Lecture 16-Morphological Operations （Chapter 9.1-9.5）

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Outline

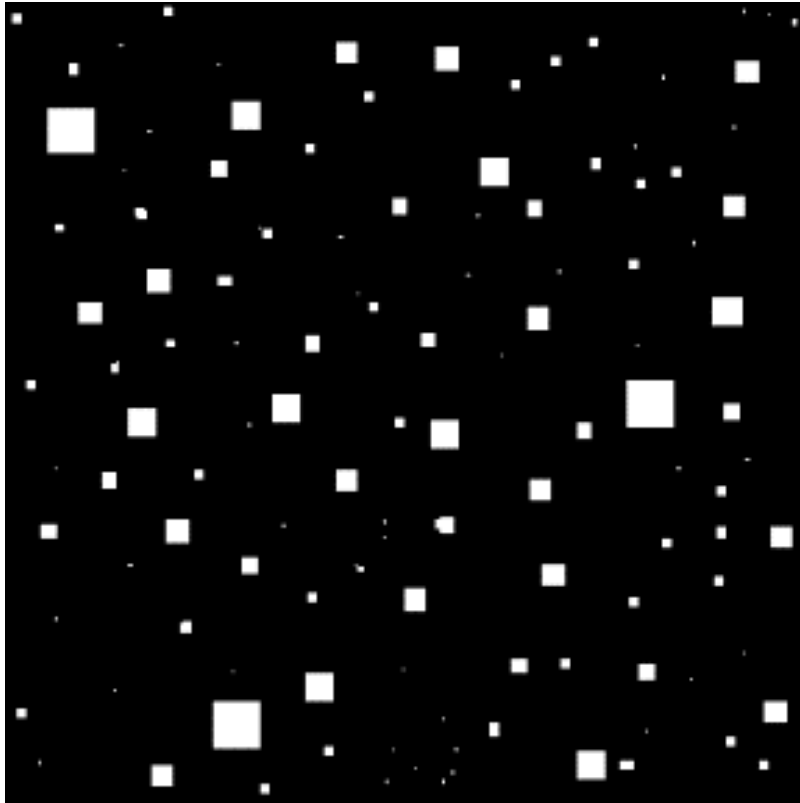This lecture will cover:

– Morphology Image Processing （形态学图像处理）

- Morphological operation.

- Morphological algorithms.

Problem try to solve

- **Imperfect from image segmentation.**

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Preliminaries ---Set Operation

A digital image $f(x, y)$ can be considered as a set A, if $w(x, y)$ in 2D integer space Z^2, then

➢$w \in A$: w is an element of A.

➢$w \notin A$: w is not an element of A.

➢$B = \{w | condition\}$: all elements which meet the specific condition.

- $A \cup B = \{w | w \in A \; or \; w \in B\}$: union (并集)

- $A \cap B = \{w | w \in A \; and \; w \in B\}$: intersection (交集)

- $A^C = \{w | w \notin A\}$: complement (补集)

- $A - B = \{w | w \in A \; and \; w \notin B\}$: difference (差集)
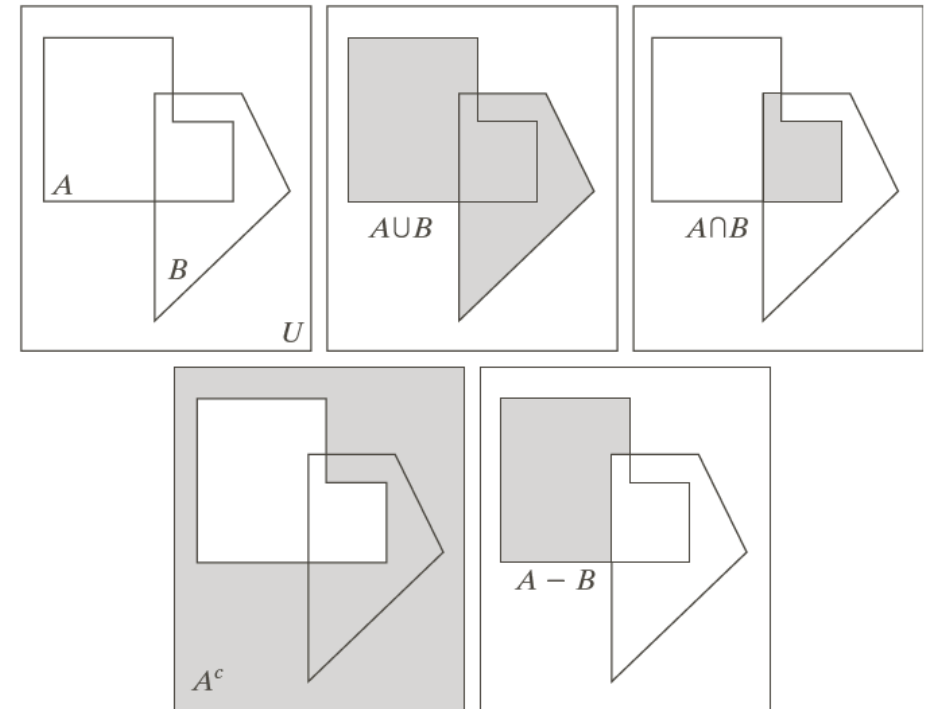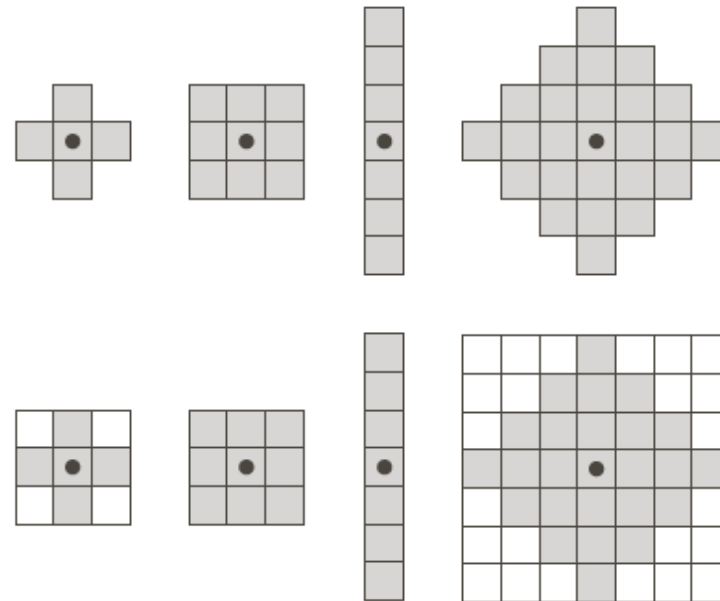
Structure element (结构元)

➢ **Structuring Element (SE) --- ** small sets or sub-images used to probe an image under study for properties of interest.

➢ **SE Selection**

- Simpler than the image
- With boundary
- Convex

➢ **Structures**

- Origin
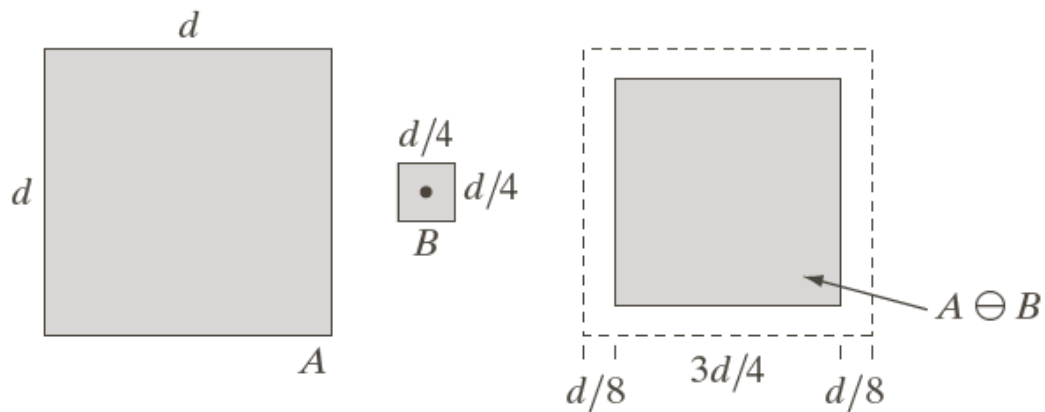- Rectangular

Erosion (腐蚀)

➢ **Definition：**

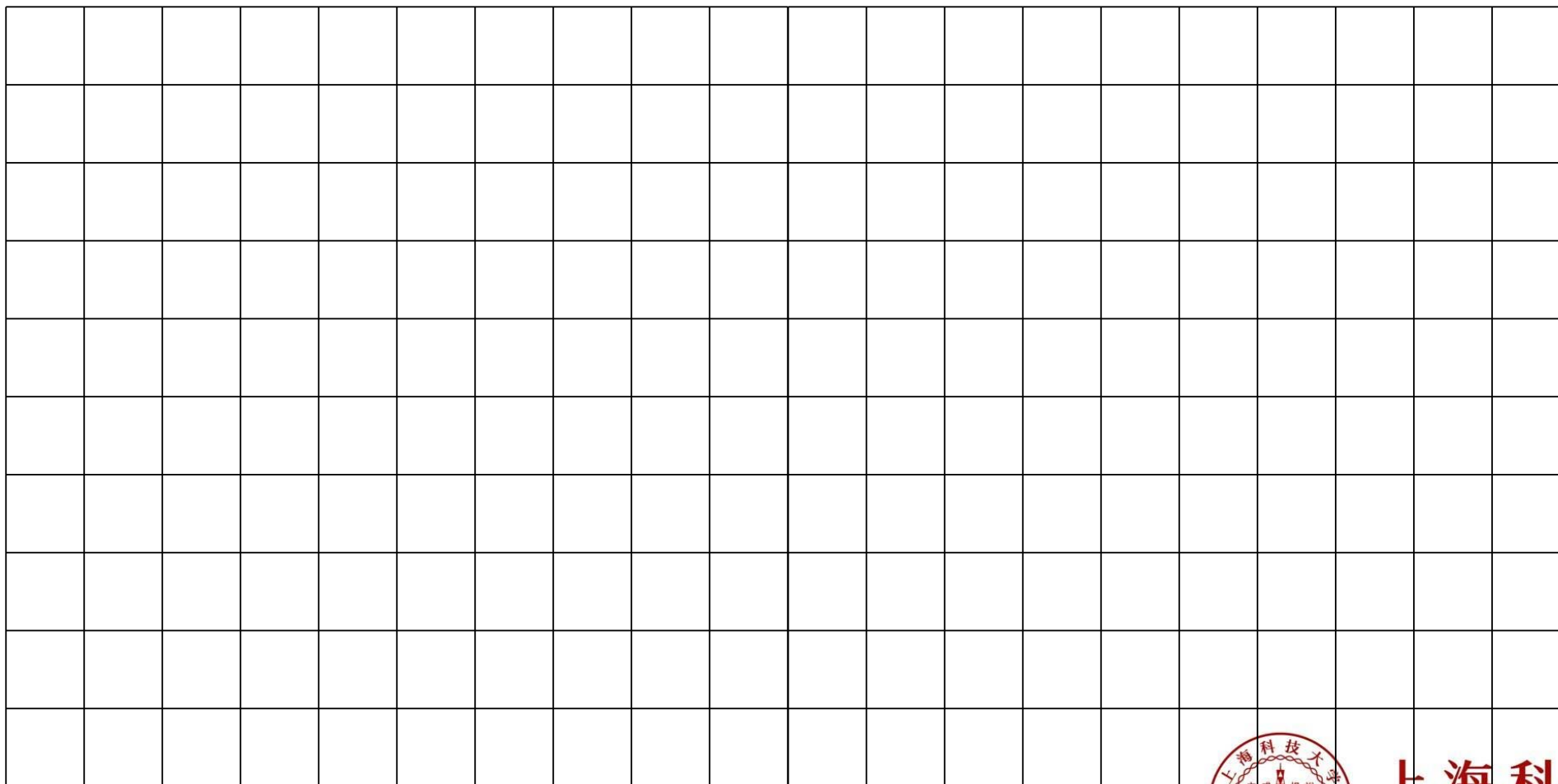$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

or

$$A \ominus B = \{z \mid (B)_z \cap A^c = \emptyset\}$$
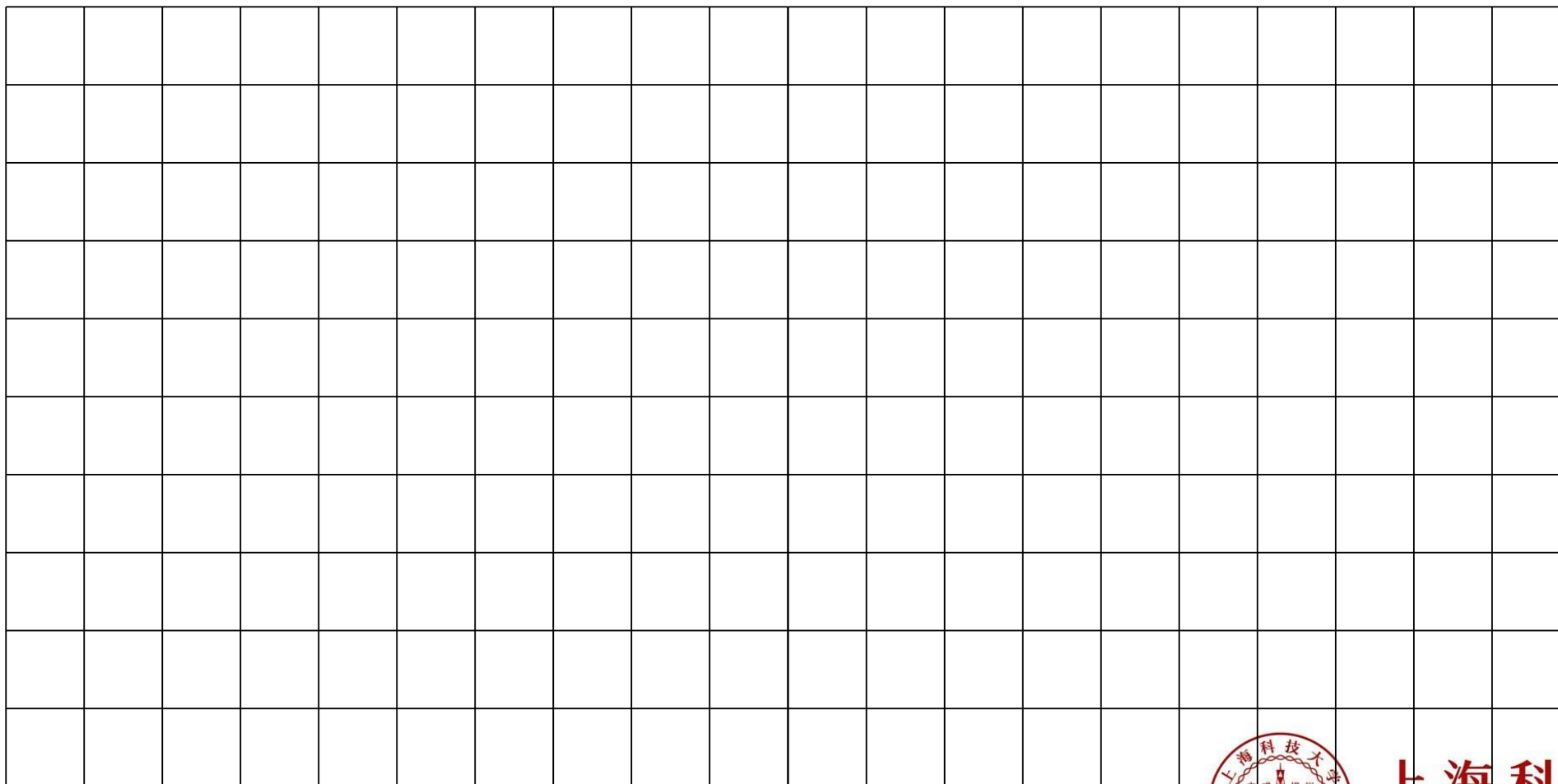
➢ **Erosion will do:**

- removes thin lines
- isolate dots
- leaves gross details
- "Peeling away" layers
- Is always a sub-set of A

Erosion (腐蚀)

Erosion (腐蚀)

Dilation (膨胀)

➢ **Definition**

$A \oplus B = \{z | \widehat{B_z} \cap A \subseteq A\}$ or

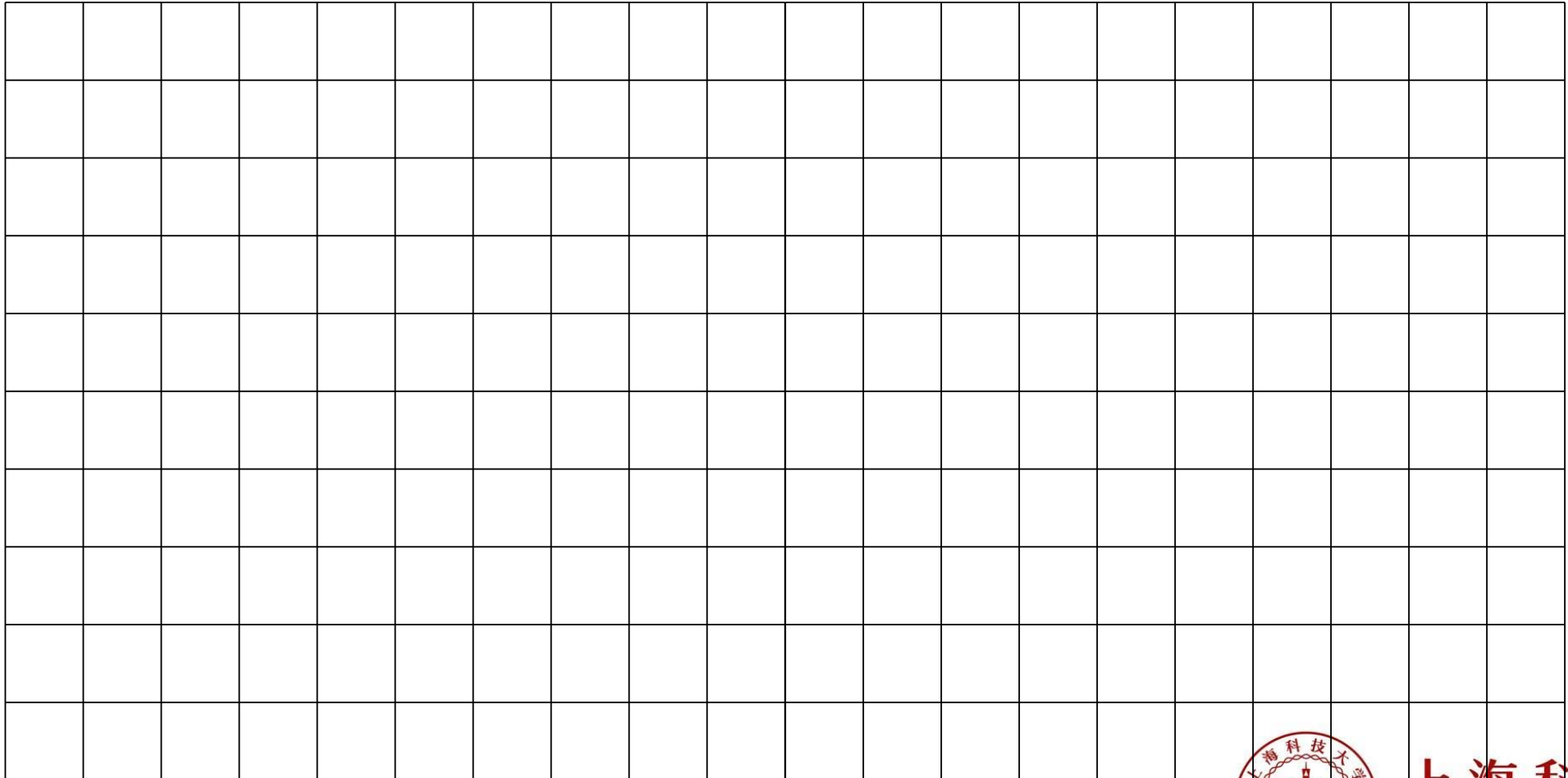$A \oplus B = \{z | \widehat{B_z} \cap A \subseteq \emptyset\}$

Dilation will do:

Fatten up. Kind of opposite of

Erosion.

Bridge gaps, fill holes, wighout change

overall size of object.

Dilation (膨胀)

Opening (开操作)

➤ **Definition:**

$$A \circ B = (A \ominus B) \oplus B$$

or

$$A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \}$$

➤ **Erode then dilate: break narrow bridges, eliminate thin structures**

➤ **Matlab Function:** J = imopen(I,SE)

➤ **Properties:**

   ① A ∘ B is a subset (subimage) of A

   ② If C is a subset of D, the C ∘ B is a subset of D ∘ B

   ③ (A ∘ B) ∘ B = A ∘ B

Closing (闭操作)

➤ **Definition:**

$$A \bullet B = (A \oplus B) \ominus B$$

➤ **Dilate, then erode: fuse narrow breaks, eliminate small holes**

➤ **Matlab Function:** J = imclose(I,SE)

➤ **Properties:**

① A is a subset (subimage) of A • B

② If C is a subset of D, the C • B is a subset of D • B

③ (A • B) • B = A • B

Opening & Closing (开操作和闭操作)

➢ **Opening（开操作）**

- Smooth the contour of an object
- Break narrow bridges.
- Eliminate thin structures.

➢ **Closing（闭操作）**

- Smooth the contour of an object
- Fuse narrow breaks and long thin gulfs
- Eliminate small holes
- Fill gaps in the contour

The Hit-or-Miss Transformation (击中或击不中变换)

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 1 1 1 1 0 0 0 0 0 0
0 1 1 1 0 0 0 0 0 0 0 0 1 1 0 0
0 0 1 0 0 0 0 0 0 0 0 0 1 1 1 0
0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0
0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

B_1

```
    1
1 [1] 1
    1
```

```
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 0 1 1 1 0 0 0 0 1 1 1 1 1 1
1 0 0 0 1 1 1 1 1 1 1 1 0 0 1 1
1 1 0 1 1 1 1 1 1 1 1 1 0 0 0 1
1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1
1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1
1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

B_2

```
1     1
  [ ]
1     1
```

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

```
1 0 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1
1 0 1 0 1 0 0 0 0 0 0 1 1 1 1 1
0 0 0 0 0 1 1 1 1 1 0 0 0 0 1
1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 1 1 1 0 0 0 0 1
1 0 1 0 0 0 0 0 1 1 0 0 0 0 0
1 1 1 0 1 0 1 0 1 1 1 1 0 1 0 1
1 1 1 0 0 0 0 0 1 1 1 1 1 1 1 1
1 1 1 0 1 0 1 0 1 1 1 1 1 1 1 1
```

The Hit-or-Miss Transformation (击中或击不中变换)

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$
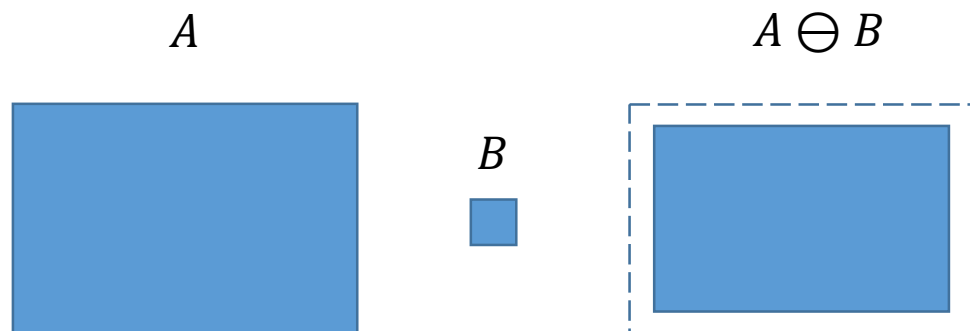
➢A method to find the location of a shape B_1 in an image A.

  -Erosion of $A \ominus B_1$ gives all places where B_1 fits in A.

➢So also require the boundary around the shape, B_2 to be empty.

  -Erosion of $A^c \ominus B_2$ gives all places where B_2 fits in empty places of A.

➢Then take the intersection:

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

Boundary Extraction (边界提取)

Morphological algorithm: $\beta(A) = A - (A \ominus B)$

A

B

$A \ominus B$

$\beta(A) = A - (A \ominus B)$

Hole Filling (孔洞填充)

- Let A be the set of 8-connected boundary points of a region

- Start with a point inside the region

- Repeatedly dilate

- At each step, the points corresponding to the region boundary are set to zero：

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1,2,3, \ldots$$
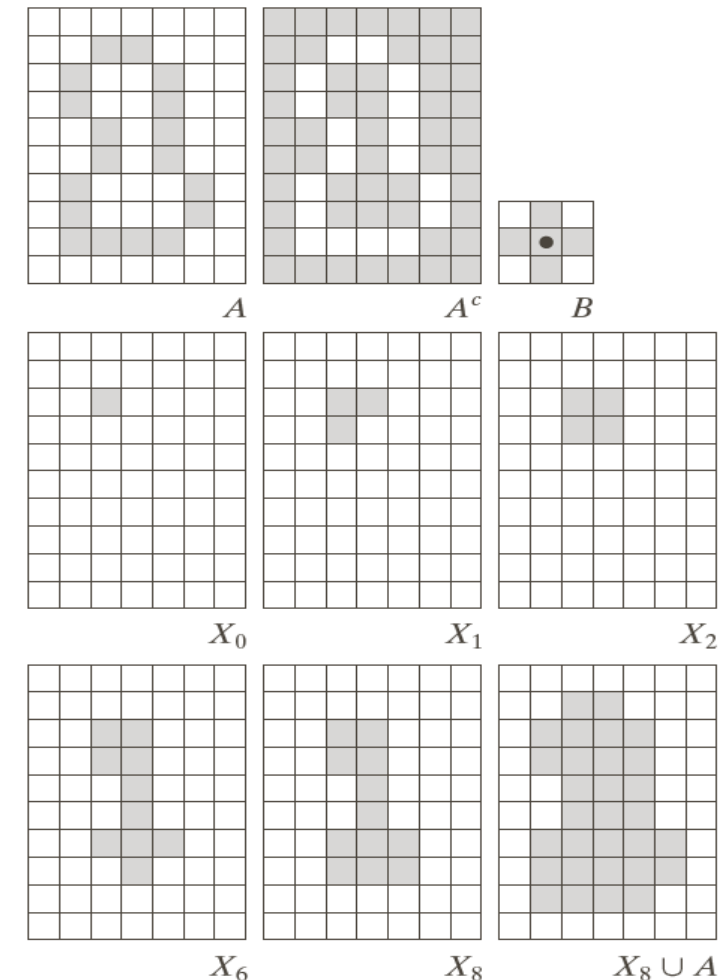
- Stop when no more changes

Figure 9.5 Region filling

Take home message

➢ **Morphological Language：Set theory (集合)**

➢ **Morphological operations take a set of pixels:**

- Key element: "structure element".
- Insensitive to noise.
- Smooth edge
- **Key operations**

- HMT (Hit or Miss Transformation)(击中与否变换)
- Dilation (膨胀）and Erosion (腐蚀)