Lecture 10 Frequency Domain Filtering （chapter 4.7-4.10 ）

Yuyao Zhang PhD
zhangyy8@shanghaitech.edu.cn
SIST Building 2 302-F

上海科技大学
ShanghaiTech University

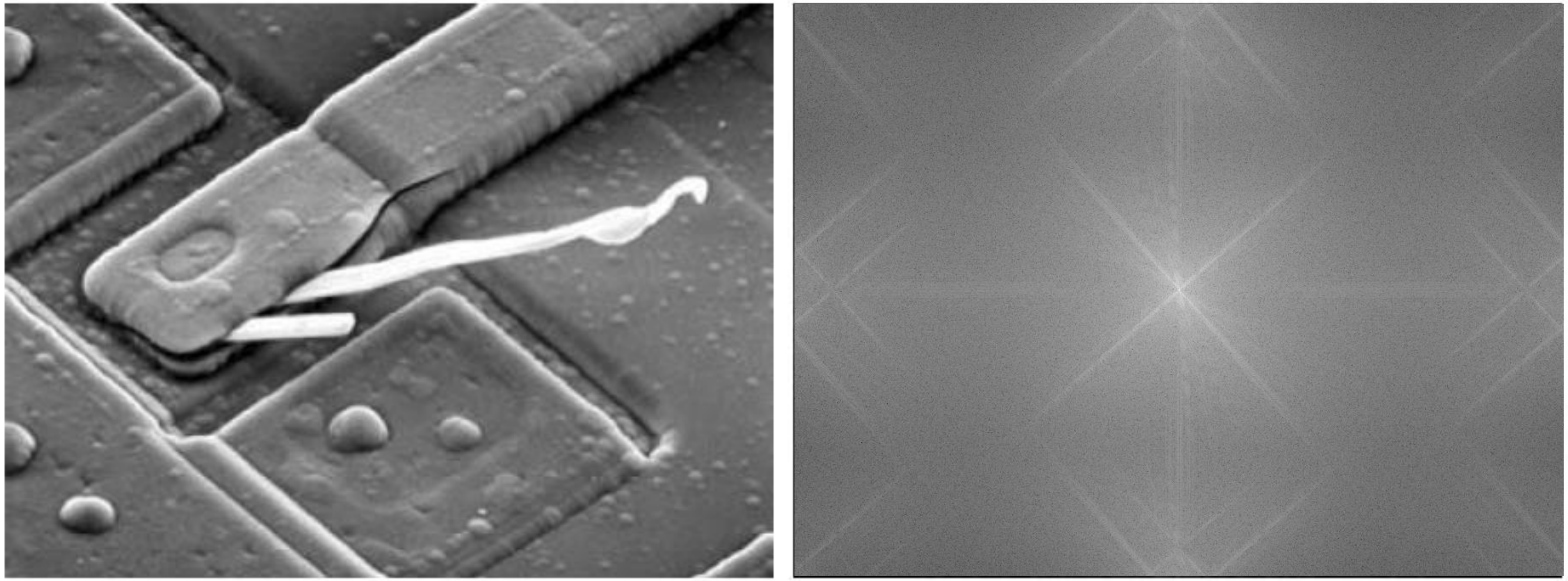
Outline

– 2D Discrete Fourier Transform （傅里叶变换）

– Frequency Domain Filtering（频率域滤波）

- Lowpass Filtering（低通滤波器）
- Highpass Filtering （高通滤波器）
- Selective Filtering（选择性滤波）

Fourier Spectrum



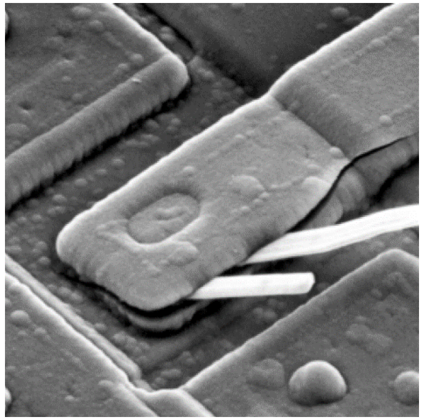
a b

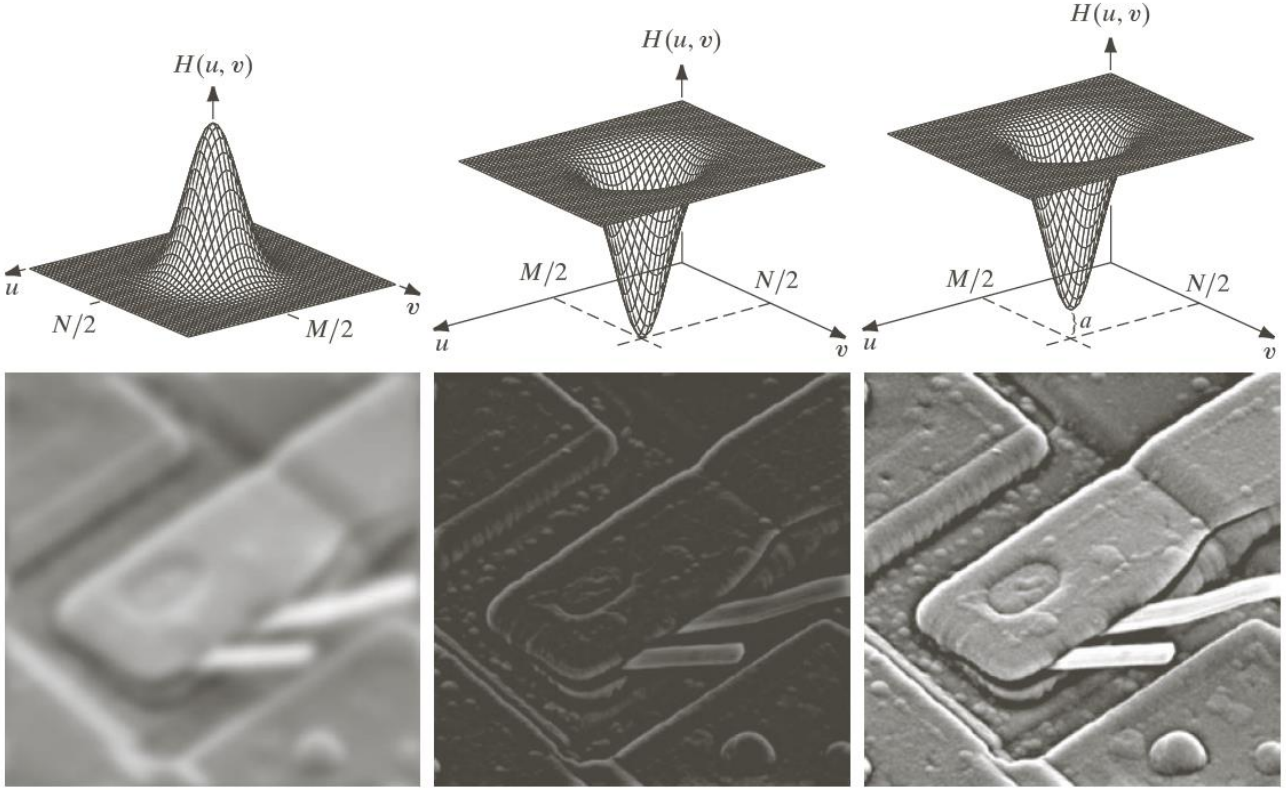
FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

Frequency Domain Filtering

Basic Filtering form: $$g(x,y) = \mathcal{F}^{-1}[H(u,v)F(u,v)]$$

Original image

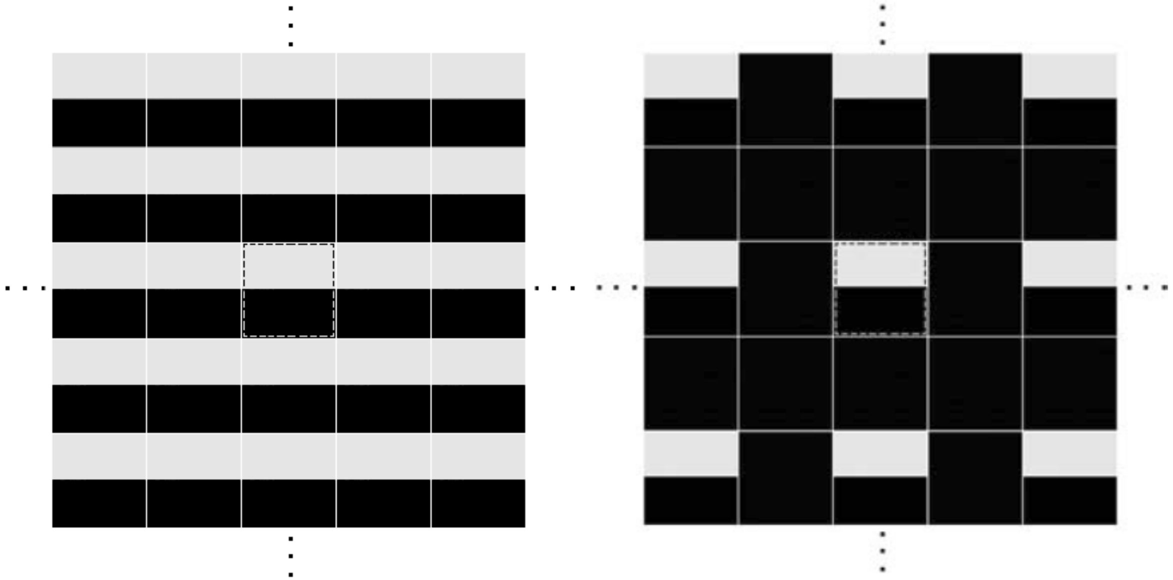




a b c
d e f

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

Padding (填充)



Filtering in Spatial and Frequency Domains

- **Frequency filters ⇒ Spatial filter** $\boldsymbol{H(u,v) \Rightarrow h(x,y)}$

Design filter in frequency domain → Compute IDFT → Full size spatial filter → Smaller spatial masks

- **Gaussian Filter**

$$H(u) = Ae^{-\frac{u^2}{2\sigma^2}} \Leftrightarrow h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2x^2}$$

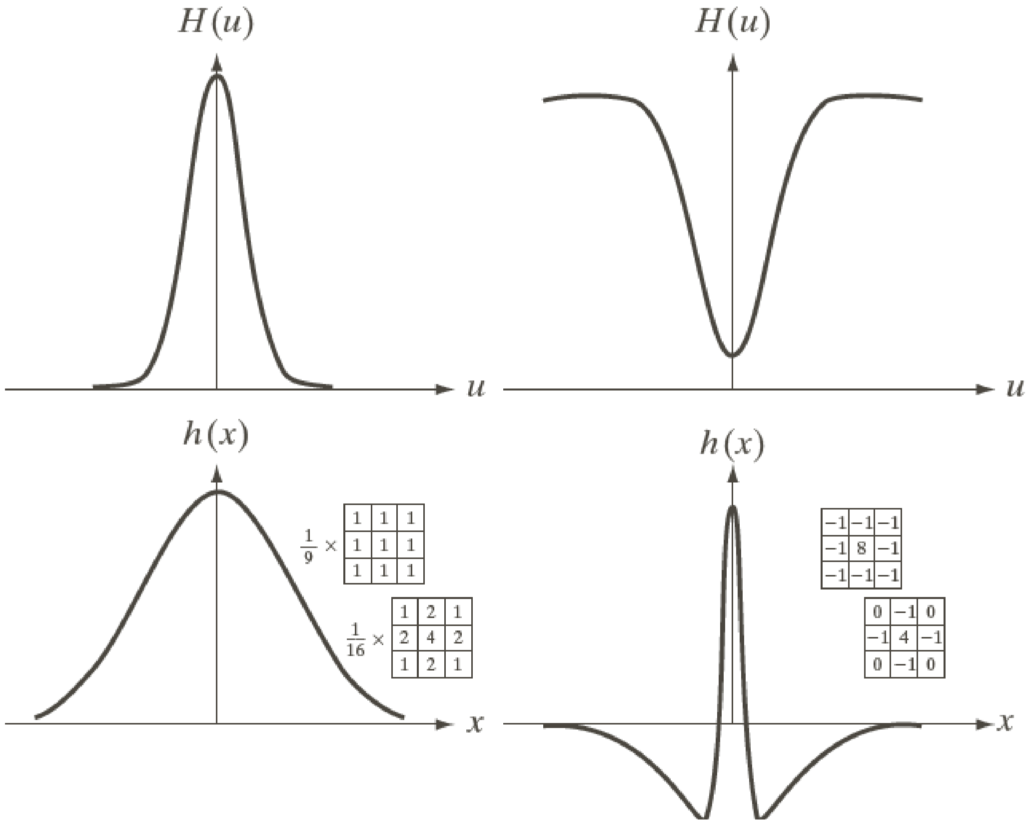
$$H(u) = Ae^{-\frac{u^2}{2\sigma_1^2}} - Be^{-\frac{u^2}{2\sigma_2^2}} \Leftrightarrow h(x) = \sqrt{2\pi}\sigma_1 A e^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma_2 B e^{-2\pi^2\sigma_2^2x^2}$$

$$H(u,v) = Ae^{-\frac{u^2+v^2}{2\sigma^2}} \Leftrightarrow h(x,y) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2+y^2)}$$

Filtering in Spatial and Frequency Domains

➢ **Frequency filters ⇒ Spatial filter** $\boldsymbol{H(u,v) \Rightarrow h(x,y)}$

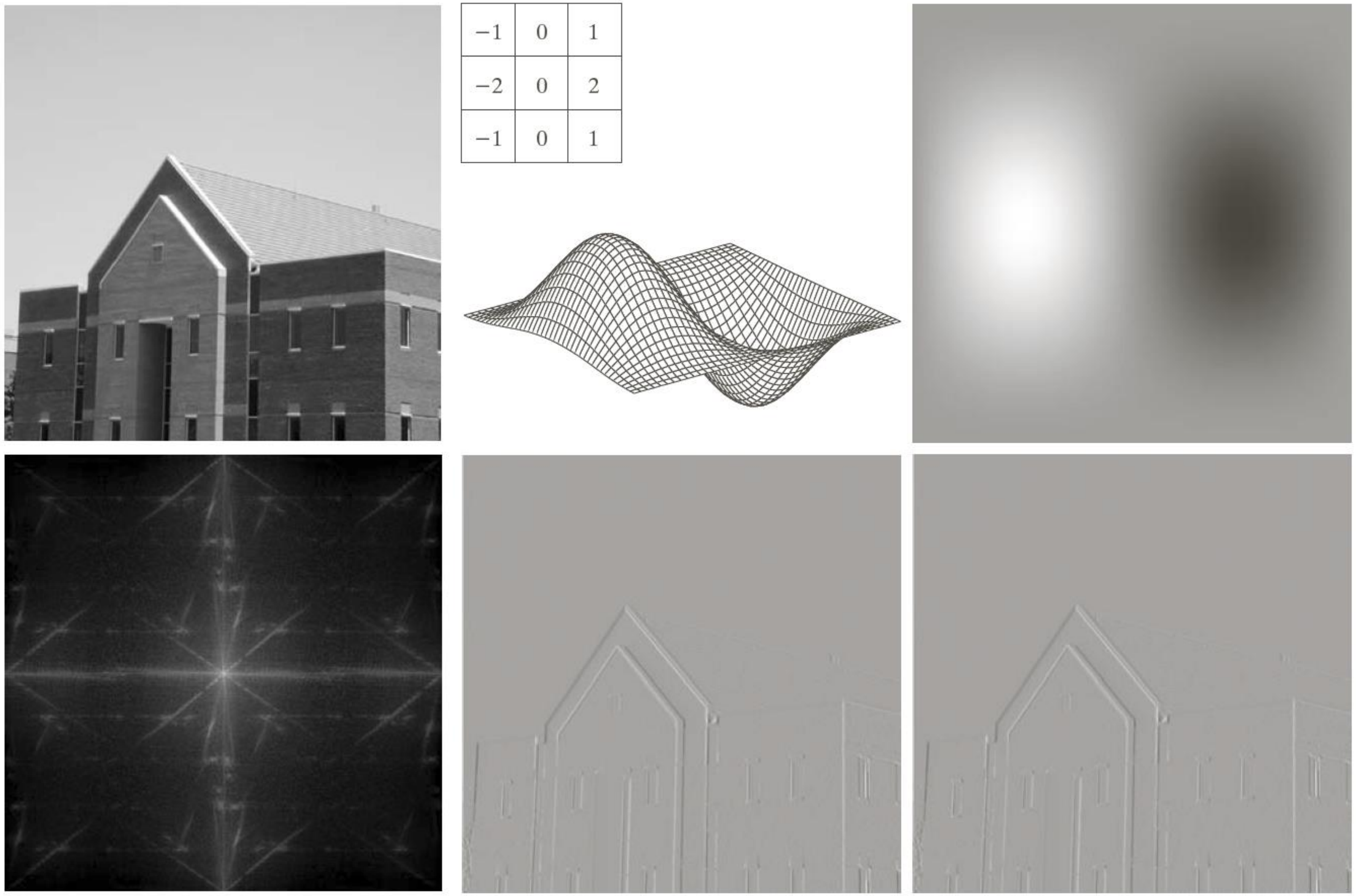
$$H(u) = Ae^{-\frac{u^2}{2\sigma^2}} \iff h(x) = \sqrt{2\pi}\sigma A e^{-2\pi^2\sigma^2x^2}$$



$$H(u) = Ae^{-\frac{u^2}{2\sigma_1^2}} - Be^{-\frac{u^2}{2\sigma_2^2}} \iff h(x) = \sqrt{2\pi}\sigma_1 A e^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma_2 B e^{-2\pi^2\sigma_2^2x^2}$$

Spatial and Frequency Filtering

−1	0	1
−2	0	2
−1	0	1



Lowpass Filtering

- Ideal Lowpass Filter（理想低通滤波器）
- Butterworth Lowpass Filter（布特沃斯低通滤波器）
- Gaussian Lowpass Filter （高斯低通滤波器）

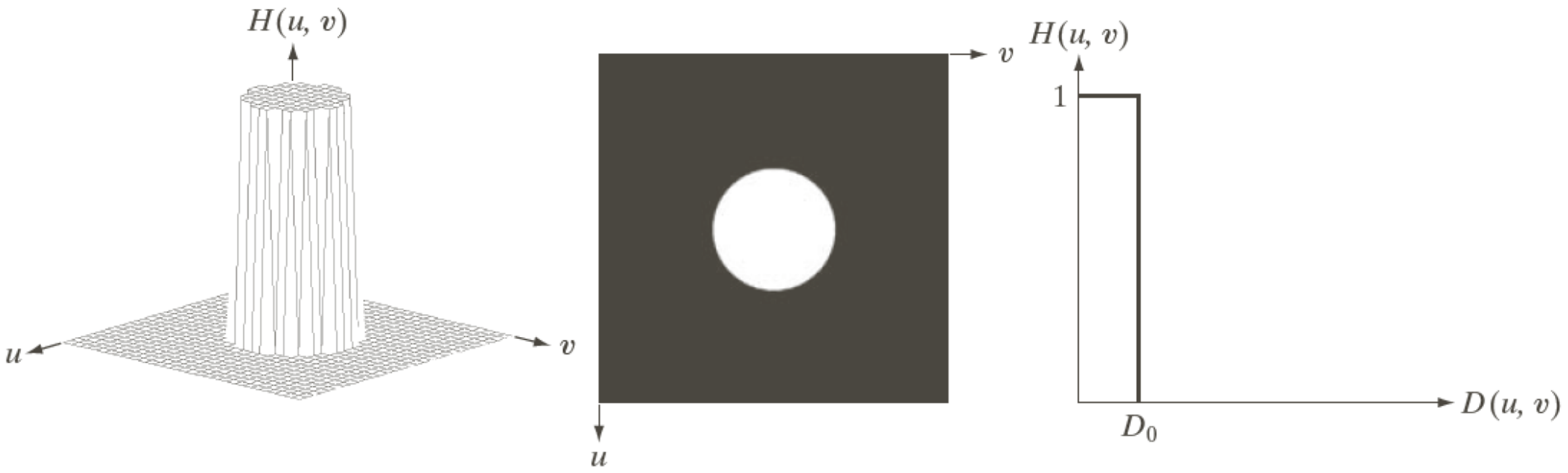
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

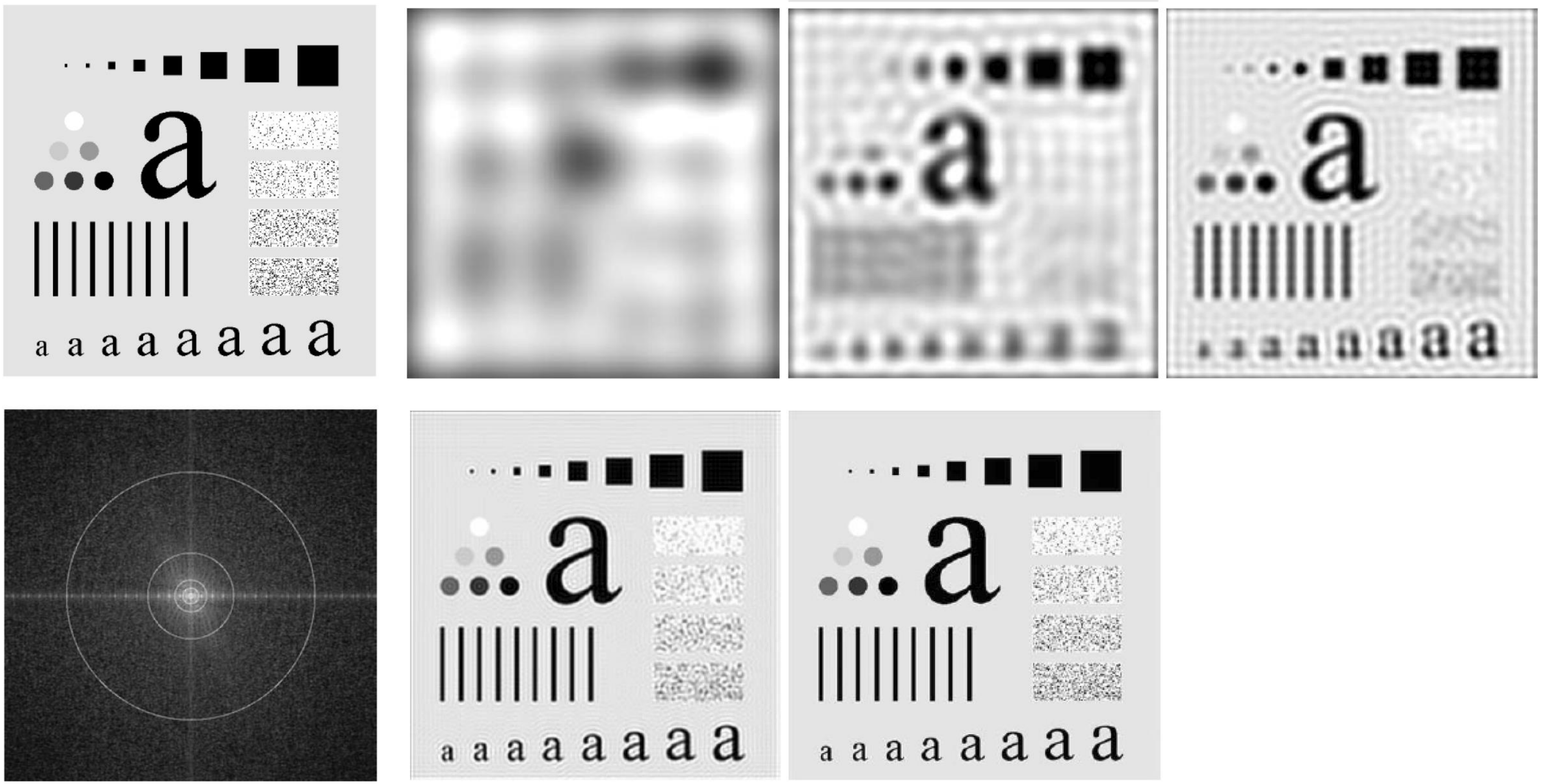
Ideal Lowpass Filter（理想低通滤波器）

Ideal Lowpass Filter (ILPF):

$$H(u,v)=\begin{cases}1, & D(u,v)\le D_0\\ 0, & D(u,v)>D_0\end{cases} \qquad D(u,v)=\left[(u-\frac{P}{2})^2+(v-\frac{Q}{2})^2\right]^{1/2}$$



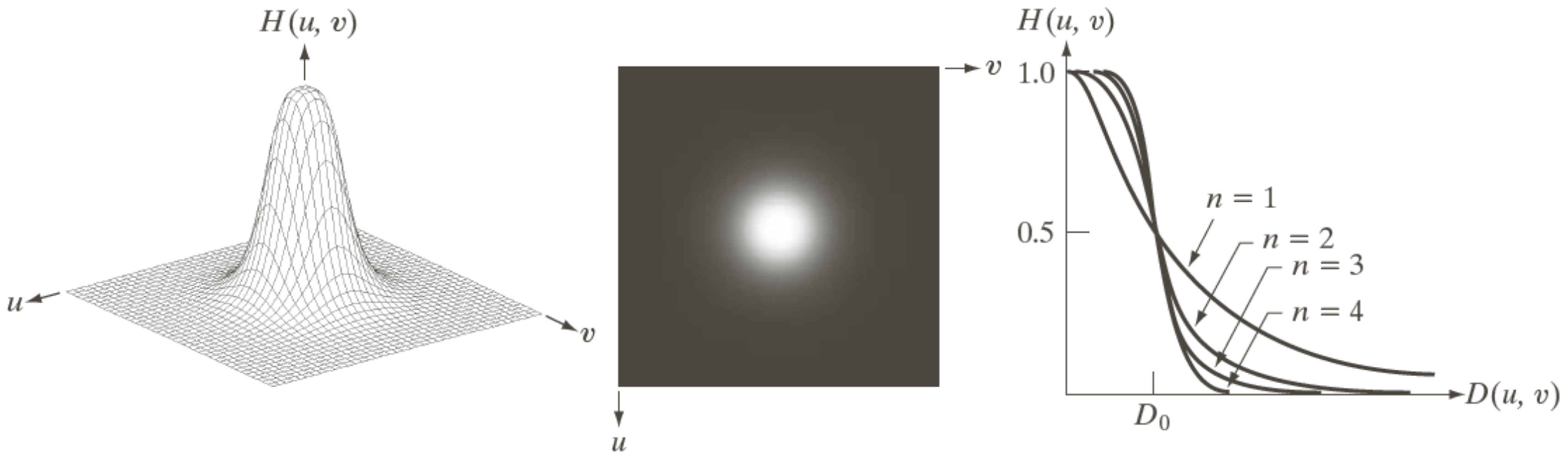
Cutoff Frequency（截止频率）



Butterworth Lowpass Filter（布特沃斯）

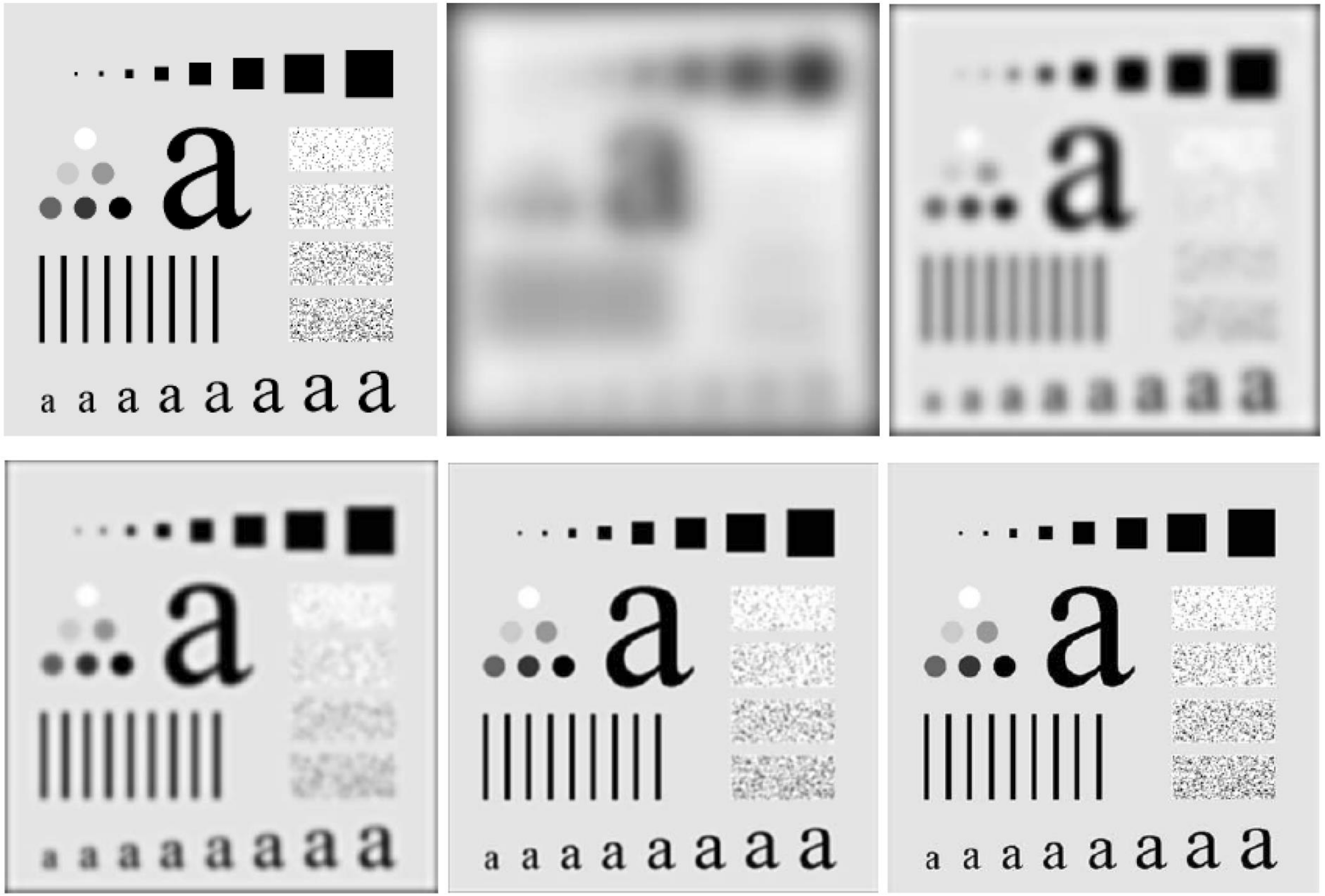
$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

Where $D(u,v) = \left[(u - \frac{P}{2})^2 + (v - \frac{Q}{2})^2\right]^{1/2}$, and $H(u,v) = 0.5$ when $D(u,v) = D_0$



上海科技大学
ShanghaiTech University

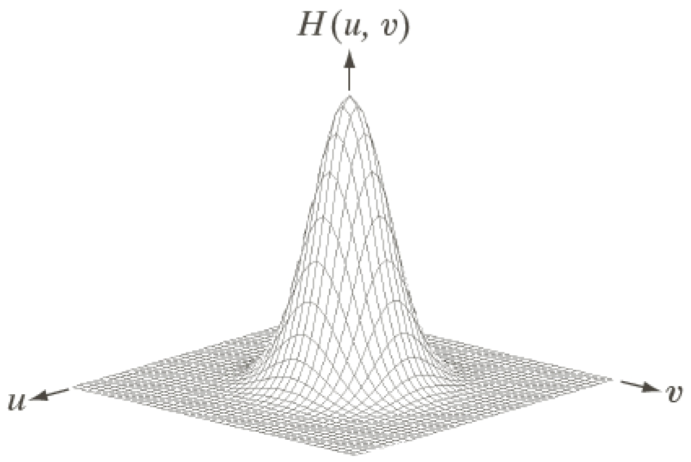
Butterworth Lowpass Filter（布特沃斯）

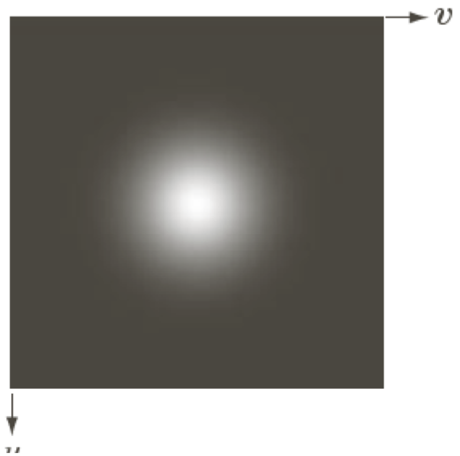


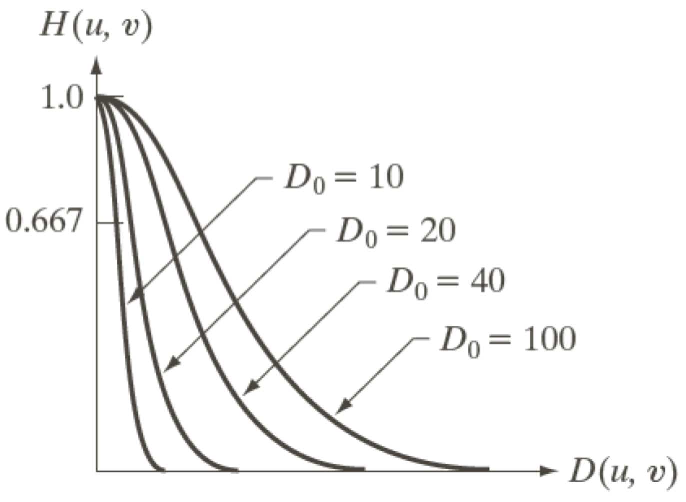
Gaussian Lowpass Filter (高斯滤波器)

$$H(u,v) = e^{-\frac{D(u,v)^2}{2D_0^2}}$$

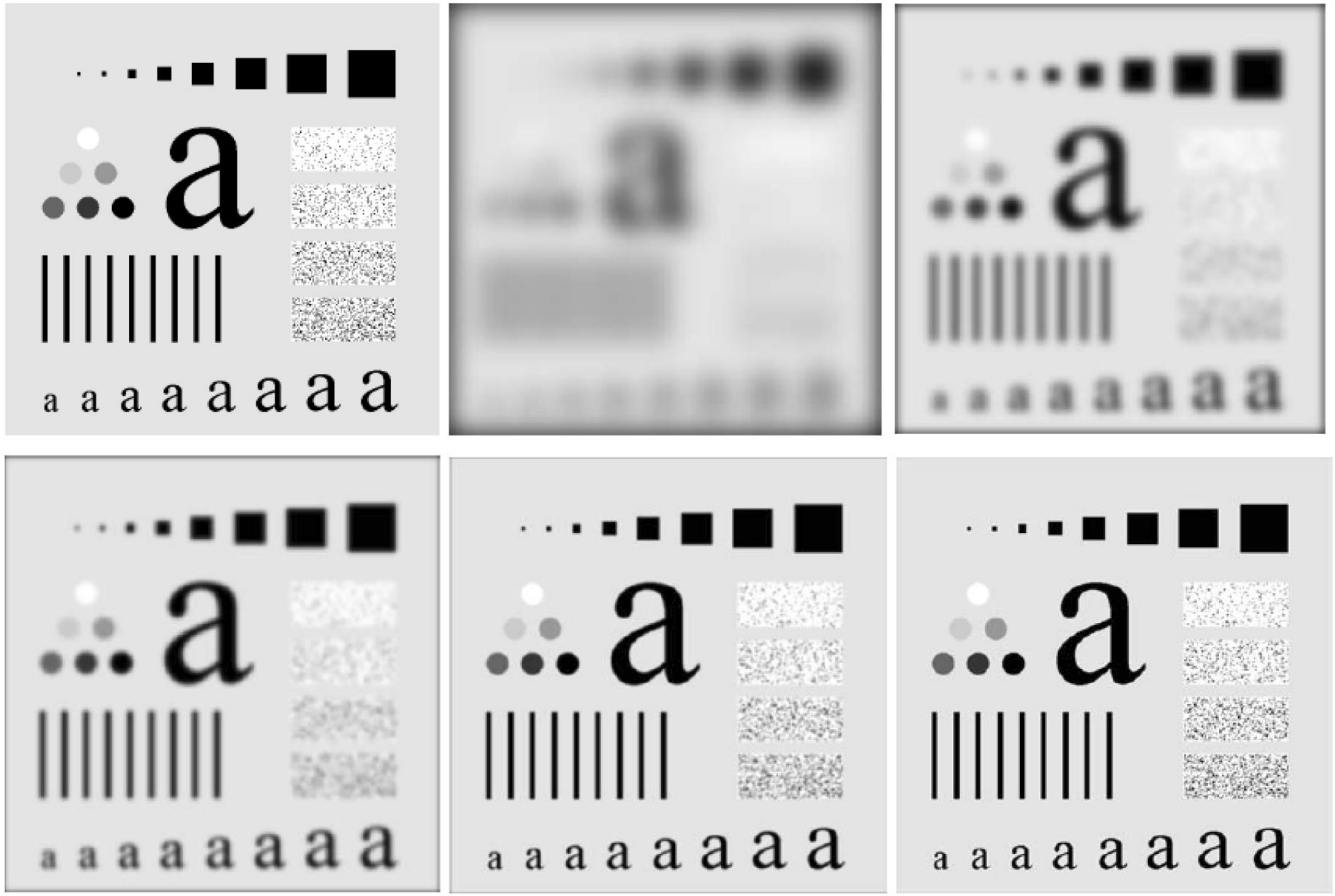
Where $H(u,v) = 0.607$ when $D(u,v) = D_0$







Gaussian Lowpass Filter (高斯滤波器)



Highpass Filtering

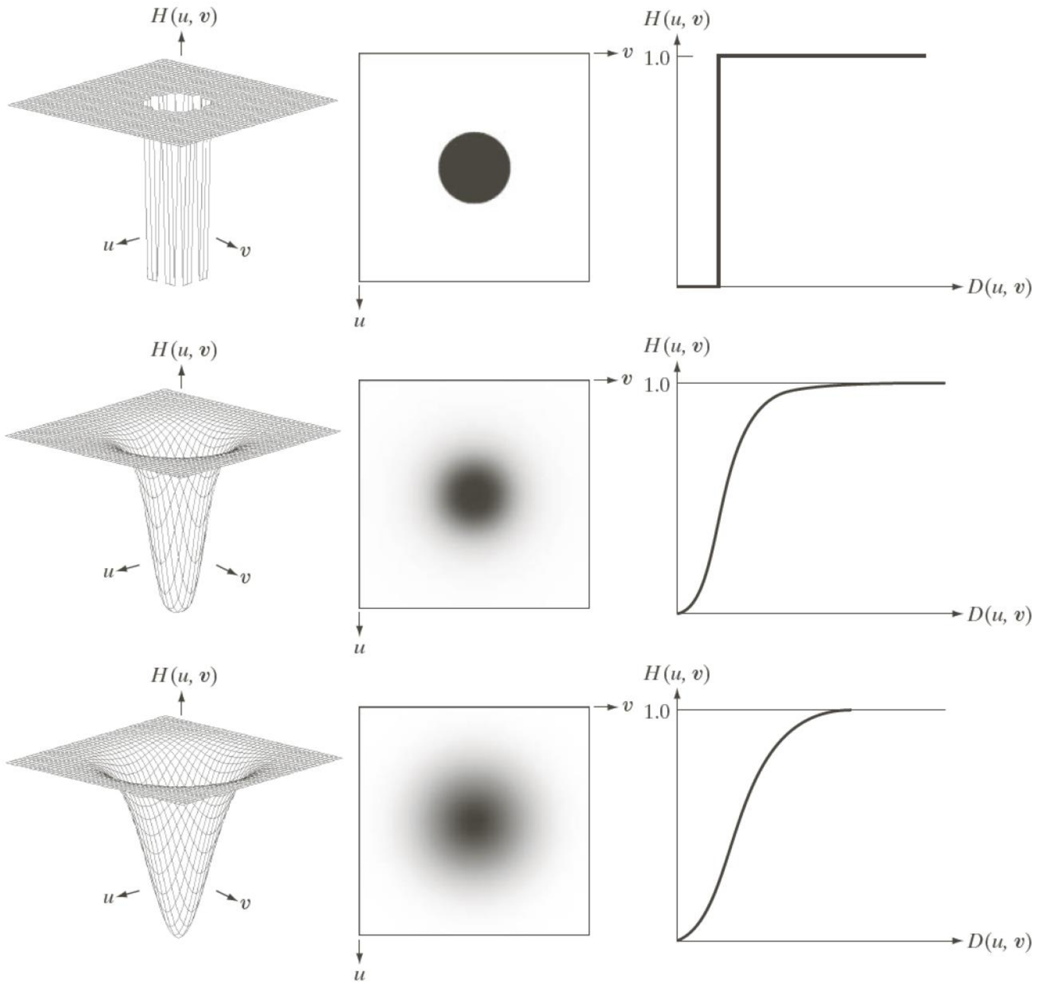
- Ideal Highpass Filter（理想高通滤波器）
- Butterworth Highpass Filter（布特沃斯高通滤波器）
- Gaussian Highpass Filter （高斯高通滤波器）

$$H_{\mathrm{HP}}(u,v) = 1 - H_{\mathrm{LP}}(u,v)$$

Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

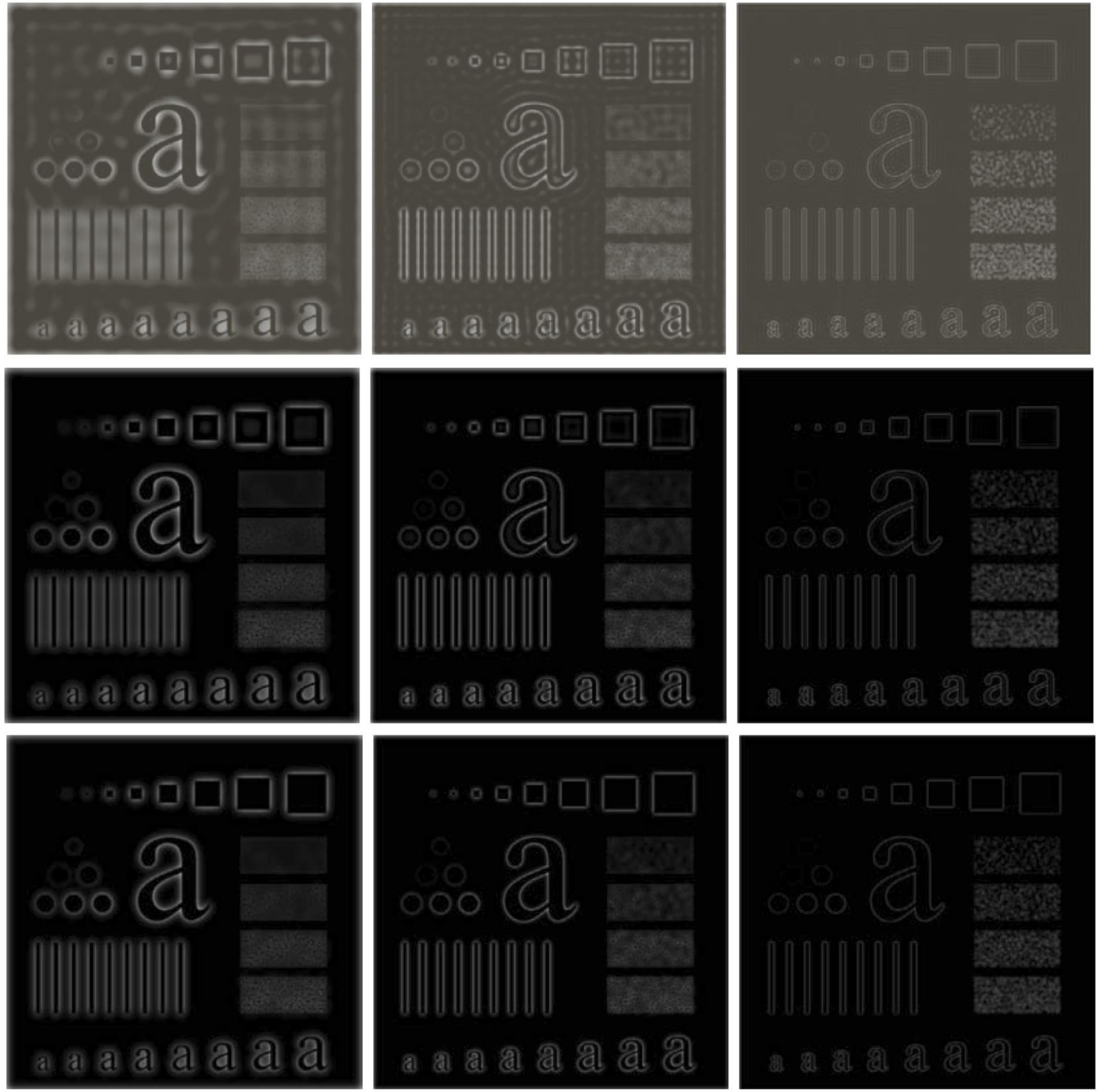
Highpass Filtering



IHPF

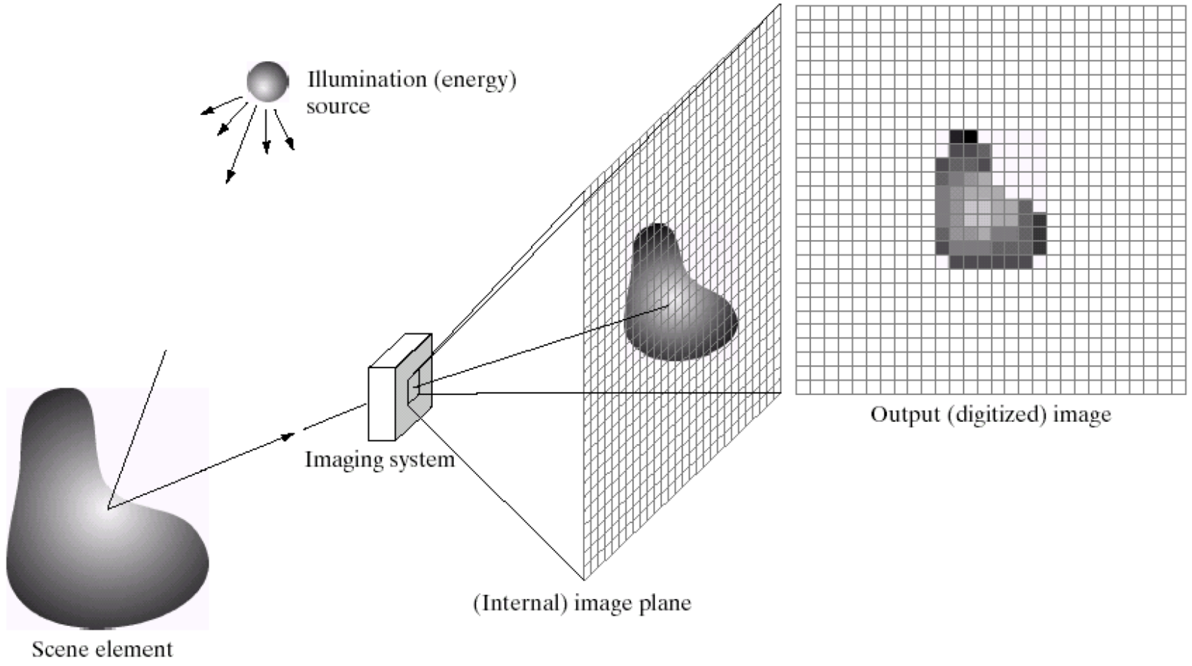
BHPF

GHPF



Highpass Filtering-Homomorphic Filtering

- Homomorphic Filtering （同态滤波）



$$f(x,y) = i(x,y)r(x,y) \qquad 0 < i(x,y) < \infty, 0 \le r(x,y) < 1$$

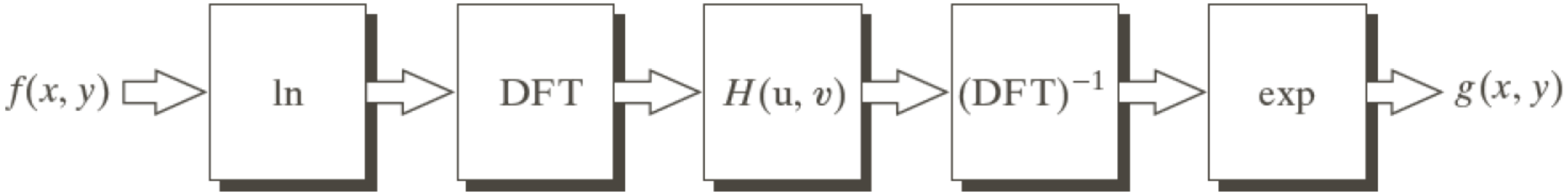
Homomorphic Filtering

- We first transform the multiplicative components to additive components by moving to the log domain.

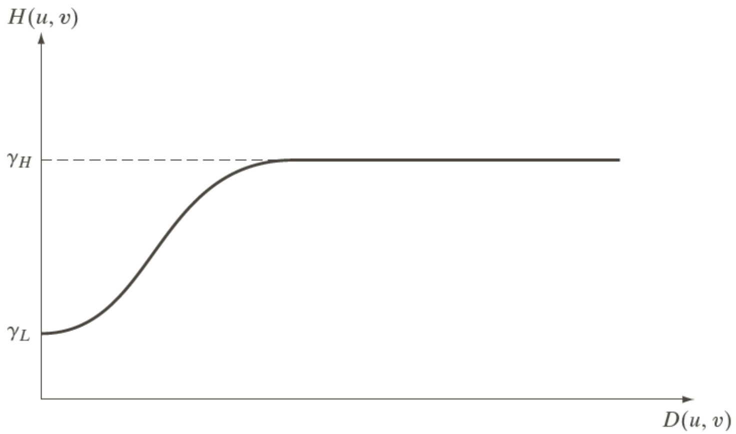
$$f(x,y) = i(x,y)r(x,y) \qquad 0 < i(x,y) < \infty, 0 \le r(x,y) < 1$$

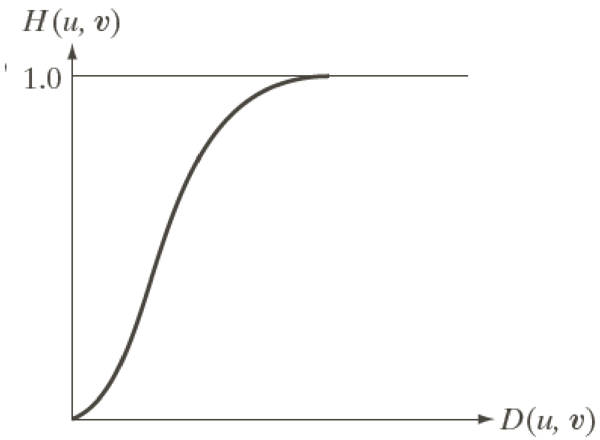
Let $z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

$$\boldsymbol{Z(u,v) = F_i(u,v) + F_r(u,v)}$$

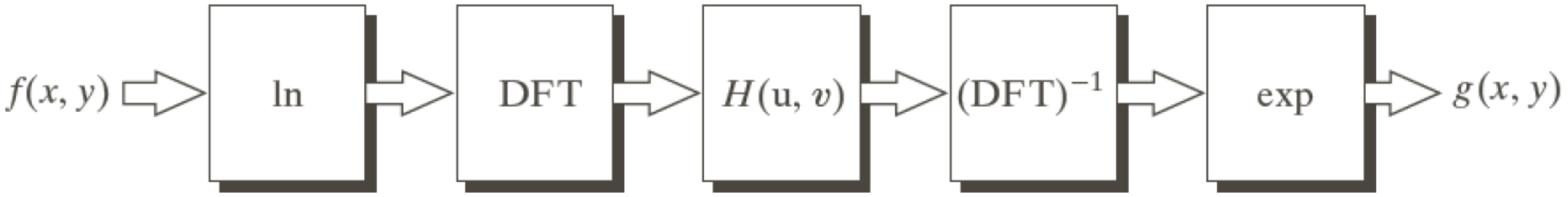


$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\left[\frac{D(u,v)}{D_0}\right]^2} + \gamma_L\right]$$





Homomorphic Filtering



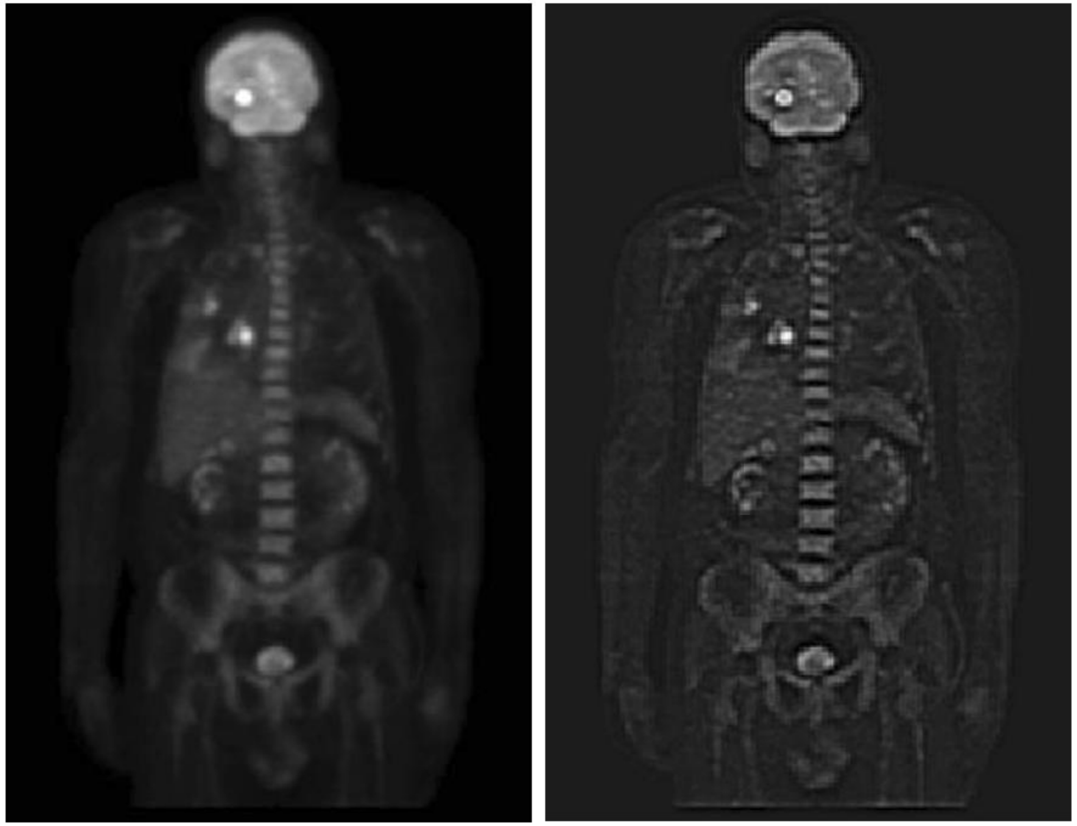
- After filtering the image is reconstructed by a inverted DFT and exponential computation.

$$
\begin{aligned}
s(x,y) &= \mathcal{F}^{-1}[H(u,v)Z(u,v)] \\
&= \mathcal{F}^{-1}[H(u,v)F_i(u,v)] + \mathcal{F}^{-1}[H(u,v)F_r(u,v)]
\end{aligned}
$$

$$
\boldsymbol{g(x,y) = e^{s(x,y)} = i_0(x,y)r_0(x,y)}
$$

Homomorphic Filtering

- Homomorphic filtering is most commonly used for correcting non-uniform illumination in images.
- Illumination typically varies slowly across the image as compared to reflectance which can change quite abruptly at object edges.
- We use a high-pass filter in the log domain to remove the low-frequency illumination component while preserving the high-frequency reflectance component.



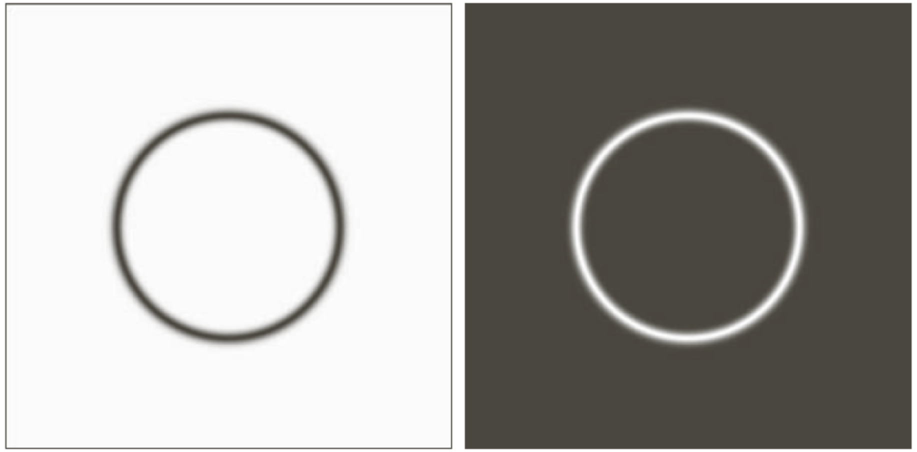

gettyimages
Ryan McVay
gettyimages
Ryan McVay

Selective Filtering

➢ **Bandreject(带阻) and Bandpass(带通) Filters**

$$H_{\text{BP}}(u,v) = 1 - H_{\text{BR}}(u,v)$$

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$



Selective Filtering

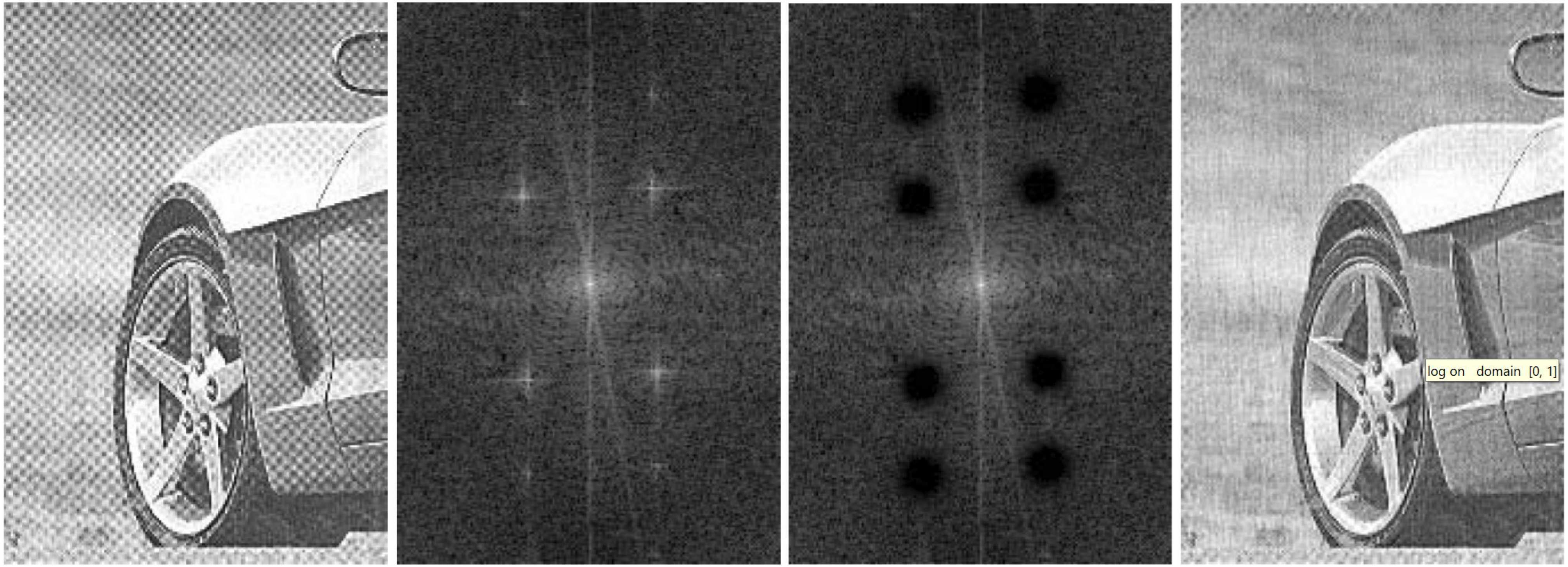
- **Notch Filter** (陷波滤波器)
  - Reject or pass frequencies in predefined neighborhood
  - Symmetric about the origin for a zero-phase shift filters
  - Selectively modify local regions of the DFT

$$H_{\mathrm{NR}}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$

$$H_{\mathrm{NP}}(u,v) = 1 - H_{\mathrm{NR}}(u,v)$$

Where $H_k(u,v)$, $H_{-k}(u,v)$ are Highpass filters with center at (u_k, v_k) and (u_{-k}, v_{-k})

Notch Filter (陷波滤波器)



Take home message

- Zero-padding in spatial domain is necessary for frequency domain filtering due to cyclic extension caused by frequency domain sampling.
- Frequency domain filtering and spatial domain filtering are related.
- Homomorphic Filtering is able to reduce abnormal illumination effect in image.