$\boldsymbol{p_s(s) = p_r(r) \cdot \frac{dr}{ds}}$$

- Since $S = T(r)$ is strictly monotonically increasing function

$\Longrightarrow$ We have $s = T(r), \mathrm{v} = T(\mathrm{w})$, if $\mathrm{v} < \mathrm{s}$ then we have $\mathrm{v} < \mathrm{s} \Leftrightarrow \mathrm{w} < \mathrm{r}$

$\Longrightarrow P(\mathrm{v} < \mathrm{s}) = P(\mathrm{w} < \mathrm{r})$

$\Longrightarrow (\int_{-\infty}^{s} P_s(v)dv)' = (\int_{-\infty}^{r} P_r(w)dw)'$

$\Longrightarrow P_s(s)ds = P_r(r)dr$

$\Longrightarrow \boldsymbol{p_s(s) = p_r(r) \cdot \frac{dr}{ds}}$

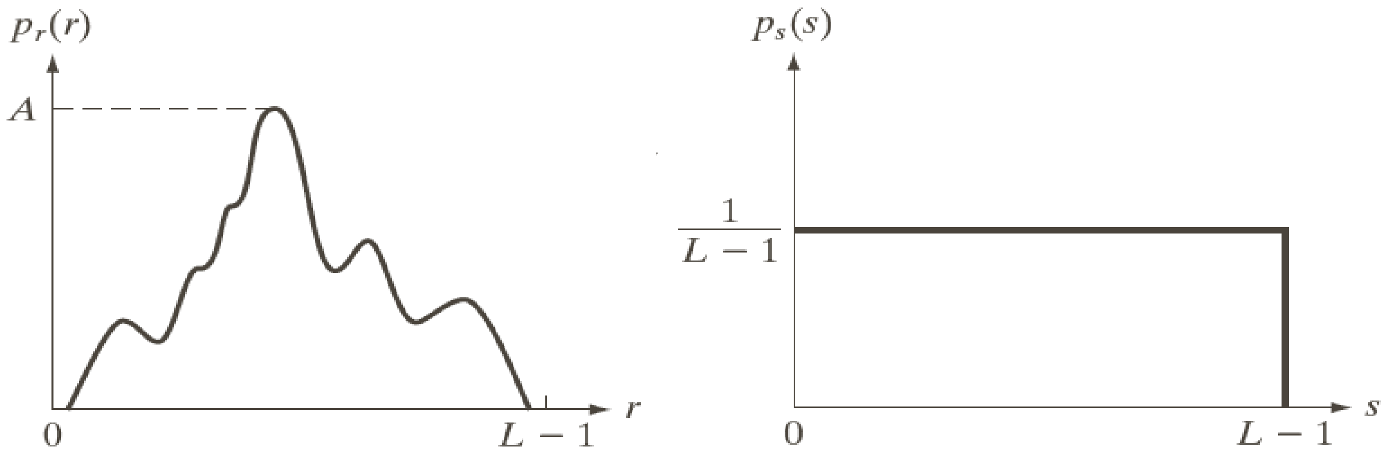
1) If $f(x)$ is continuous on [a,b], then $F(x) = \int_a^x f(t)dt$ is differentiable, and $F'(x) = f(x)$.
2) If $f(x)$ is continuous on [a,b], and $\varphi(x)$ is differentiable, then $(\int_a^{\varphi(x)} f(t)dt)' = f[\varphi(x)]\varphi'(x)$.

Histogram Equalization

- Uniform Probability density function : $p_s(s) = \frac{1}{L-1}$
- The probability density function (PDF) of s is

$$p_s(s) = p_r(r) \cdot \frac{dr}{ds} \Longrightarrow p_r(r) \cdot \frac{dr}{ds} = \frac{1}{L-1} \Longrightarrow (L-1)p_r(r) \cdot dr = ds$$

- Transformation function : $s = T(r) = (L-1)\int_0^r p_r(w)dw$

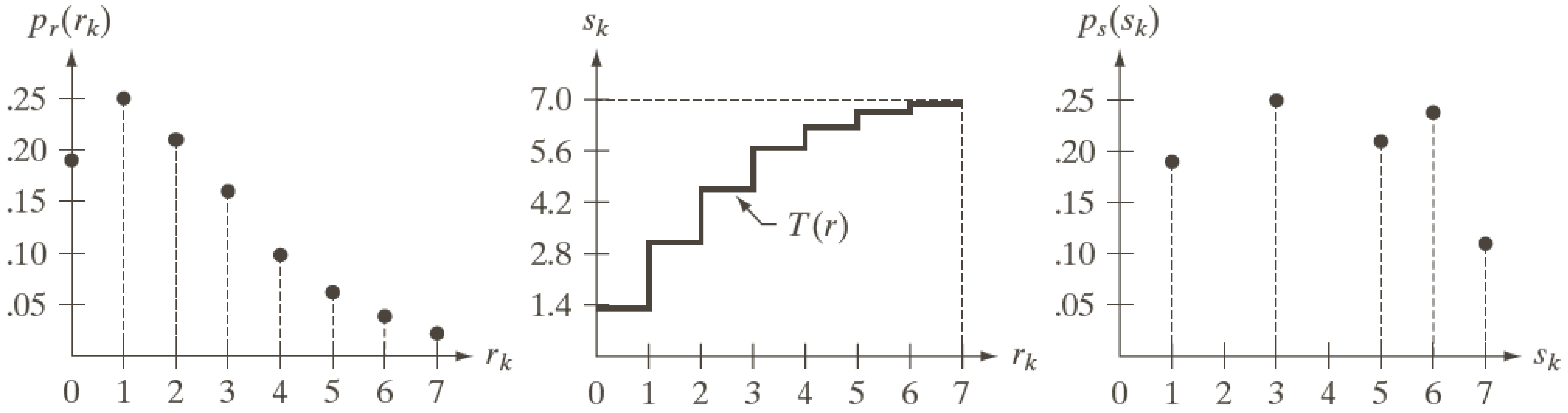


Histogram Equalization

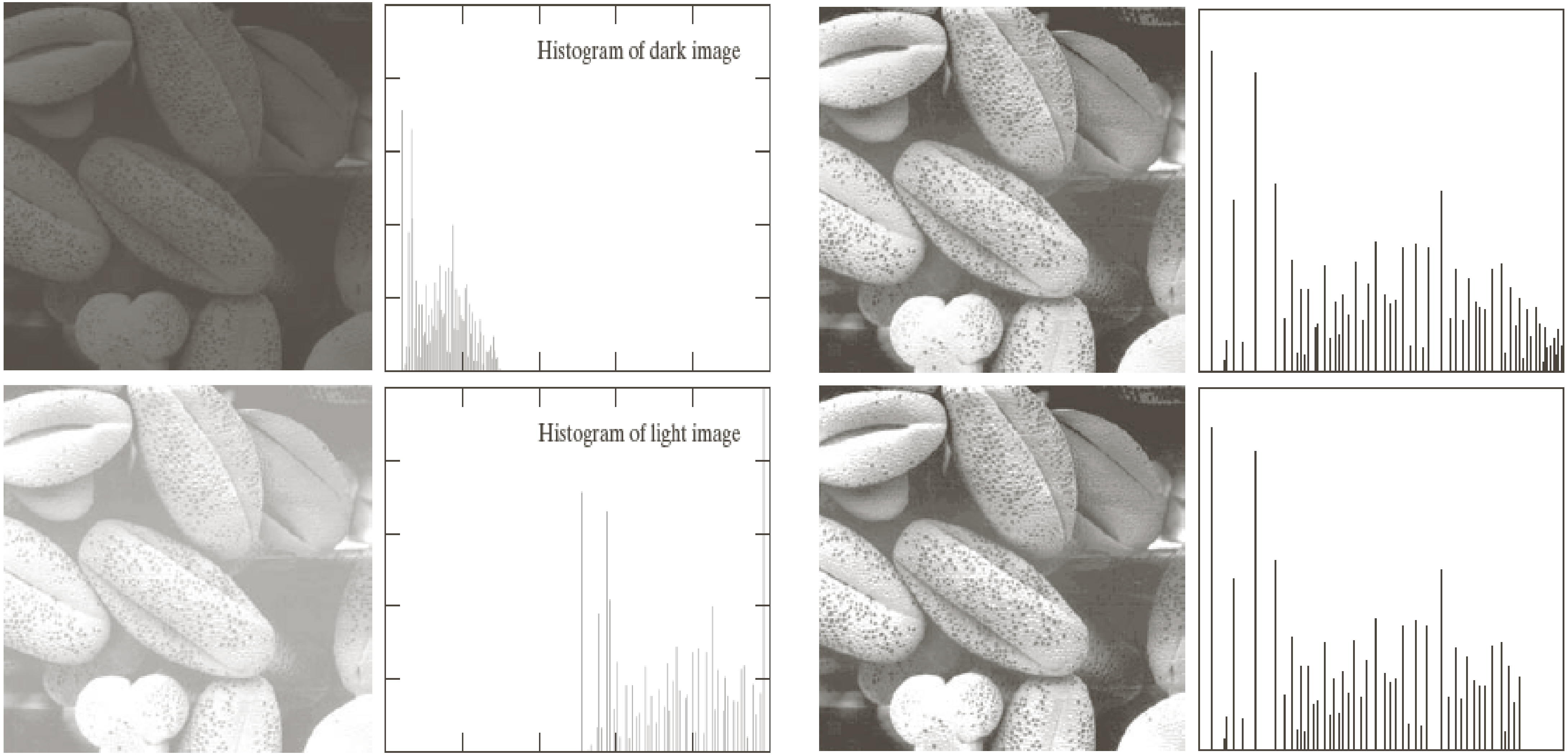
$$s = T(r) = (L-1)\int_0^r p_r(w)dw = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k}\frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \text{ k}$$

r_k	n_k	$p_r(r_k)$	s_k		s_k	$p_s(s_k)$
0	790	0.19	1.33	1	0	0
1	1023	0.25	3.08	3	1	0.19
2	850	0.21	4.55	5	2	0
3	656	0.16	5.67	6	3	0.25
4	329	0.08	6.23	6	4	0
5	245	0.06	6.65	7	5	0.21
6	122	0.03	6.86	7	6	0.24
7	81	0.02	7.00	7	7	0.11

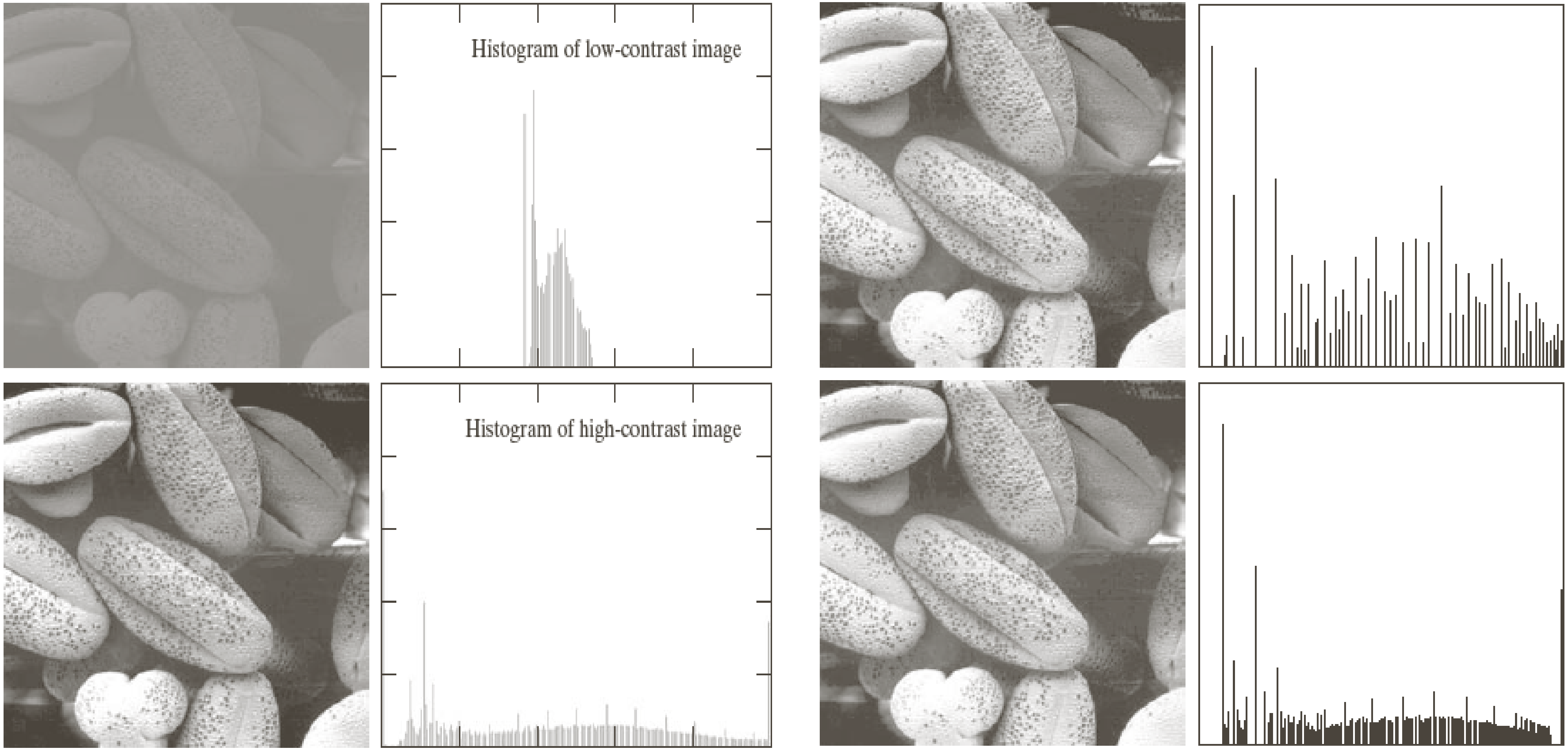
Example


$p_r(r_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
r_k
s_k
7.0
5.6
4.2
2.8
1.4
$T(r)$
0 1 2 3 4 5 6 7
r_k
$p_s(s_k)$
.25
.20
.15
.10
.05
0 1 2 3 4 5 6 7
s_k

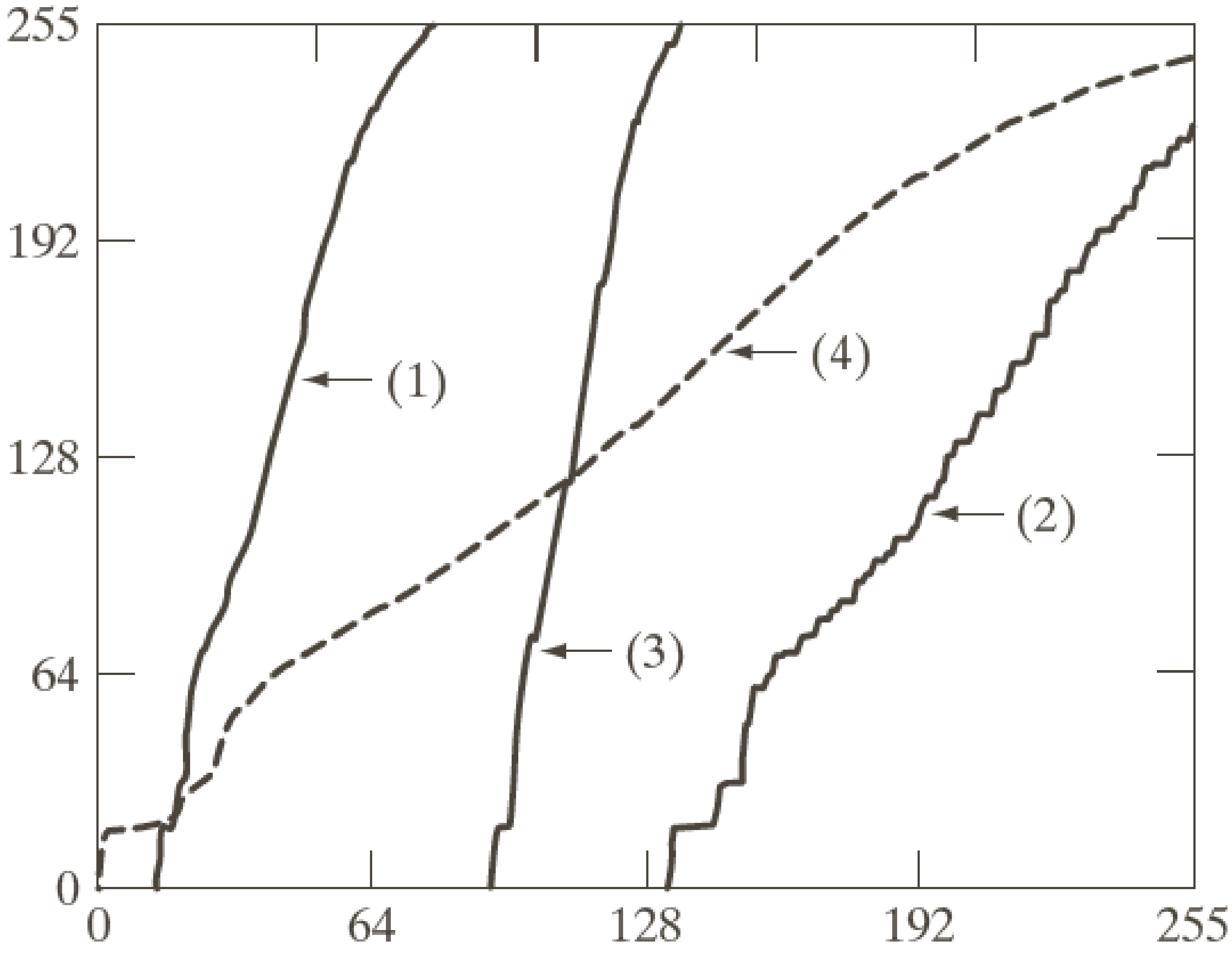
Example



Example


Histogram of low-contrast image
Histogram of high-contrast image

Transformation Function



Histogram Matching

Generate a processed image with a specified histogram

For input : $s = T(r) = (L-1)\int_0^r p_r(w)dw$

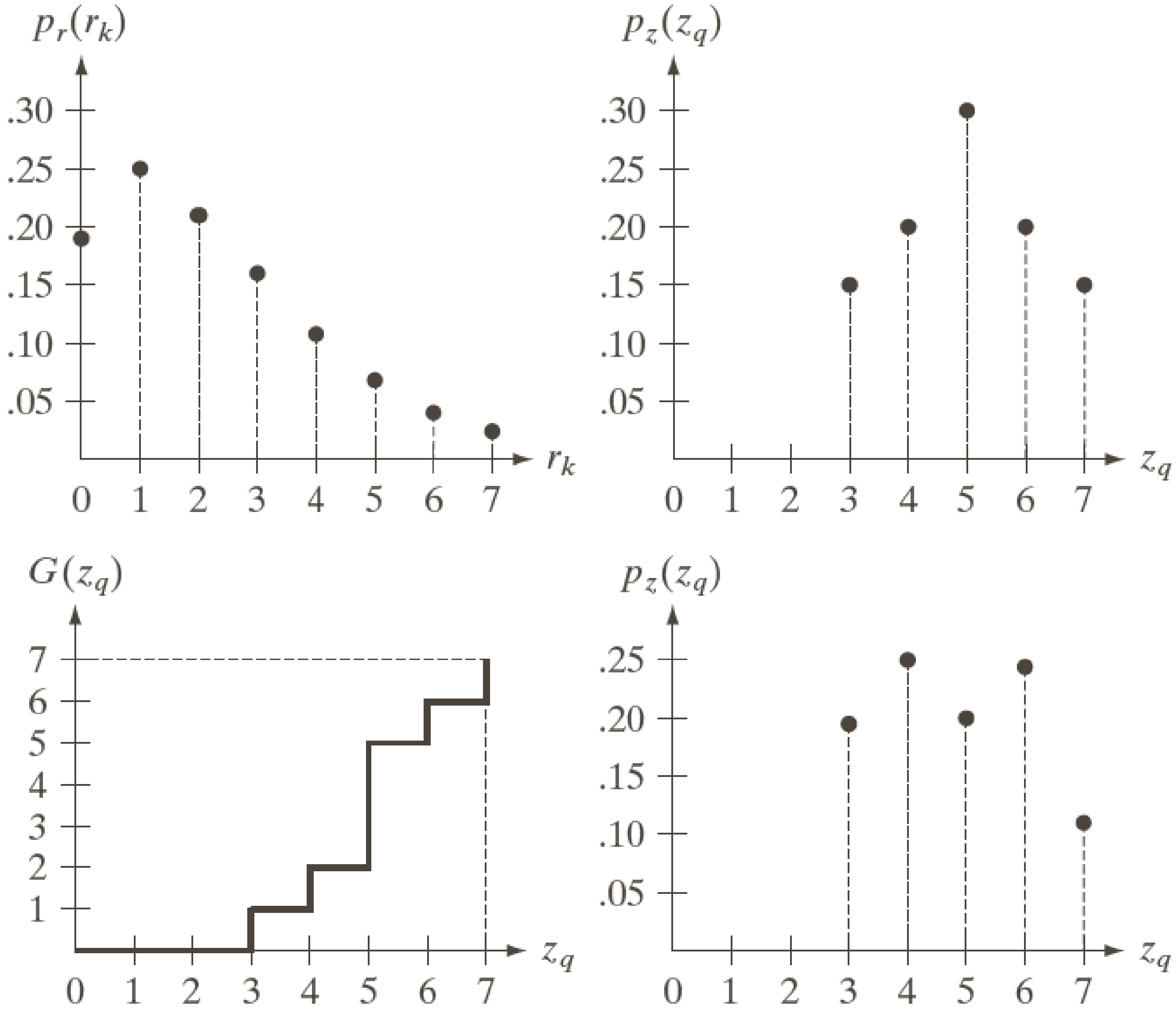
For output : $G(z) = (L-1)\int_0^z p_z(t)dt = s$

Therefore $\quad z = G^{-1}(s) = G^{-1}[T(r)]$

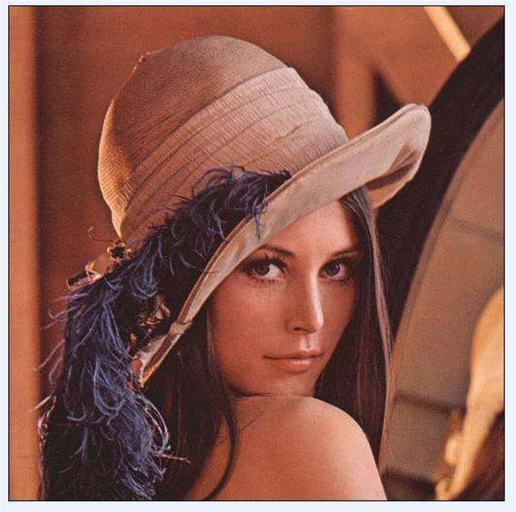
Histogram Matching

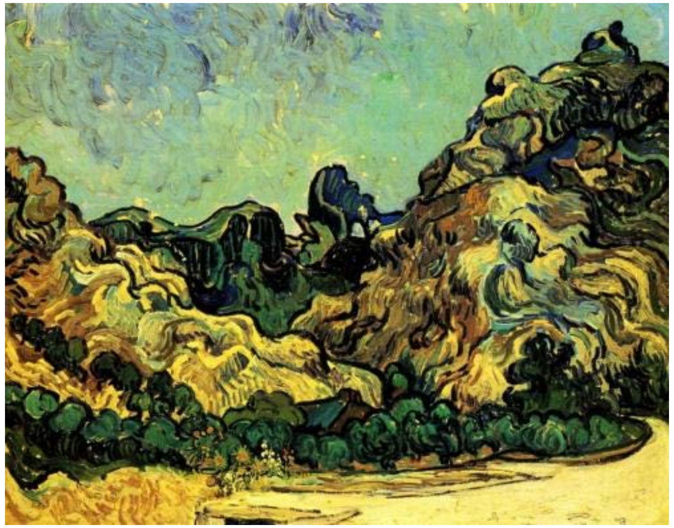
r_k	$p(r_k)$	$s_k=T(r_k)$	z_q	$p(z_q)$	$s_k = G(z_q)$	$s_k \to z_k$	$r_k \to z_k$	z_k	$p(z_k)$
0	0.19	1	0	0	0	0→0, 1, 2	0→3	0	0
1	0.25	3	1	0	0	1→3	1→4	1	0
2	0.21	5	2	0	0	2→4	2→5	2	0
3	0.16	6	3	0.15	1		3→6	3	0.19
4	0.08	6	4	0.20	2		4→6	4	0.25
5	0.06	7	5	0.30	5	5→5	5→7	5	0.21
6	0.03	7	6	0.20	6	6→6	6→7	6	0.24
7	0.02	7	7	0.15	7	7→7	7→7	7	0.11

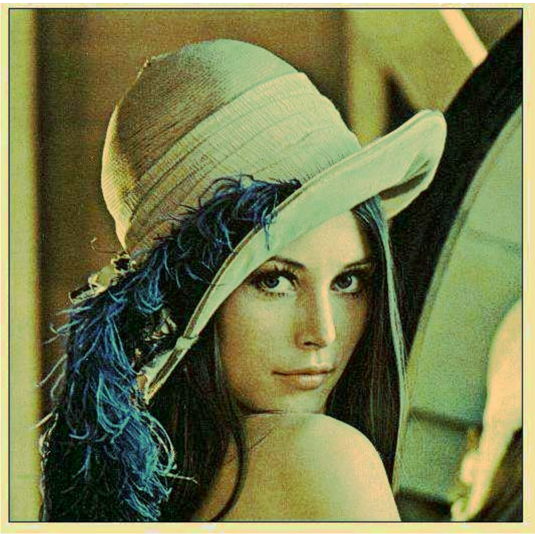
Histogram Matching



Histogram Matching Application







Take home message

- 1. Histogram describes intensity property of image, NO location (spatial) information.
- 2. Simplest image processing technique- intensity transform.

$$s = T(r)$$

- 3. The main purpose of intensity transform is to modify image histogram, to make the image contrast looks more comfortable.
- 4. Common intensity transform: contrast stretching, log, gamma, histogram equalization, histogram matching.