Introduction to *Monte Carlo Tree Search*

By Xi Huang @SIST, ShanghaiTech

Lecture Flow

Motivation

Brief History of Monte Carlo Tree Search (MCTS)

A Close Look at MCTS

Remarks

MCTS for Combinatorial Optimization Problems

MCMC HW Q10

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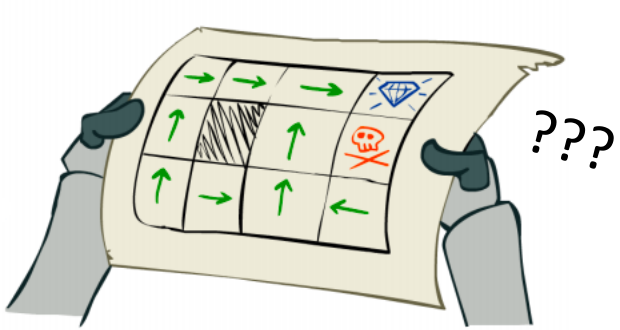
Markov Decision Process

Policy π : for each state s, $\pi(\cdot\,|s)$ as a mapping from A to [0, 1]



A Markov Decision Process is defined by a tuple $< S, A, P, R, \gamma >$, where

☆ S is a (finite) set of states

☆ A is a finite set of actions

☆ P is a state transition probability matrix

$$P^{a}_{s,s'} = P[S_{t+1} = s' | S_t = s, A_t = a]$$

☆ R is a reward function

$$R^{a}_{s} = E[R_{t+1} | S_t = s, A_t = a]$$

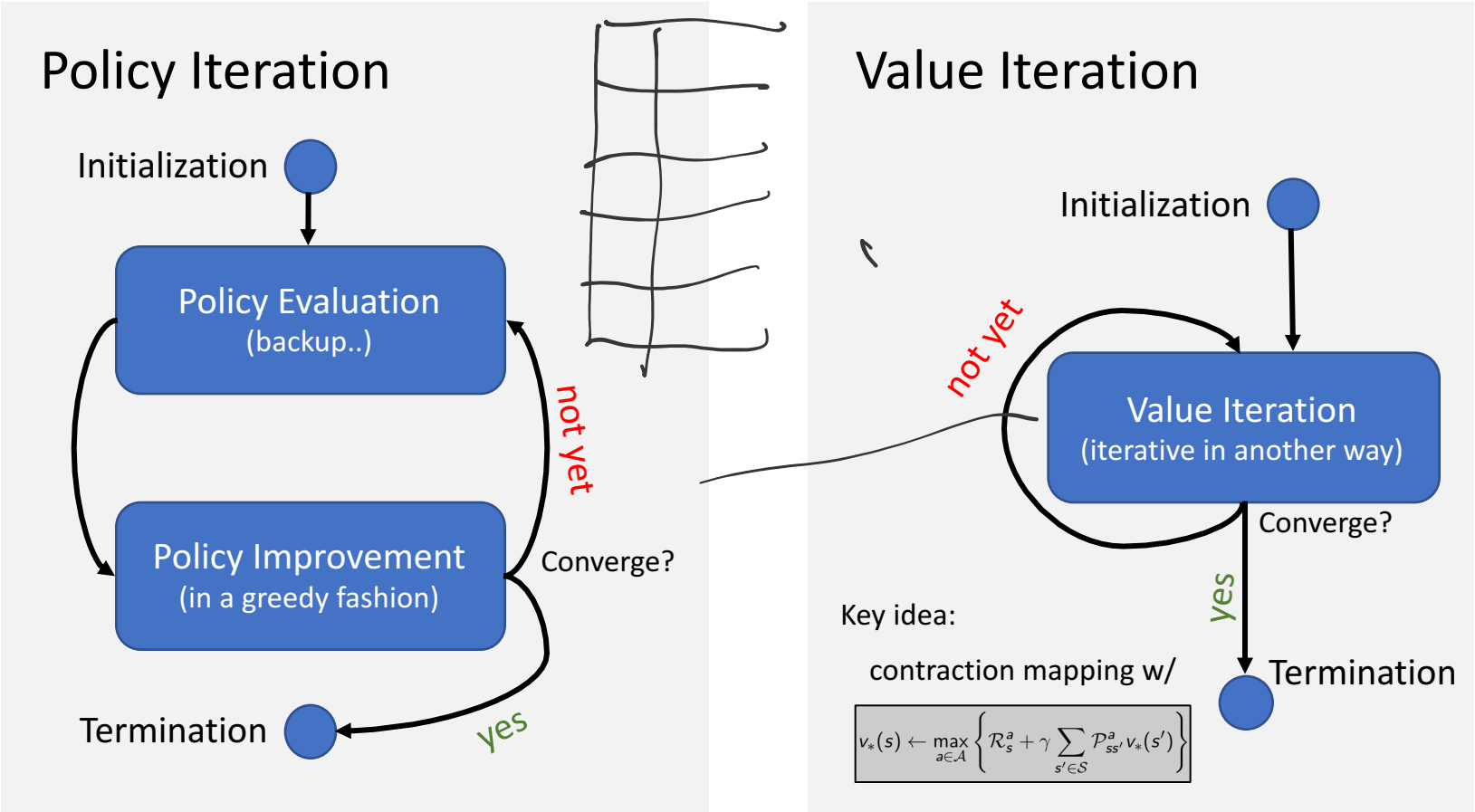
☆ γ is a discount factor such that $\gamma \in [0, 1]$

Solving MDP via DP

Tabular Methods

☆ State-value function: $v_\pi(s) = E[R_{t+1} + \gamma R_{t+2} + \cdots | S_t = s]$

☆ Action-value function: $q_\pi(s, a) = E[R_{t+1} + \gamma R_{t+2} + \cdots | S_t = s, A_t = a]$



Limitations of Pure DP Algs

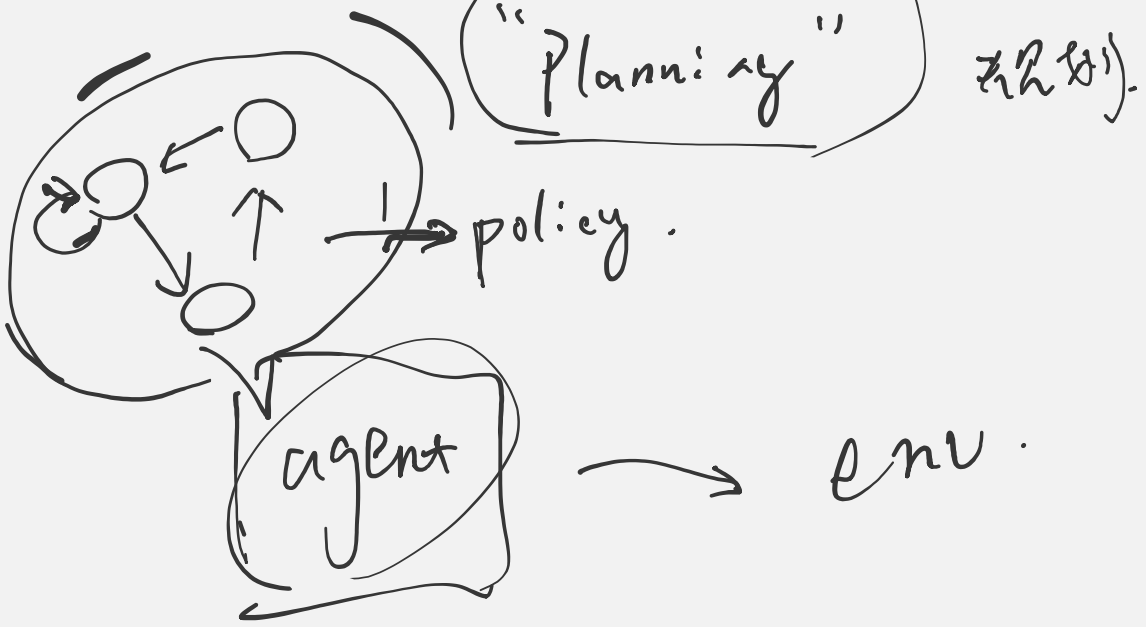
☆ Efficiency (should be addressed; better if independent of state space size)

- Computational complexity polynomial in the state space size (*e.g.,* ~10^{170})
- "Curse of dimensionality" vs. **[resource scarcity & time limit of decision-making]**

Limitations of Pure DP Algs

☆ Availability of dynamics (leverage those given right on the spot)

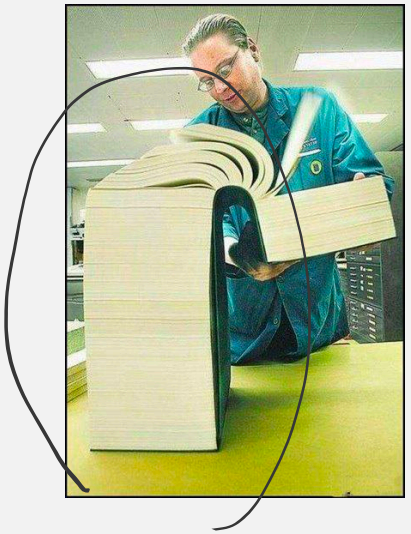
- Full knowledge of environment dynamics (required to be *Markovian*) should be explicitly given for decision making
- Usually incomplete or acquired through (simulated) interaction w/ the env.



Limitations of Pure DP Algs

☆ Necessity (why not do it in a lazy fashion)

- Only a small portion of states are frequently encountered, not ALL of them, so..



DOES the agent really need

a "THICK manual" beforehand?

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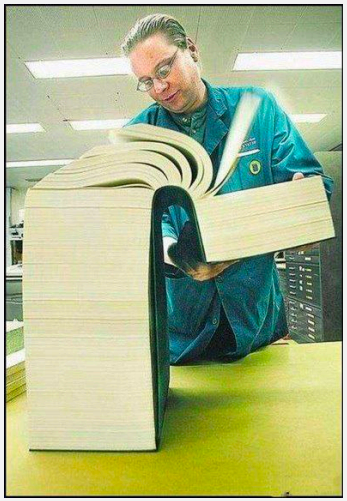
DOES it really need a "THICK manual" beforehand?



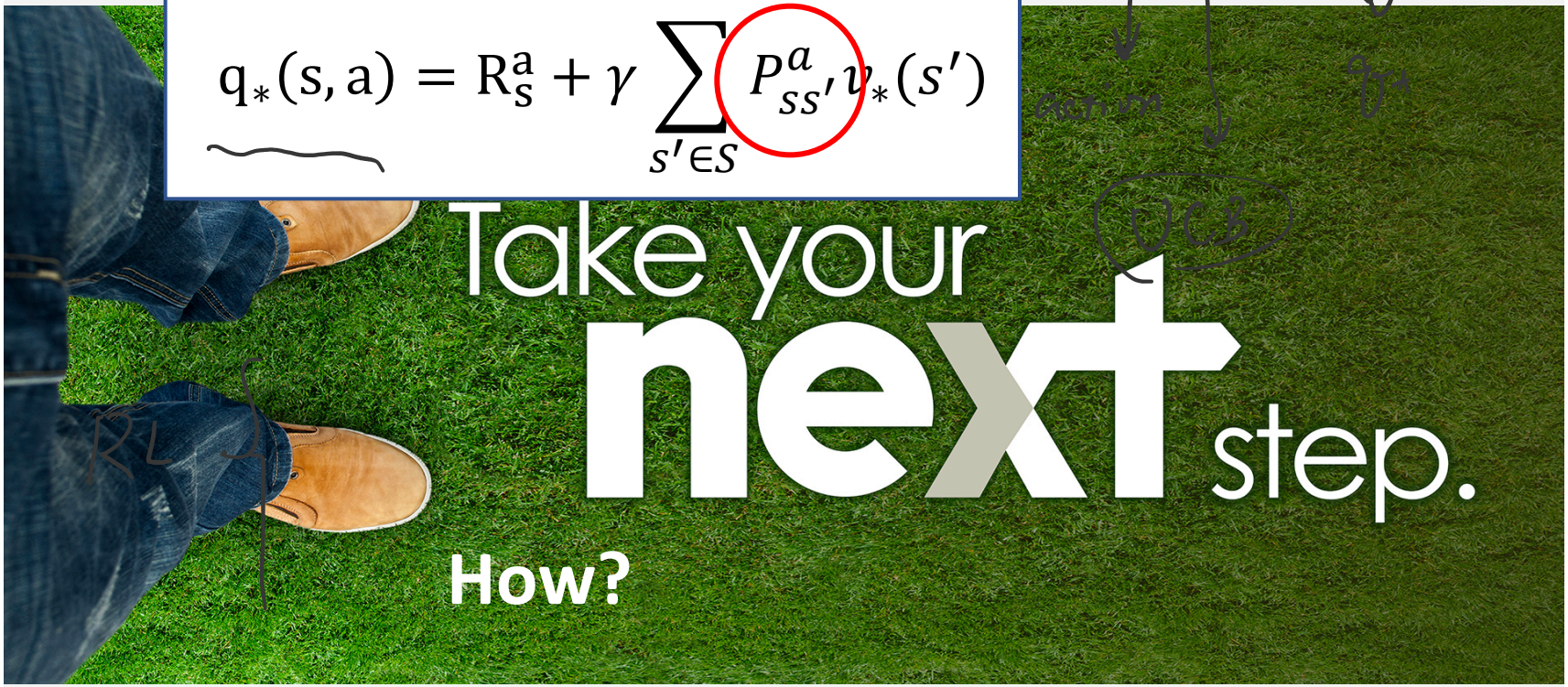
Key Idea of Monte Carlo Tree Search

☆ How the decision is made

$$v_*(s) = \max_{a \in A} q_*(s, a)$$

1. How to approximate?
   Monte Carlo Simulation
2. Exploration-exploitation?
   UCB method

$$q_*(s, a) = R_s^a + \gamma \sum_{s' \in S} P_{ss'}^a v_*(s')$$



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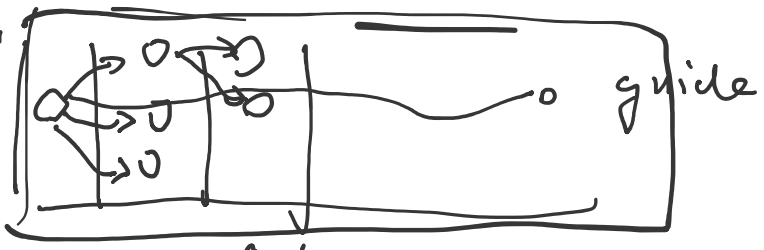
History of MCTS



Timeline of RL

1987 — 1992 — 1997 — 2002 — 2005 — 2006 — 2008

MAB

2002

Prof. Auer *et al.* developed a series of UCB methods w/ log regrets (theoretically opt.)

2005

H. S. Chang *et al.* integrates bandit learning into simulation-based tree search for the approximate computing of optimal value functions in MDP

Prof. Remi Coulom

MCTS

Success in
- MoGo
- Zen
- AlphaGo
…

Timeline of game-tree search

1987

Dr. Bruce Abramson in his PhD thesis, CU

1992

Dr. Bernd Brügmann firstly applied in Go

1997

Deep Blue vs. Kasparov

2006

Dr. K. Levente & Prof. C. Szepesvári introduced UCB method to MCTS *a.k.a.* UCT

2016

Source: https://medium.com/point-nine-news/

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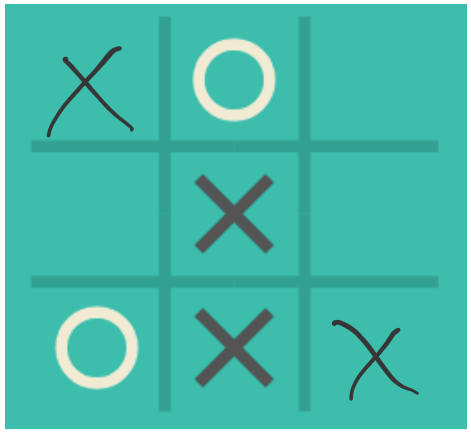
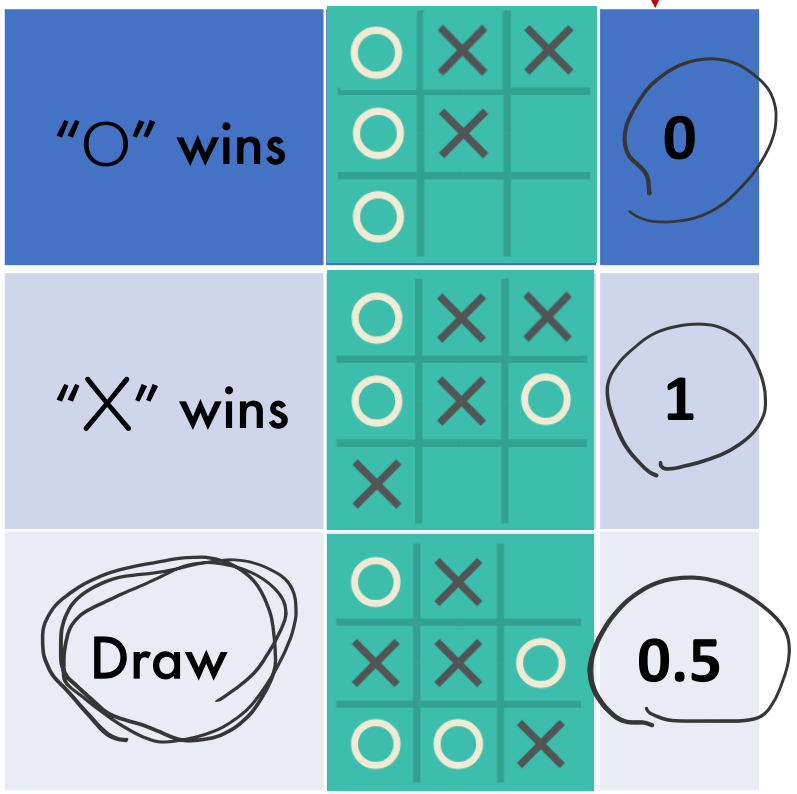
Monte Carlo Tree Search (MDP)

Sequential Decision-Making Problems as Tree-Search problems

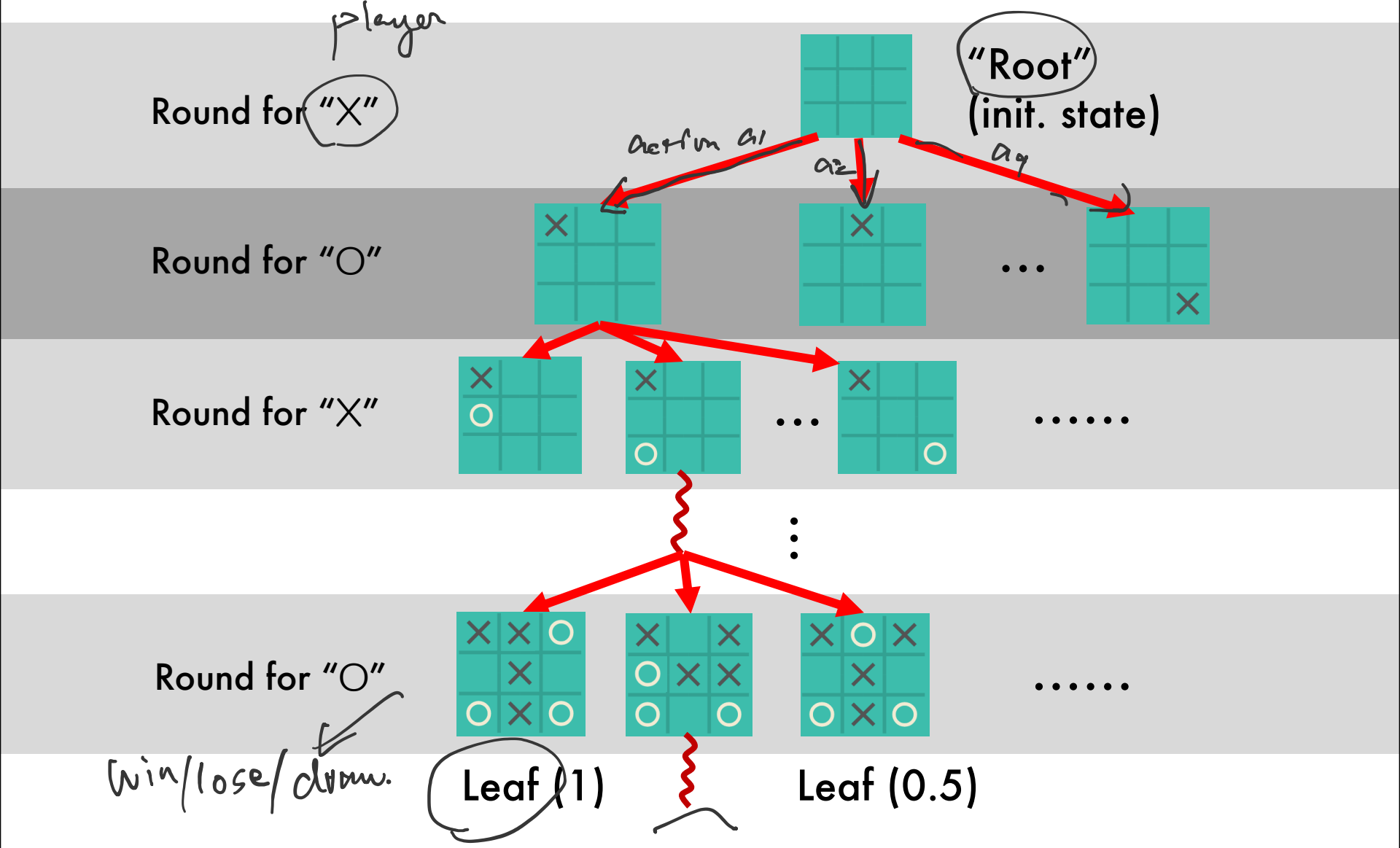
- Sequential Decision-Making Problems



Tic-Tac-Toe



Game-play Tree for "Tic-Tac-Toe"



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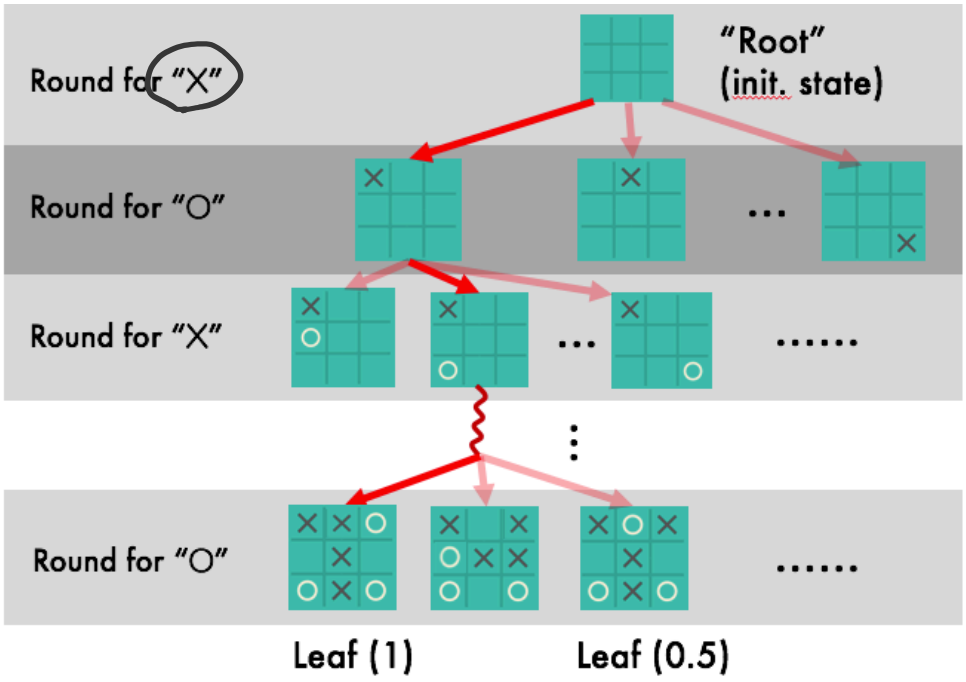
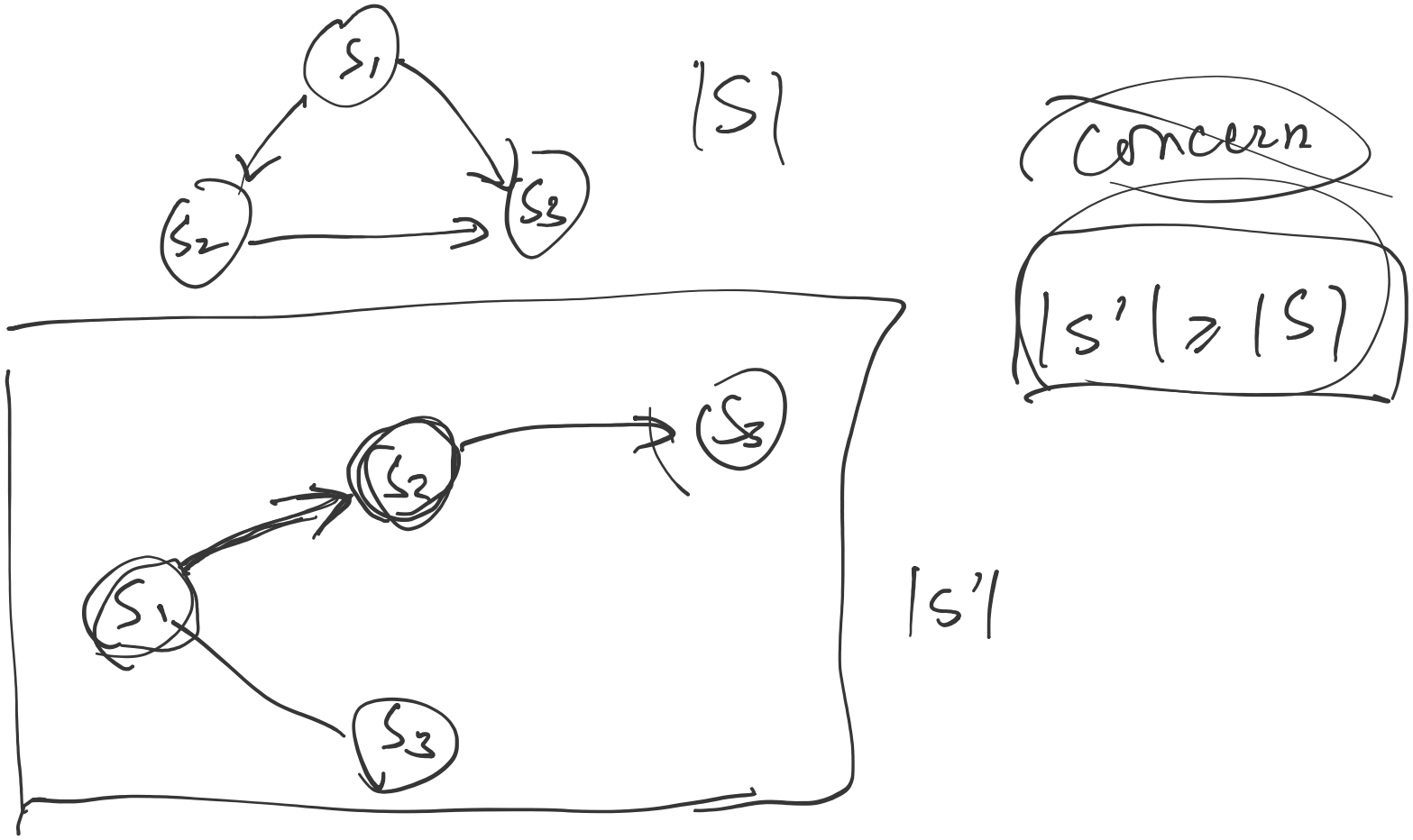
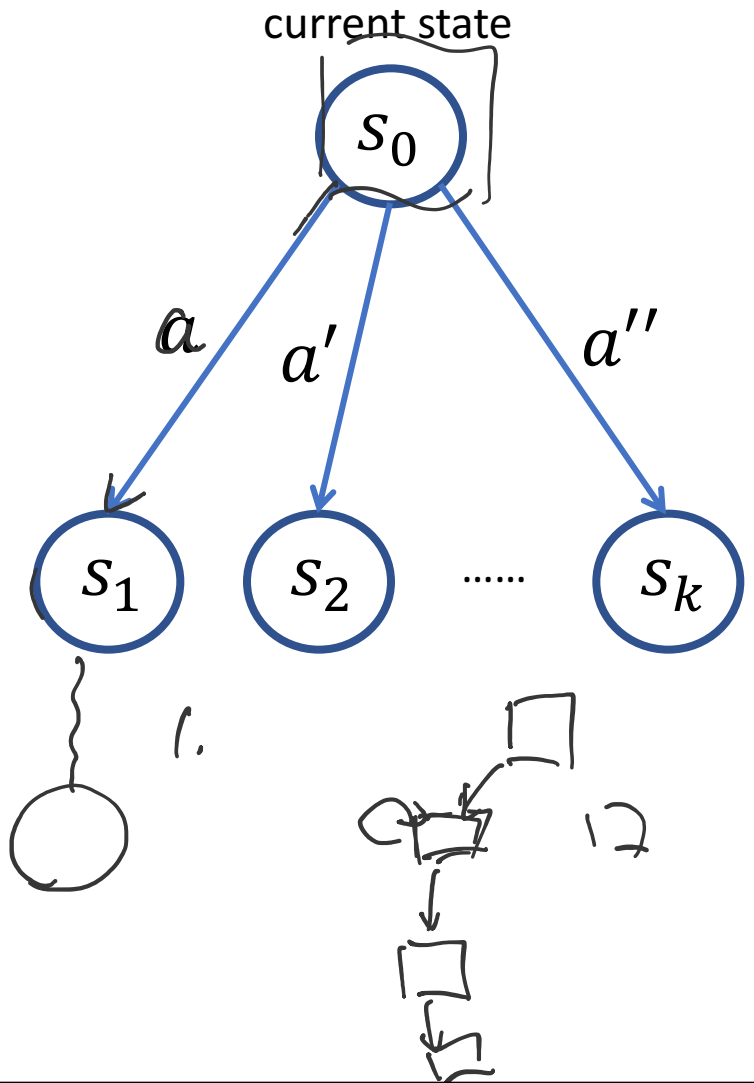


Optimal Policy: Find a path that leads to the best possible outcome (*a.k.a. planning*)

What about MDP?



Simplified Assumptions for Illustration



$$\max_{a \in A} q_*(s_0, a)$$

$$q_*(s_0, a) = R^a_{s_0} + \gamma \sum_{s' \in S} P^a_{s_0 s'} v_*(s')$$

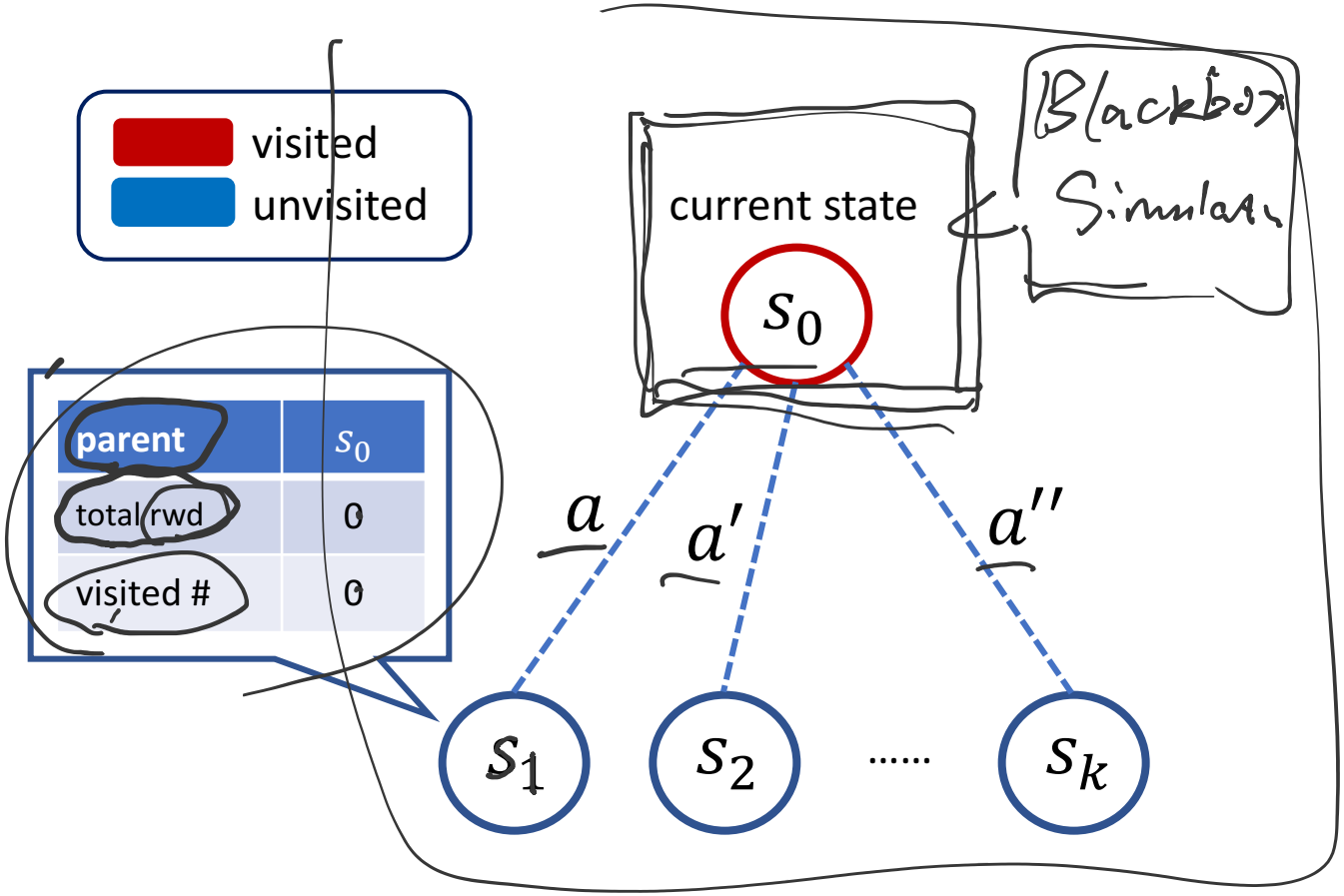
Reward

0

0

1

MCTS: Initial trials



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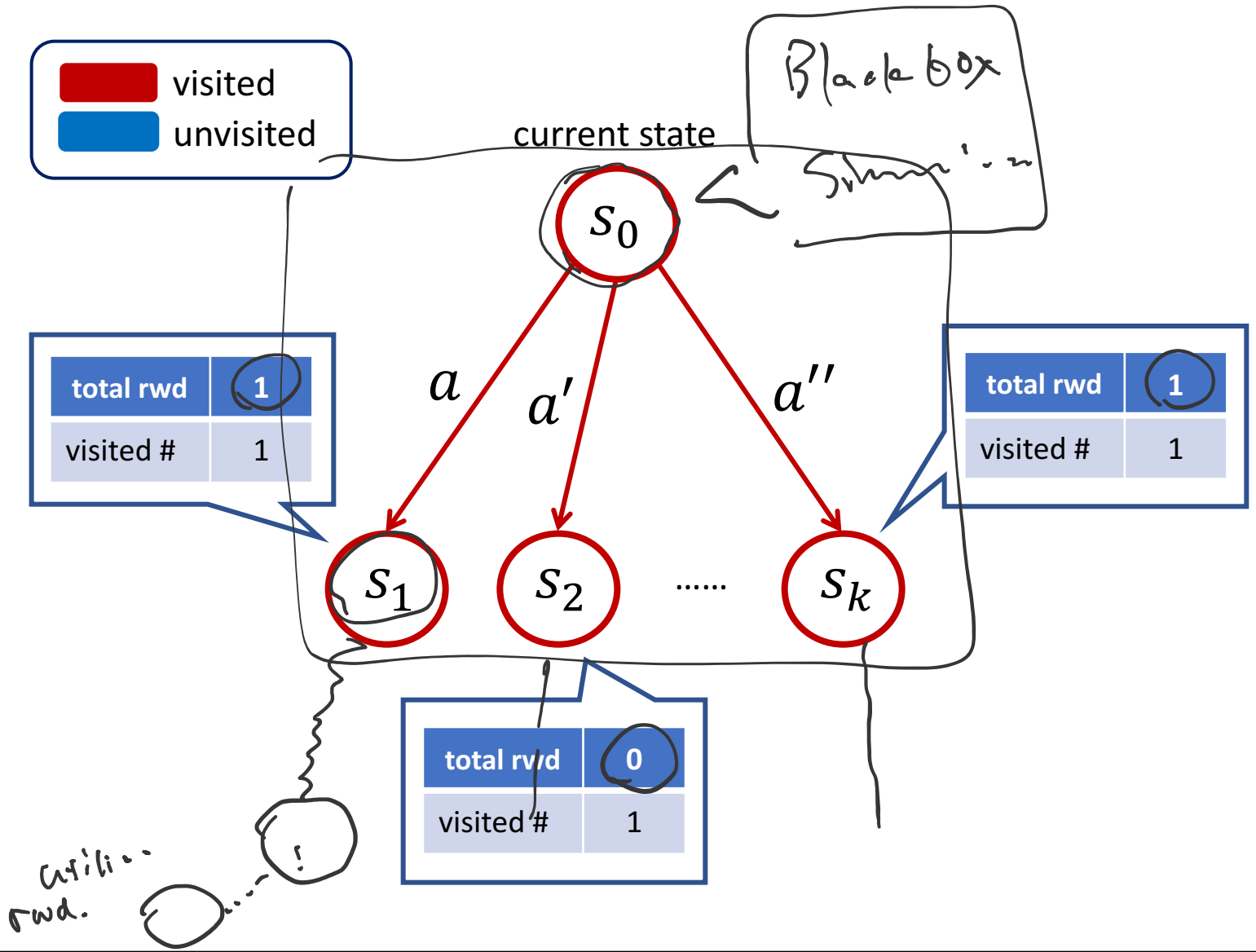
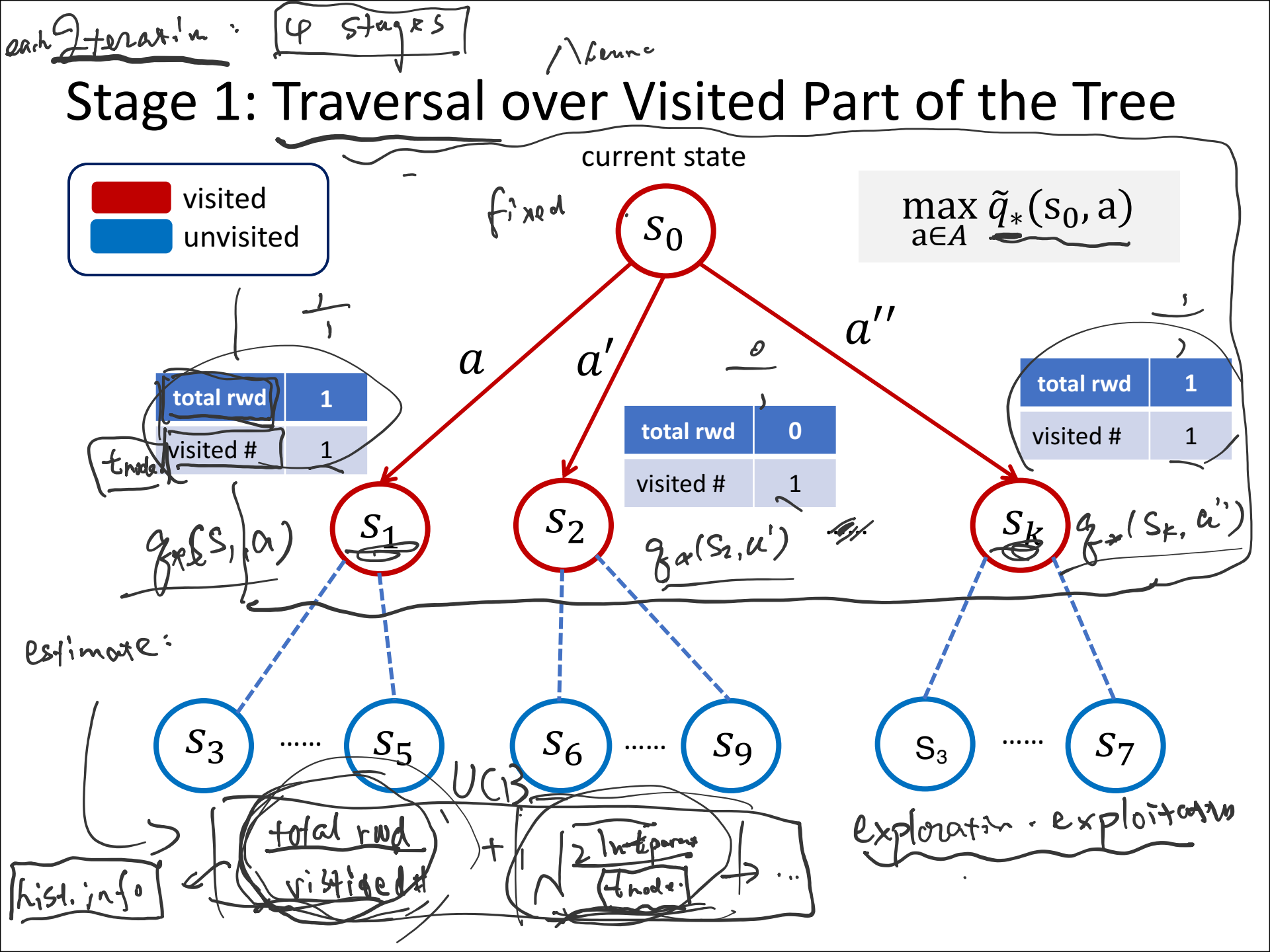


Stage 1: Traversal over Visited Part of the Tree



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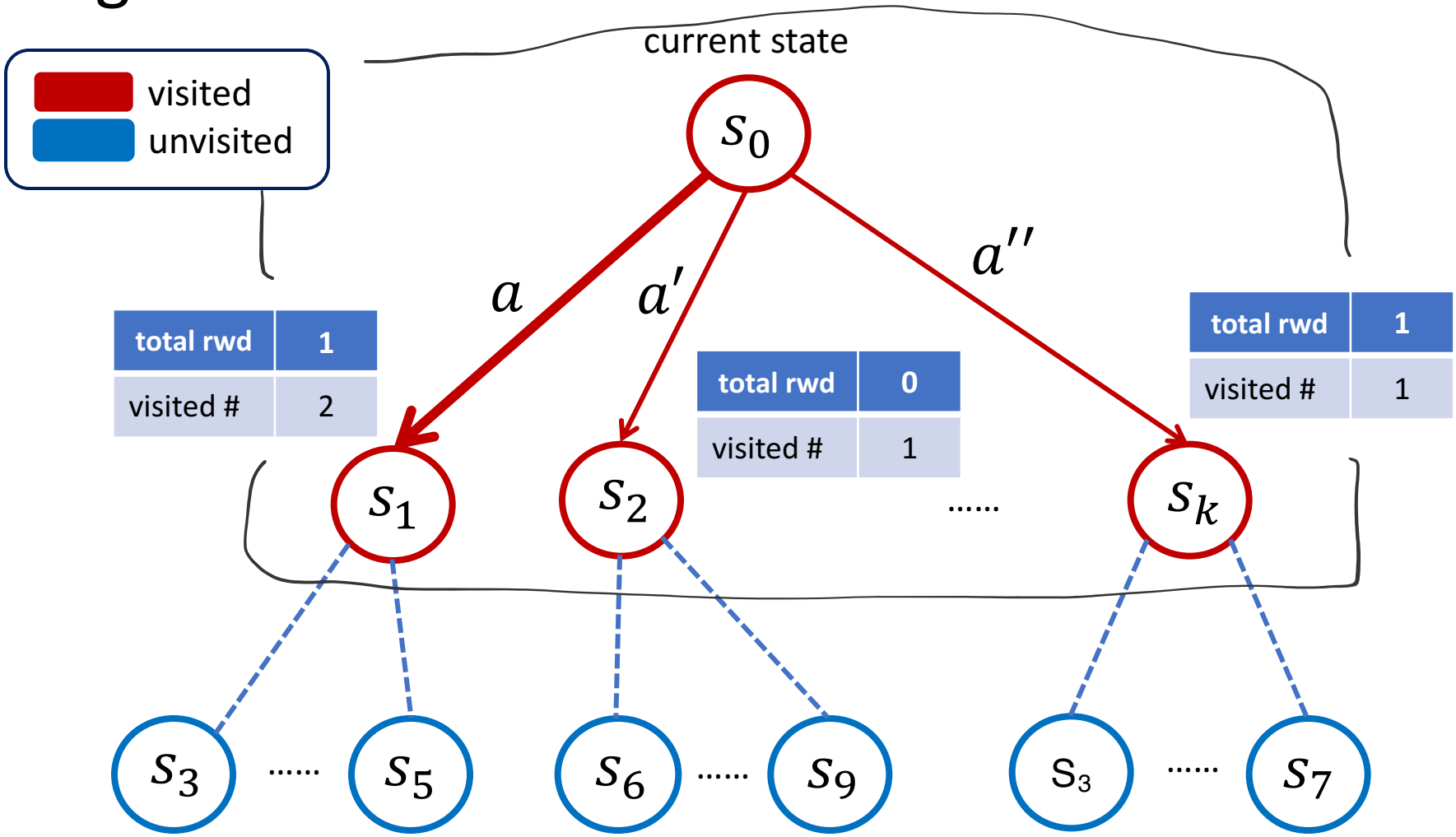
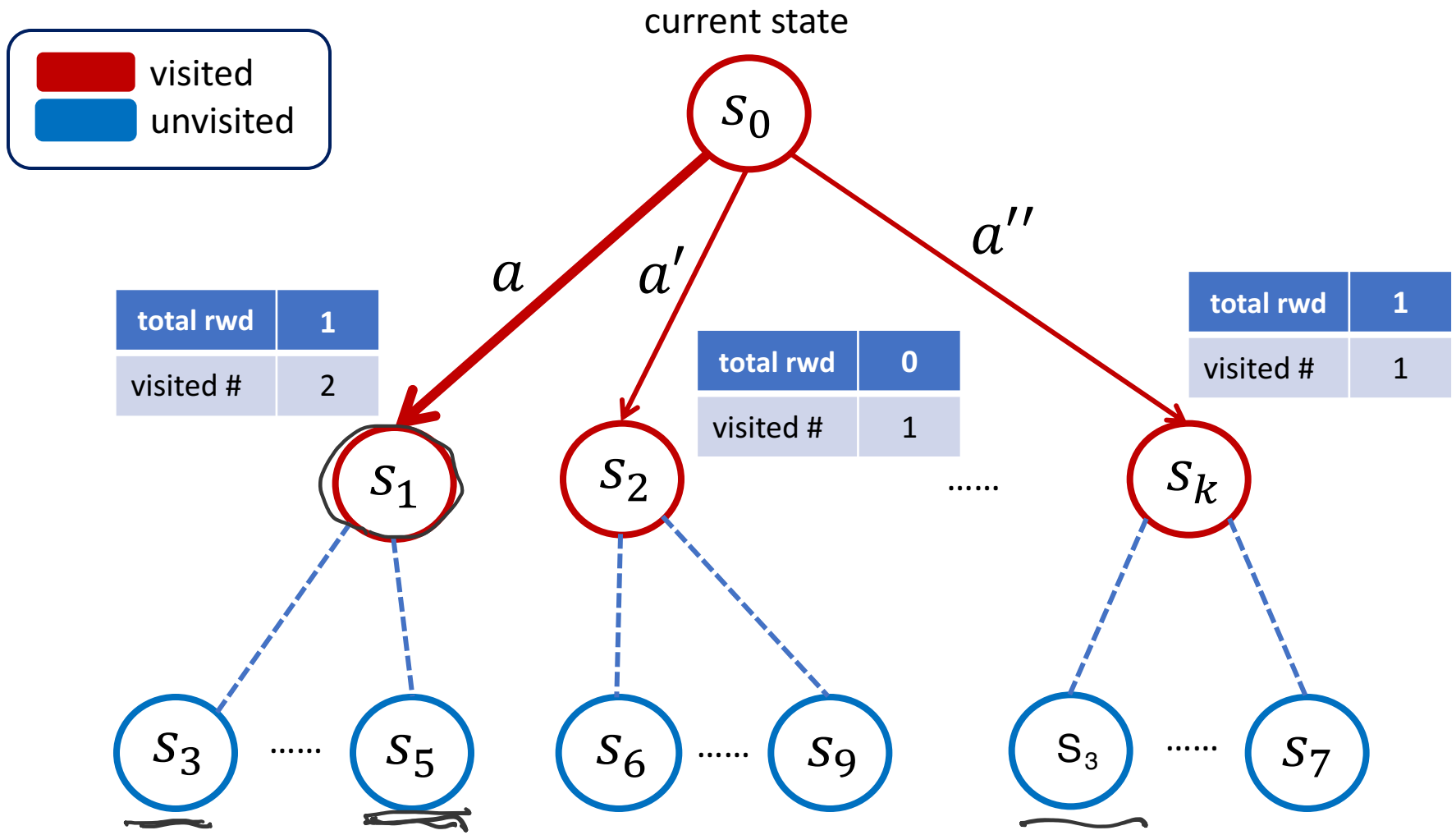


Stage 2: Expand Unvisited 'Child Node'



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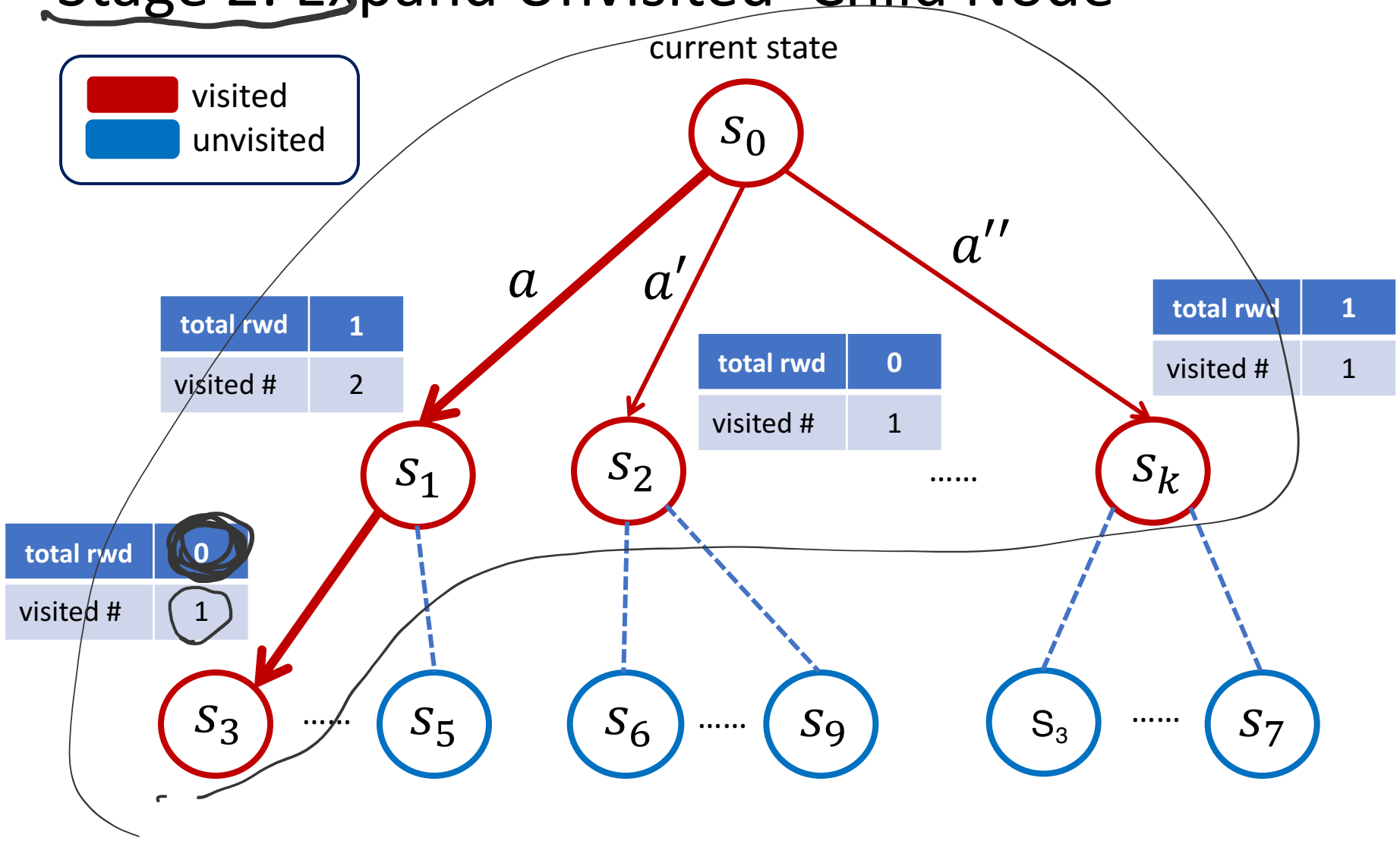
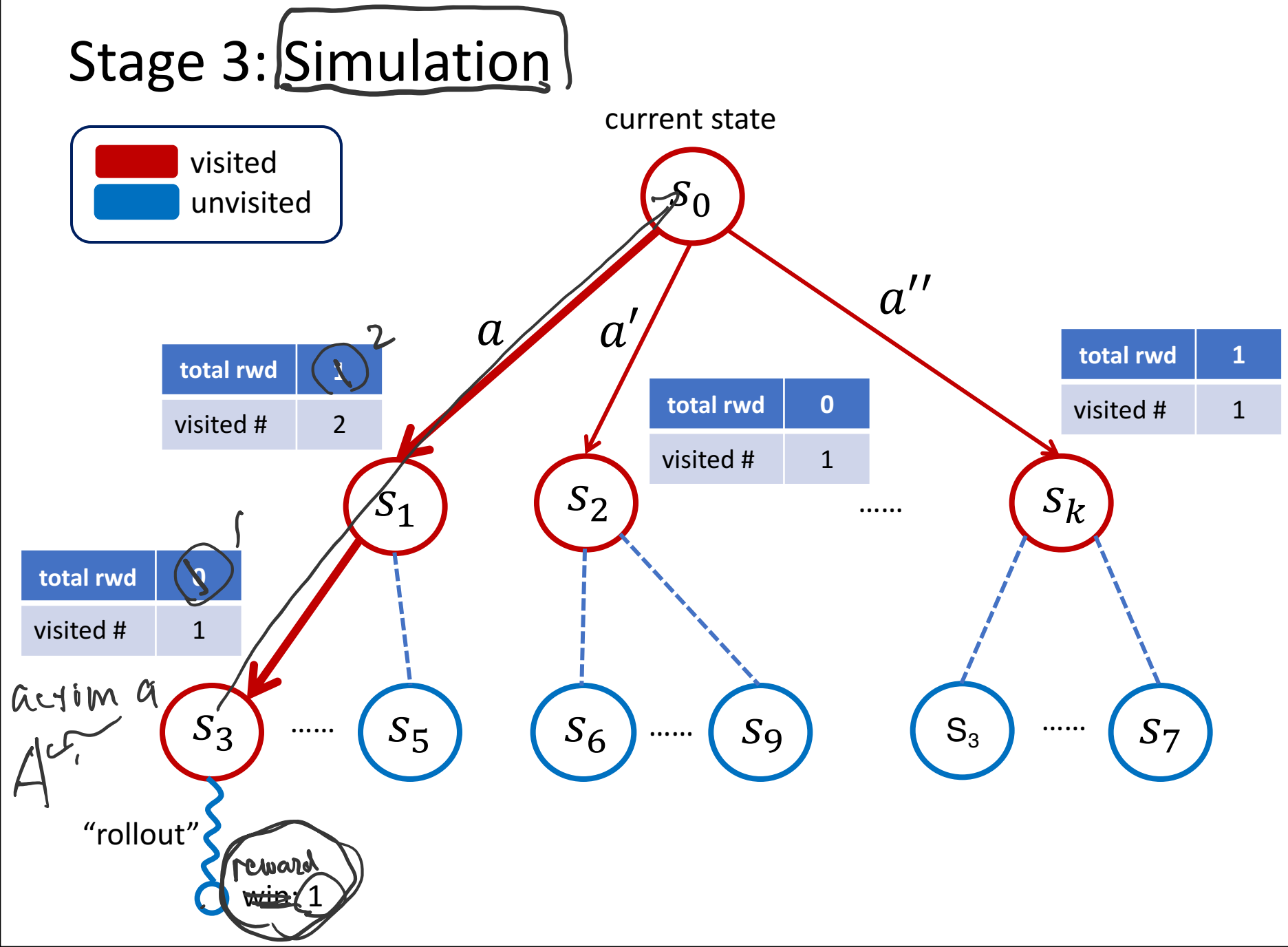
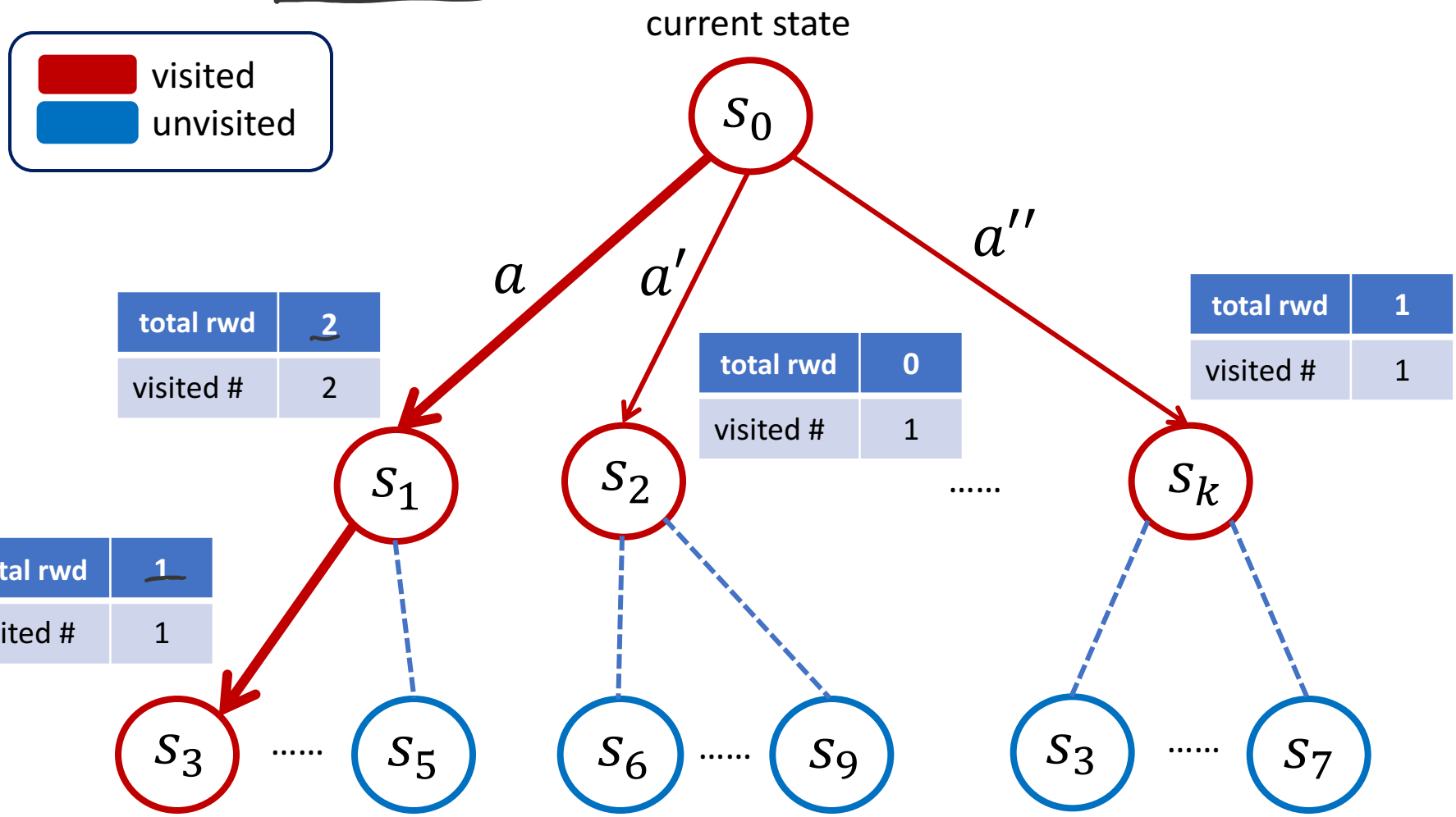


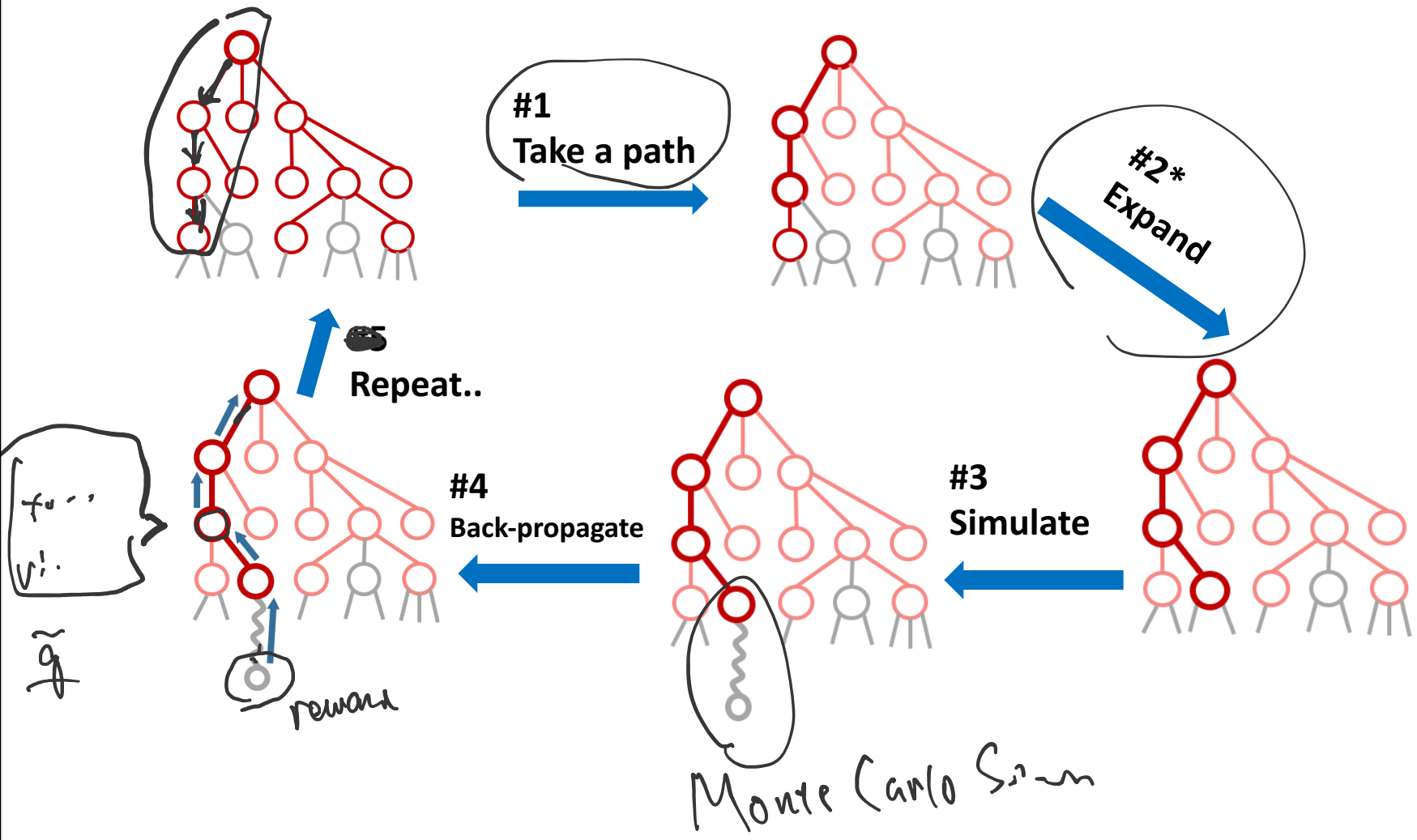
Stage 3: Simulation



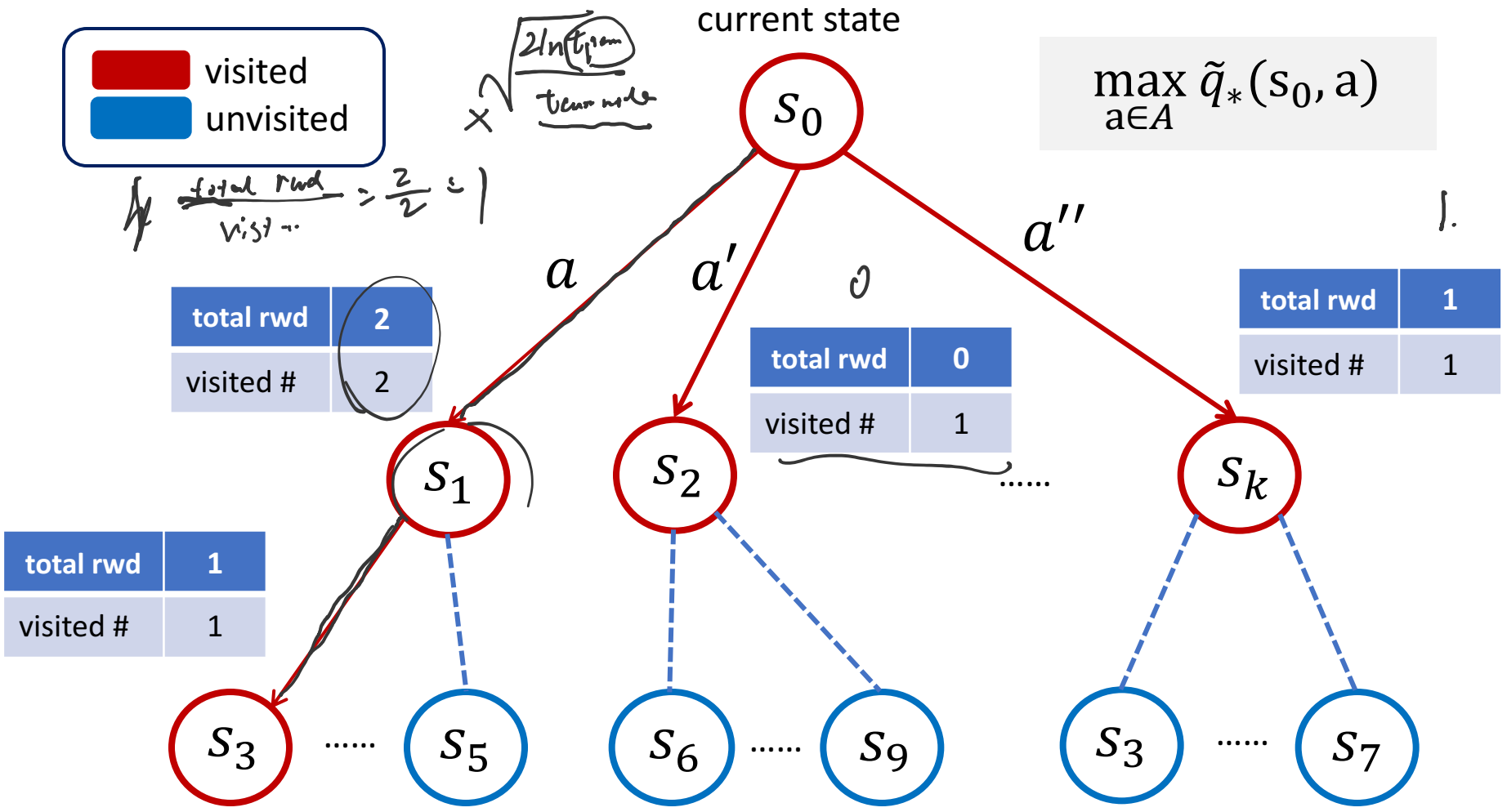
Stage 4: Backpropagation



We just finished one iteration of MCTS!



Stage 1: Traversal over Visited Part of the Tree



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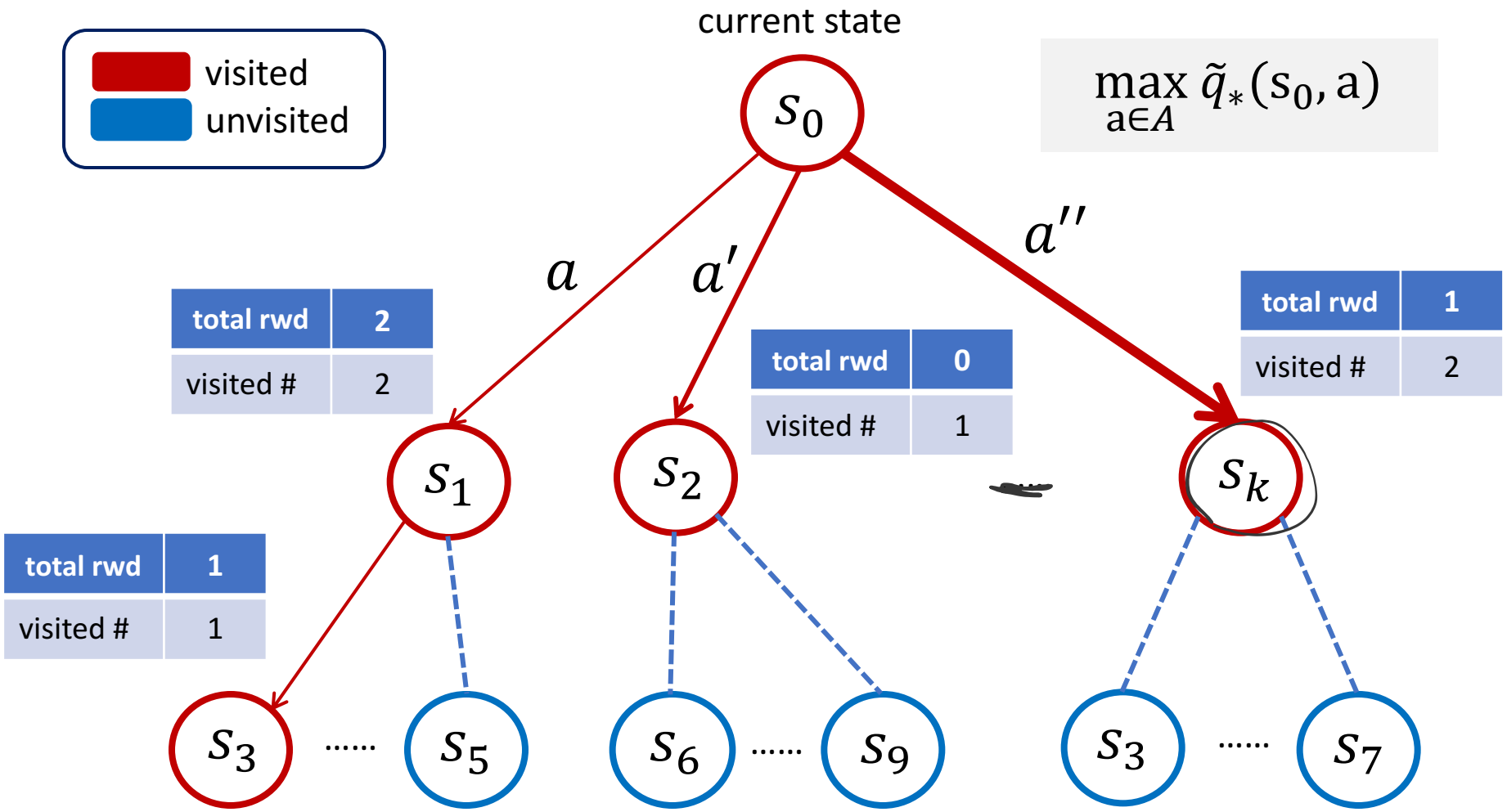

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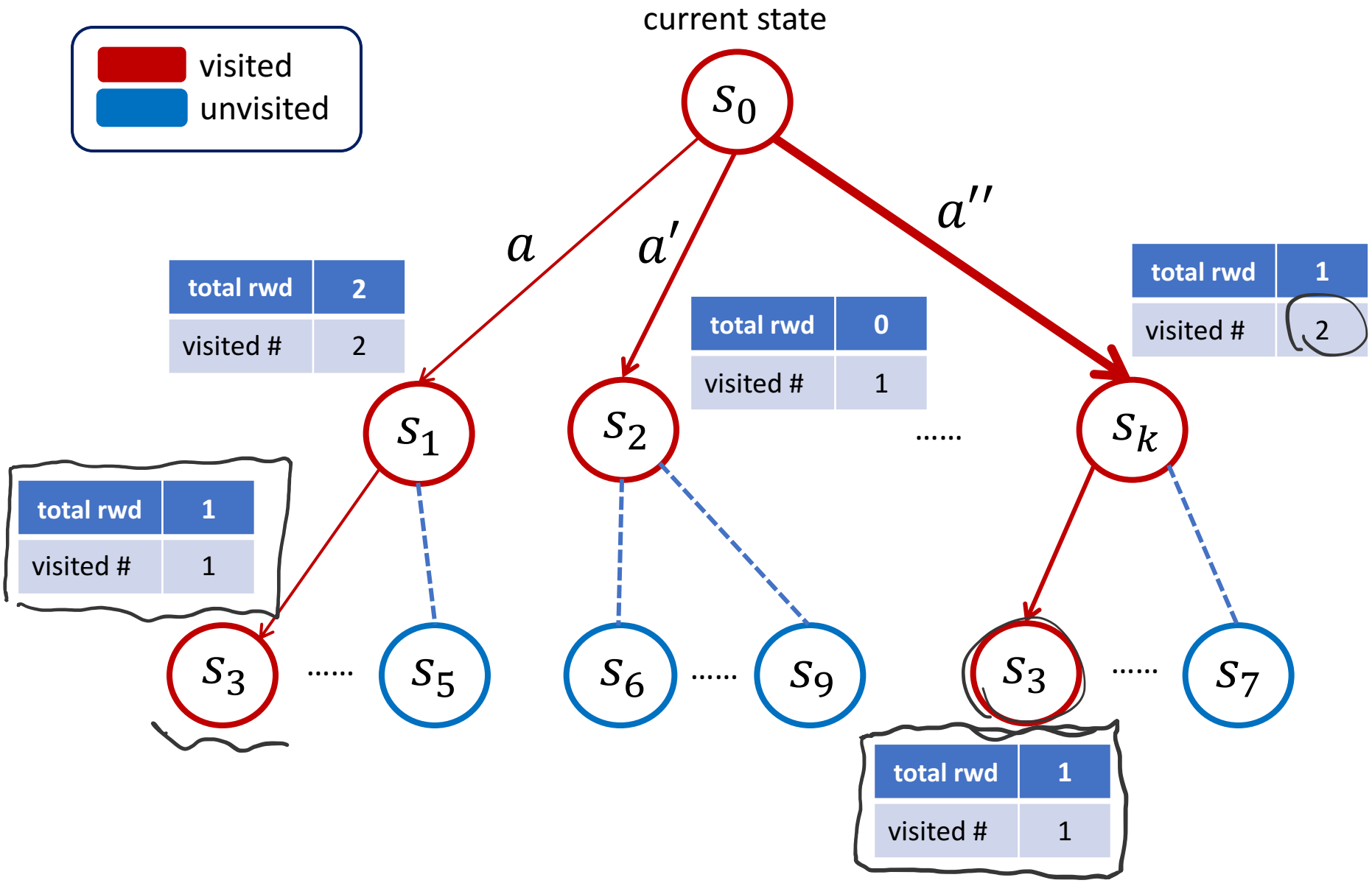
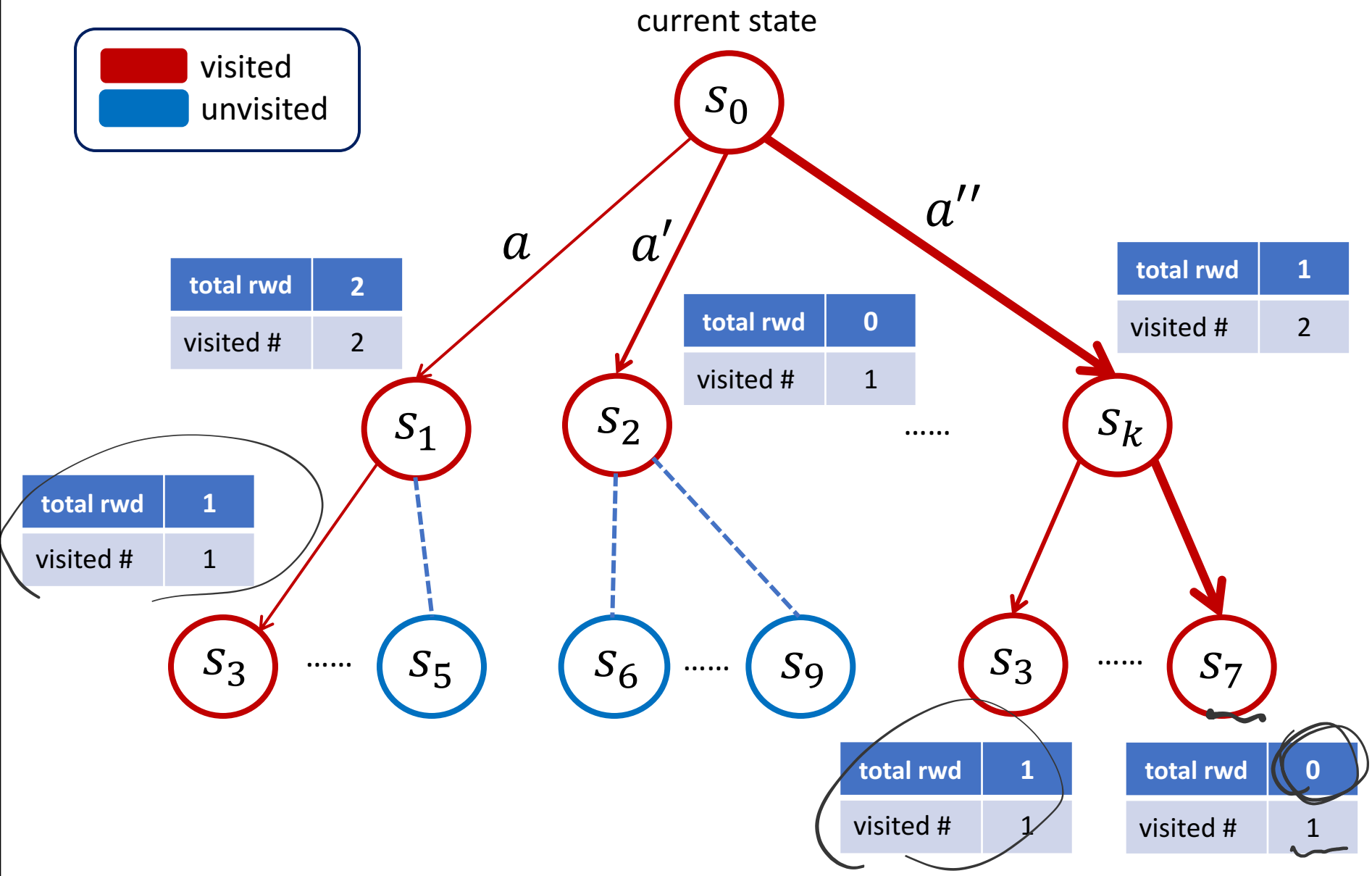
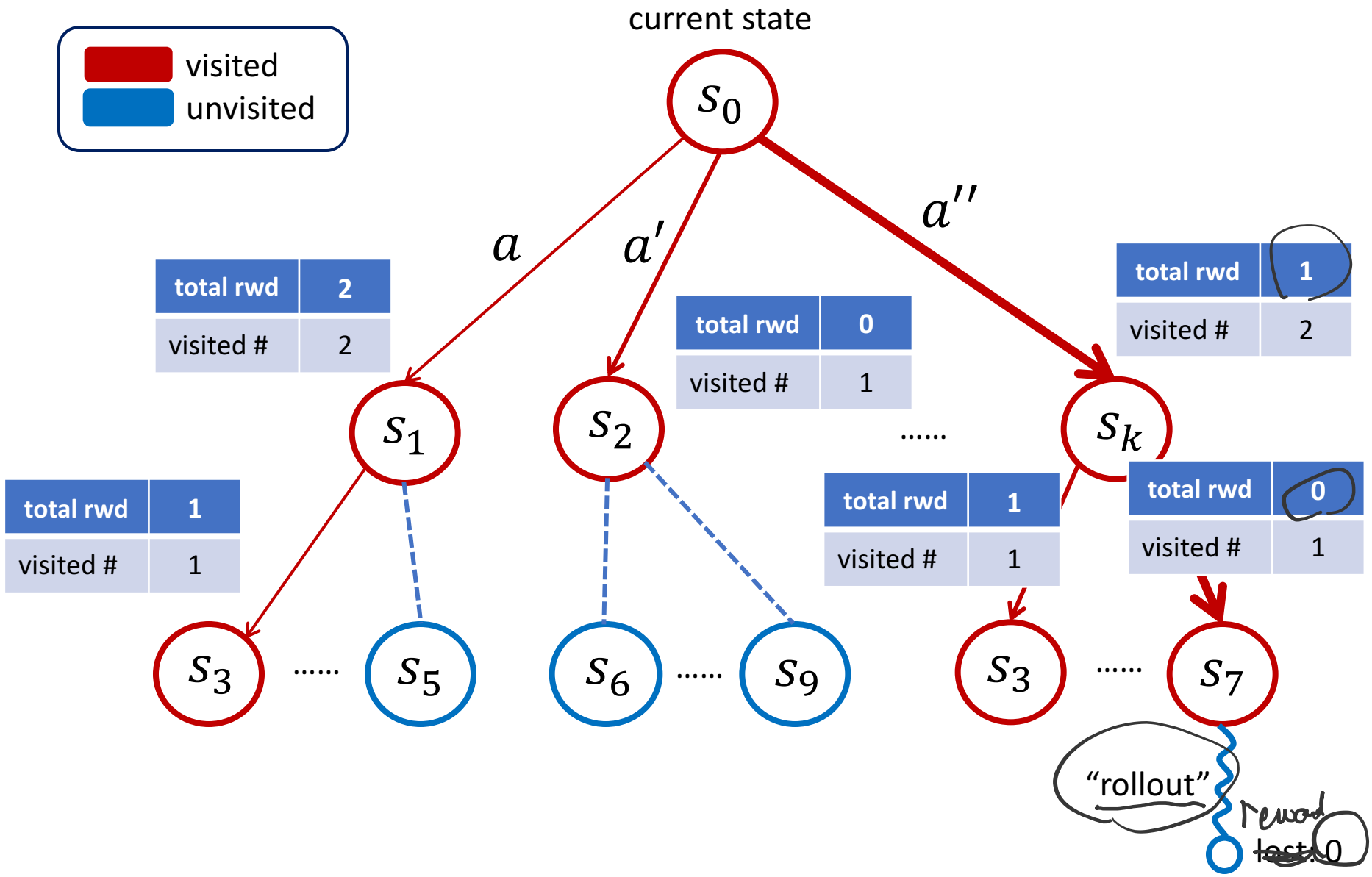


Stage 2: Expand Unvisited 'Child Node'



Stage 3: Simulation



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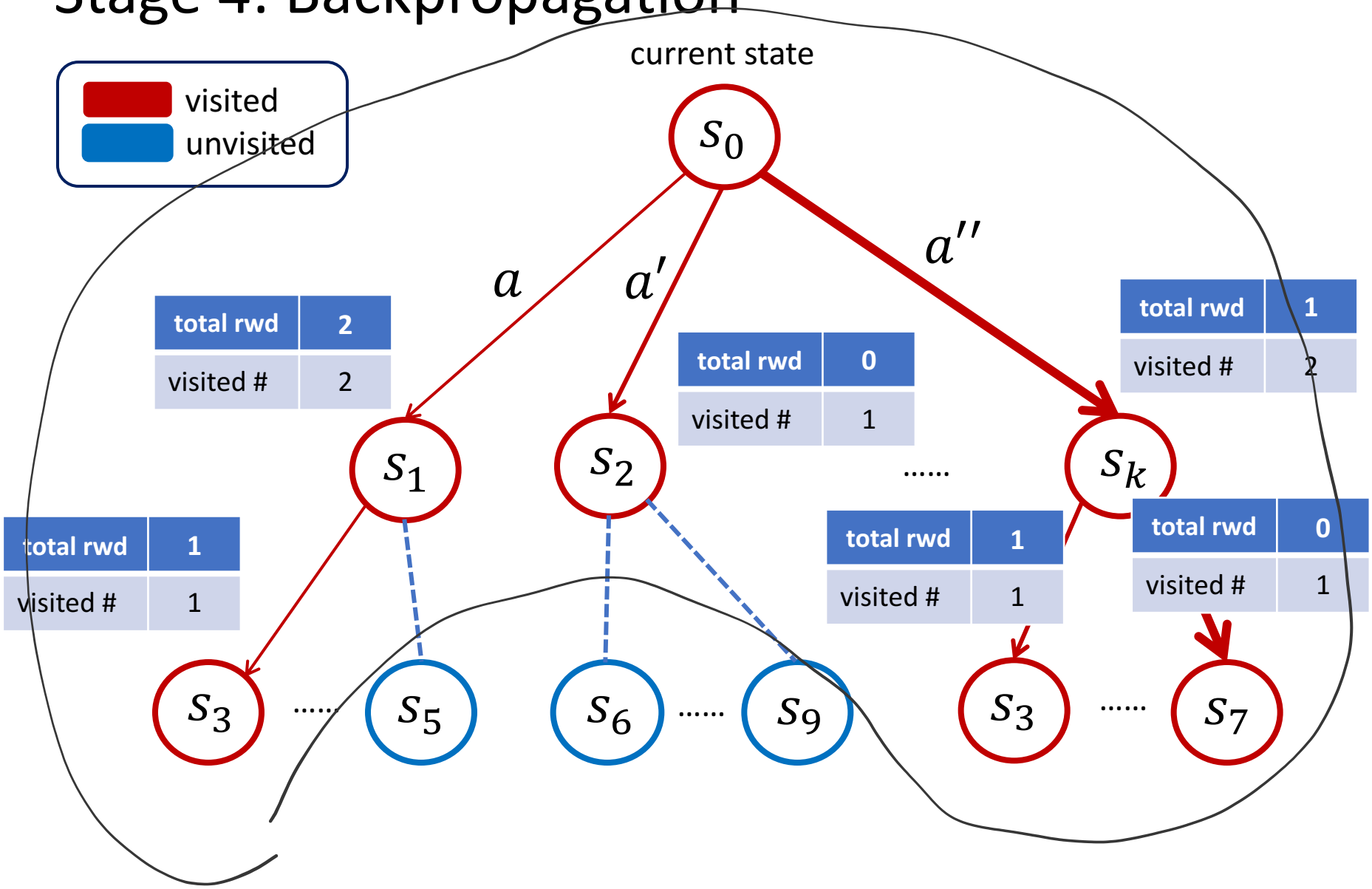
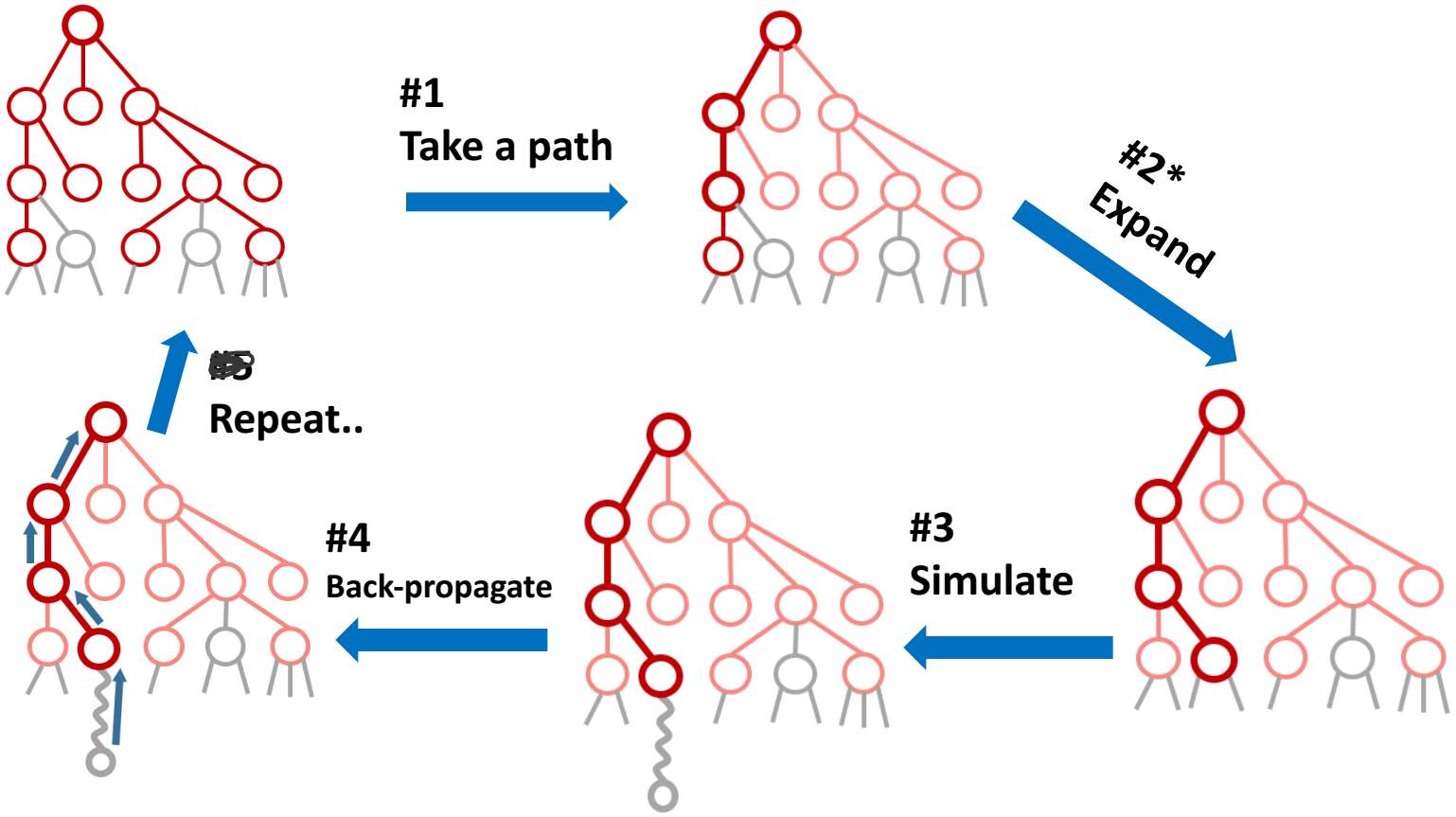
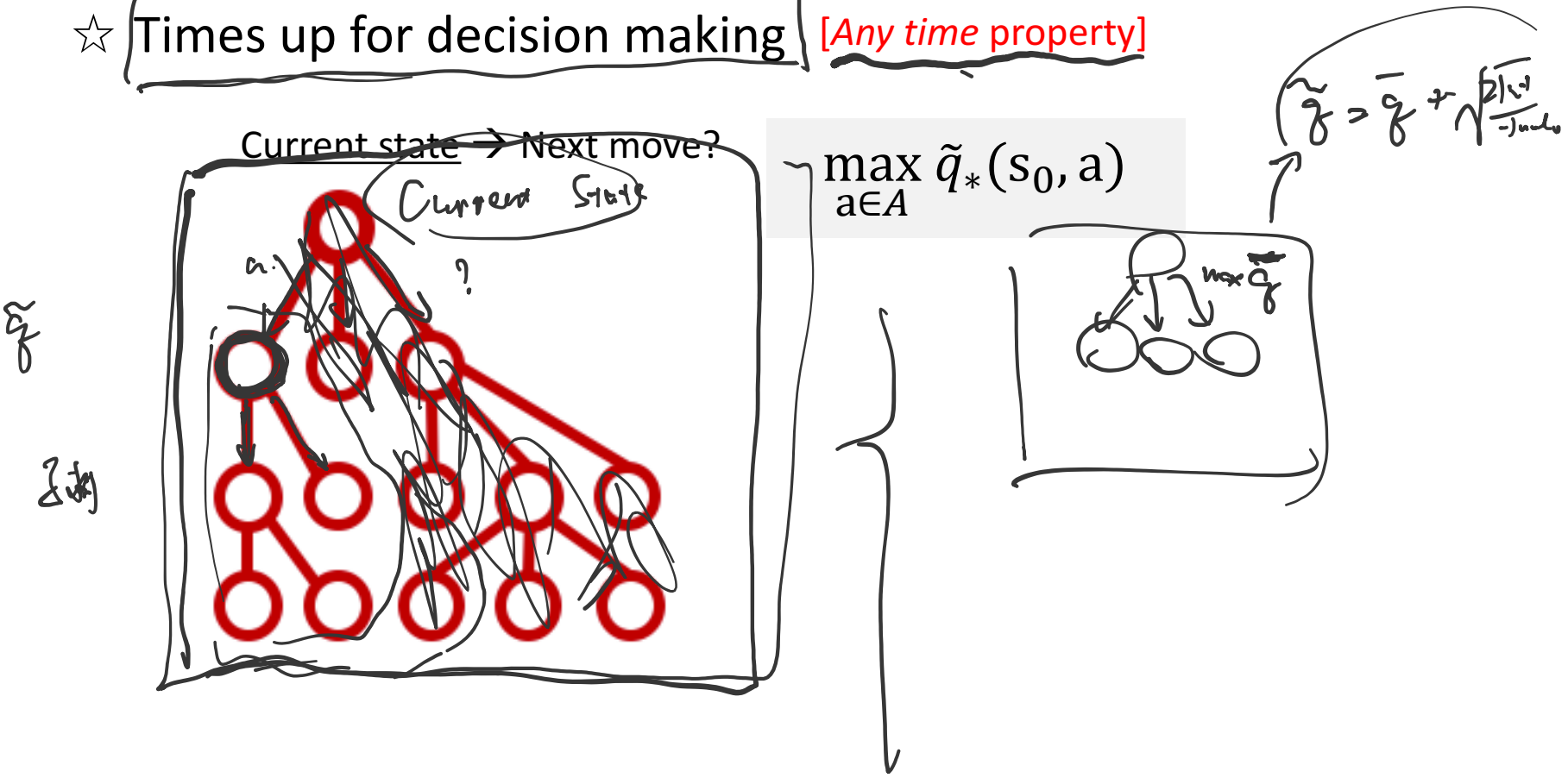


Another iteration is DONE!



Repeat iterations until …

- ☆ Reaching a pre-defined threshold for # of iterations
- ☆ Times up for decision making [*Any time* property]



Performance of MCTS

- ☆ For each state *s,* the estimated value-function is an unbiased *estimate of* the actual value-function as # of samples → ∞
- ☆ The bias converges to zero at rate of $O(\sum_{i=0}^{H-1} \ln(N_i)/N_i)$

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Advantages of MCTS

- ☆ Online w/ *any-time* property
- ☆ Requires no domain knowledge
- ☆ Structed, thus allows various degrees of freedom

  for improvements in each stage
- ☆ Requires no full knowledge of the env. dynamics

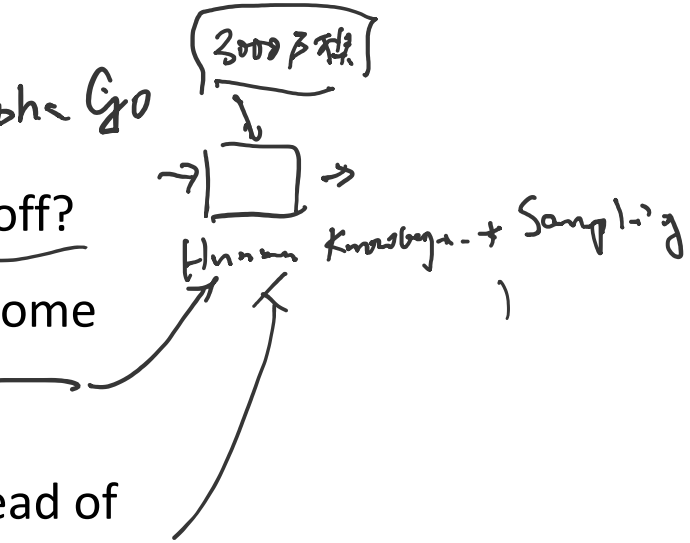
Limitations & Possible Improvements

☆ Traversal

☆ Does UCB achieve the best *e-e* tradeoff?

☆ What if samples are inadequate for some upcoming state?

☆ Can it be taught w/ experiences instead of sampling from scratch?



☆ Expansion

☆ When the branching factor is big..

☆ Simulation

☆ *Purely* random rollouts may not be a good idea

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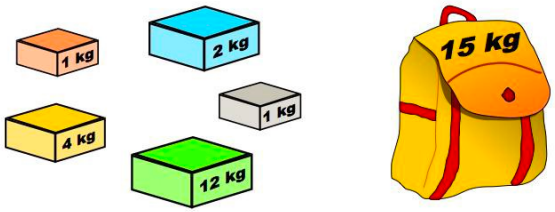
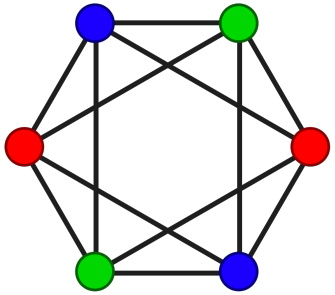
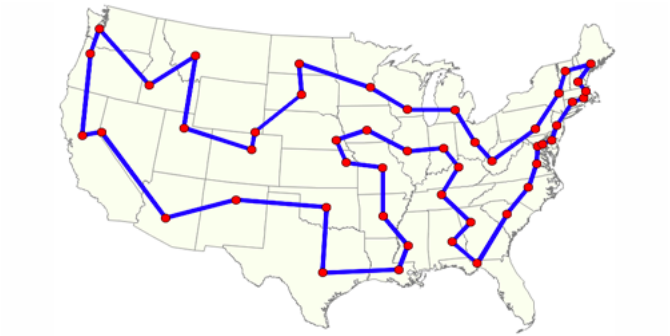
Combinatorial Optimization Problems



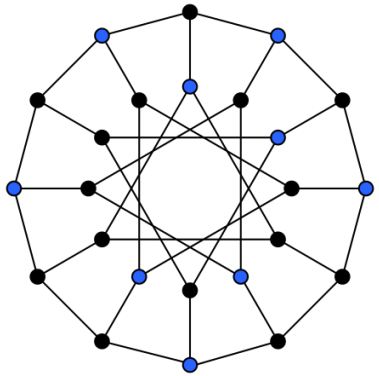
Knapsack Problem



Graph Coloring Problem

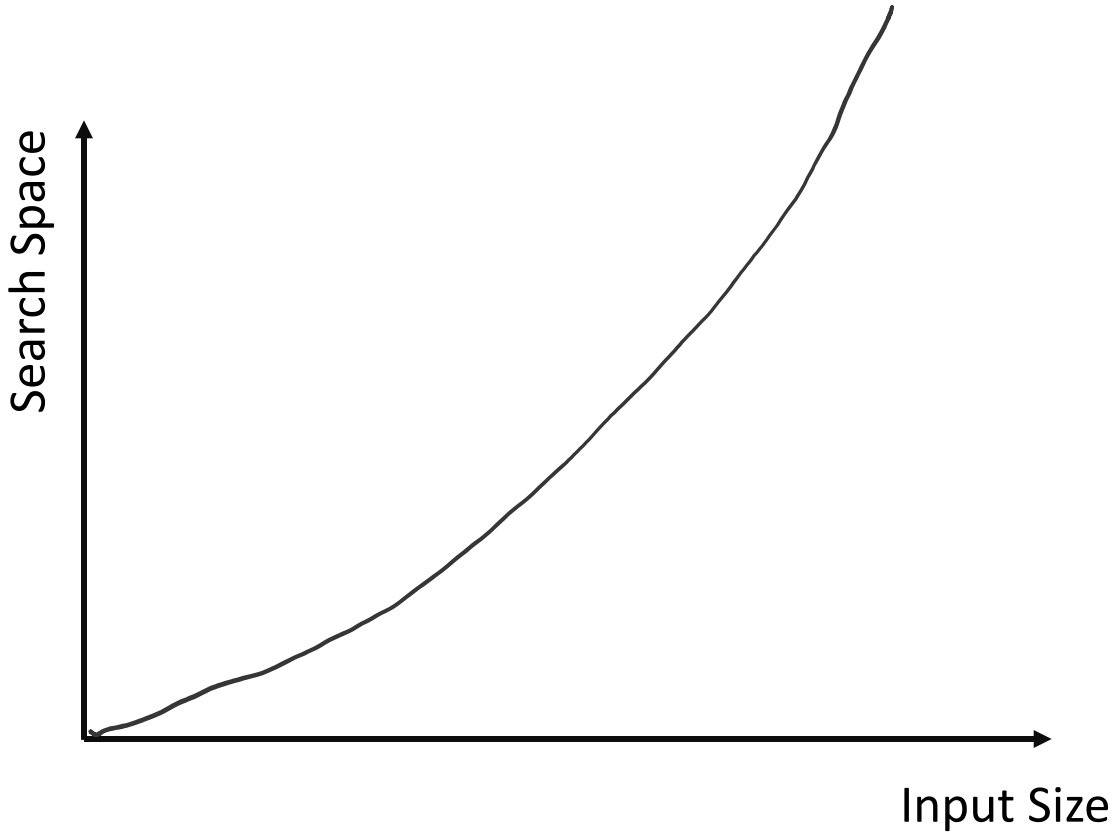


Traveling Salesman Problem



Maximum Independent Set Problem

Challenges in Problem Solving



Efficient & optimal solutions to only minority

Most are NP-hard

Curse-of-dimensionality

Zillions of problems in engineered networks

- CoFlow scheduling in DCN flow scheduling
- Maximum Weighted Independent Set in wireless networks
- Virtual Network Embedding in net. virtualization
- Function Placement in net. function virtualization
- Instance/Task Placement in stream processing systems
- …

Available Arsenal

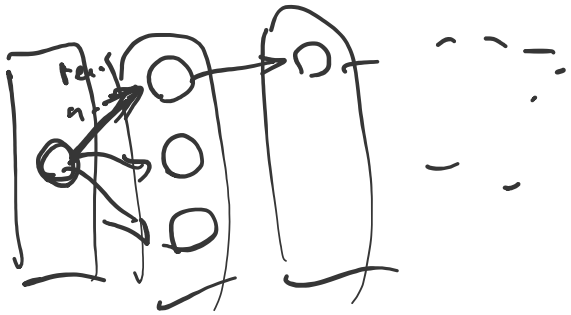
- Approximation algorithms
  - Greedy methods, DP, relaxation techniques, random sampling methods
- Exact algorithms
  - Branch-and-bound, branch-and-cut, *etc.*
- Heuristic
  - Most w/o performance guarantee but in pursuit of low complexity

etc.

MCTS for Combinatorial Optimization Problems

Convert to a seq. decision-making problem

Problem instance

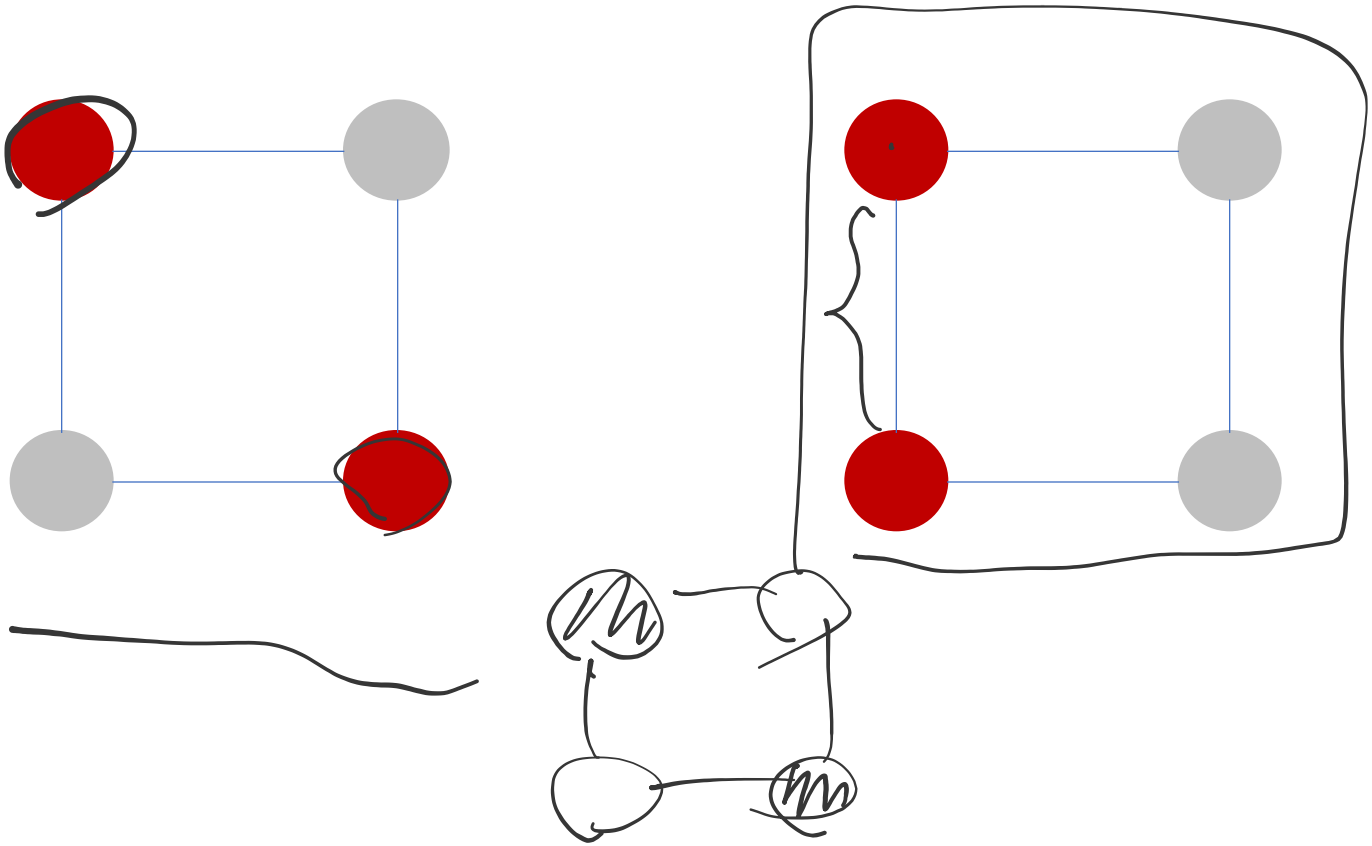


Define the “state”, ”action” & “value”

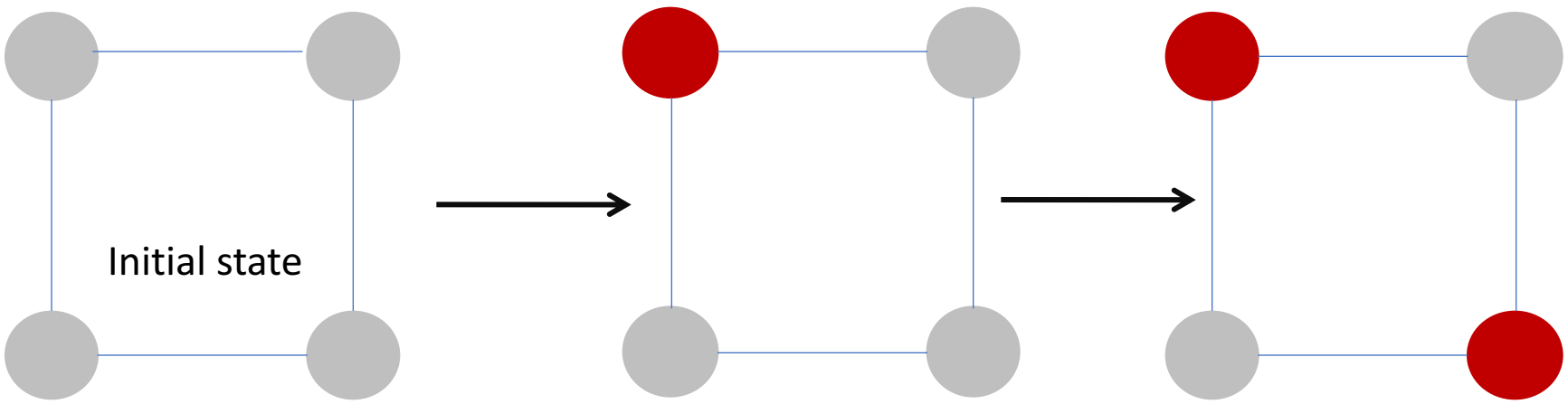
Conceive the search tree

MCTS comes into play

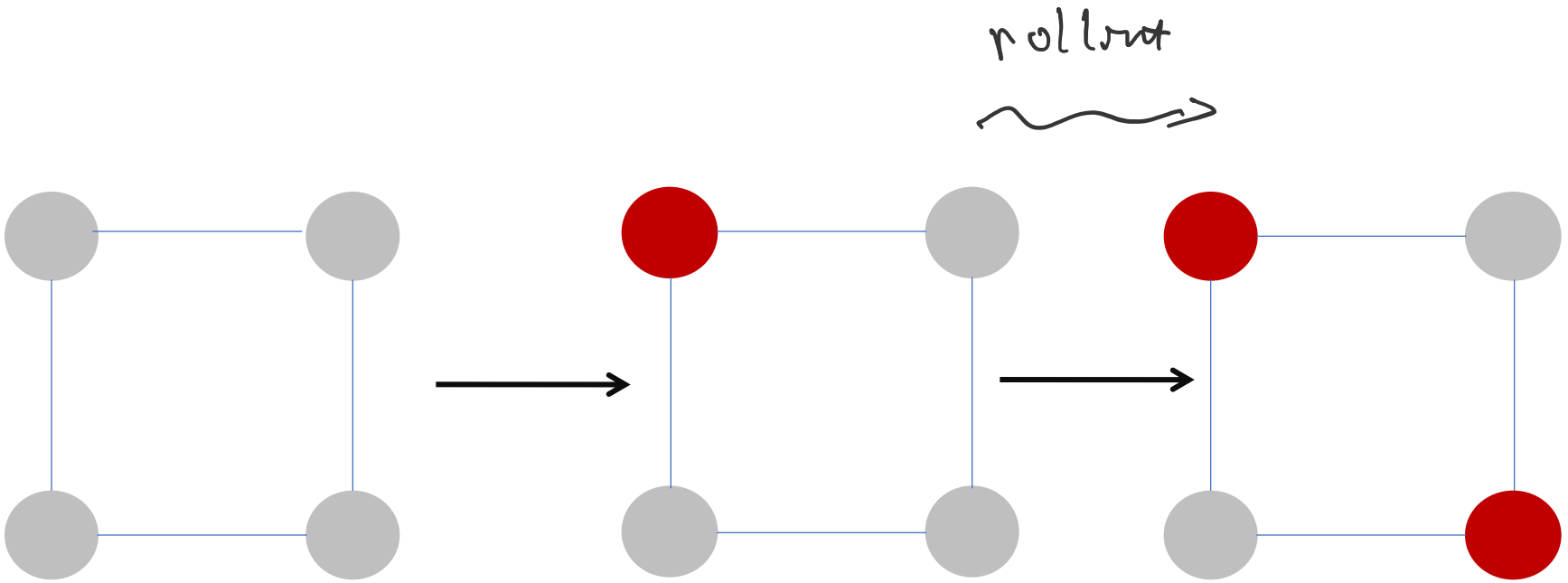
MCTS for ~~Maximum~~ Independent Set Problem



MIS as a Seq. Decision-making Problem



Search Tree for MIS



Lecture Flow

Motivation

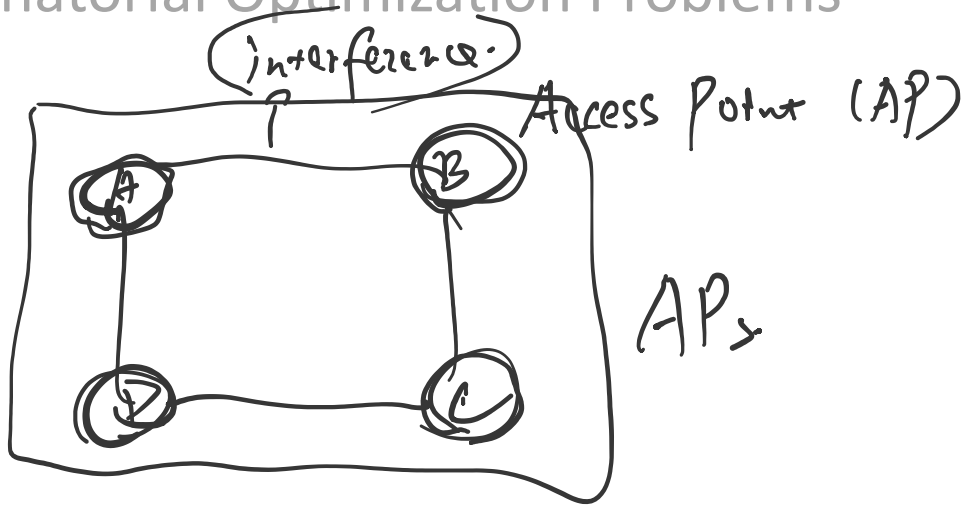
Brief History of Monte Carlo Tree Search (MCTS)

A Close Look at MCTS

Remarks

MCTS for Combinatorial Optimization Problems

MCMC HW Q10



MIS.

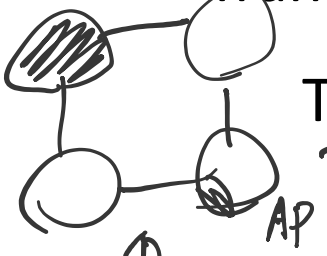
Design as a Discrete-Time Markov Chain

Desired PMF: $\pi_f^* = C^{-1}\exp(\beta|f|) \quad \forall f \in F$

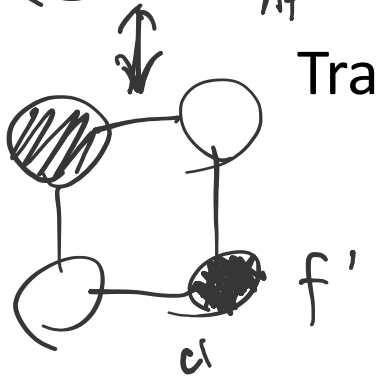
State space: Set of all feasible independent sets

Transition between states:

Topology of the chain:



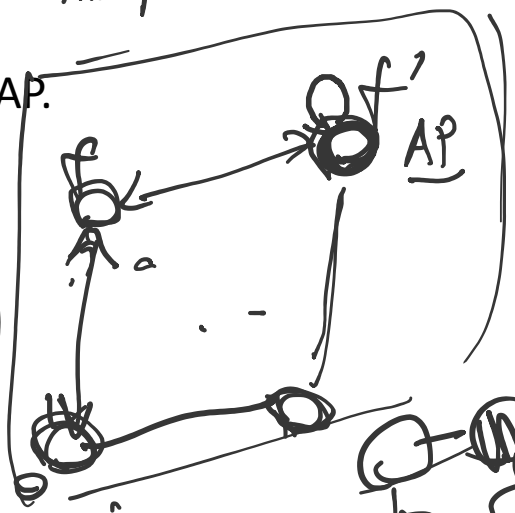
Any two adjacent states differ by one AP.



Transition probability:

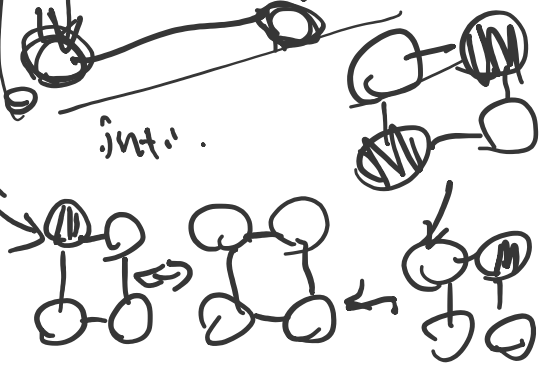
$$p_{f,f'} = \frac{\min\{1, \exp(\beta(|f'| - |f|))\}}{N}$$

$$p_{f,f} = 1 - \sum_{f' \in N(f)} p_{f,f'}$$

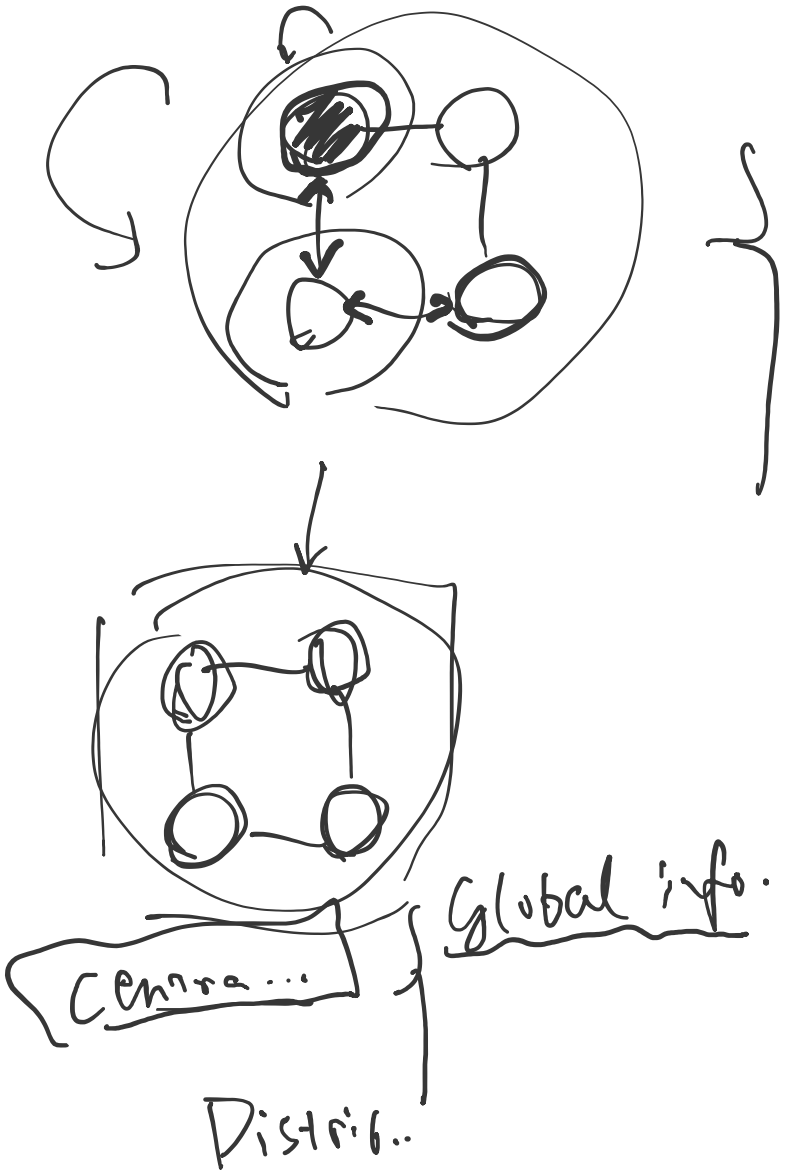


Chain Design?

$$\pi_f^* P_{f,f'} = \pi_{f'}^* P_{f',f}$$



Design as a Discrete-Time Markov Chain



Design as a Continuous-Time Markov Chain

Desired PMF: $\pi_f^* = C^{-1}\exp(\beta|f|), \forall\, f \in F$

State space: Set of all feasible independent sets

Transition between states:

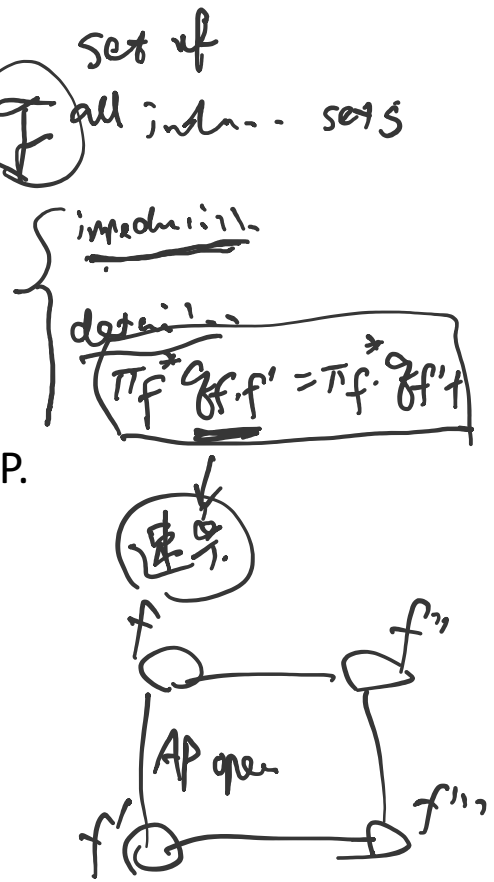
Topology of the chain:

Any two adjacent states differ by one AP.

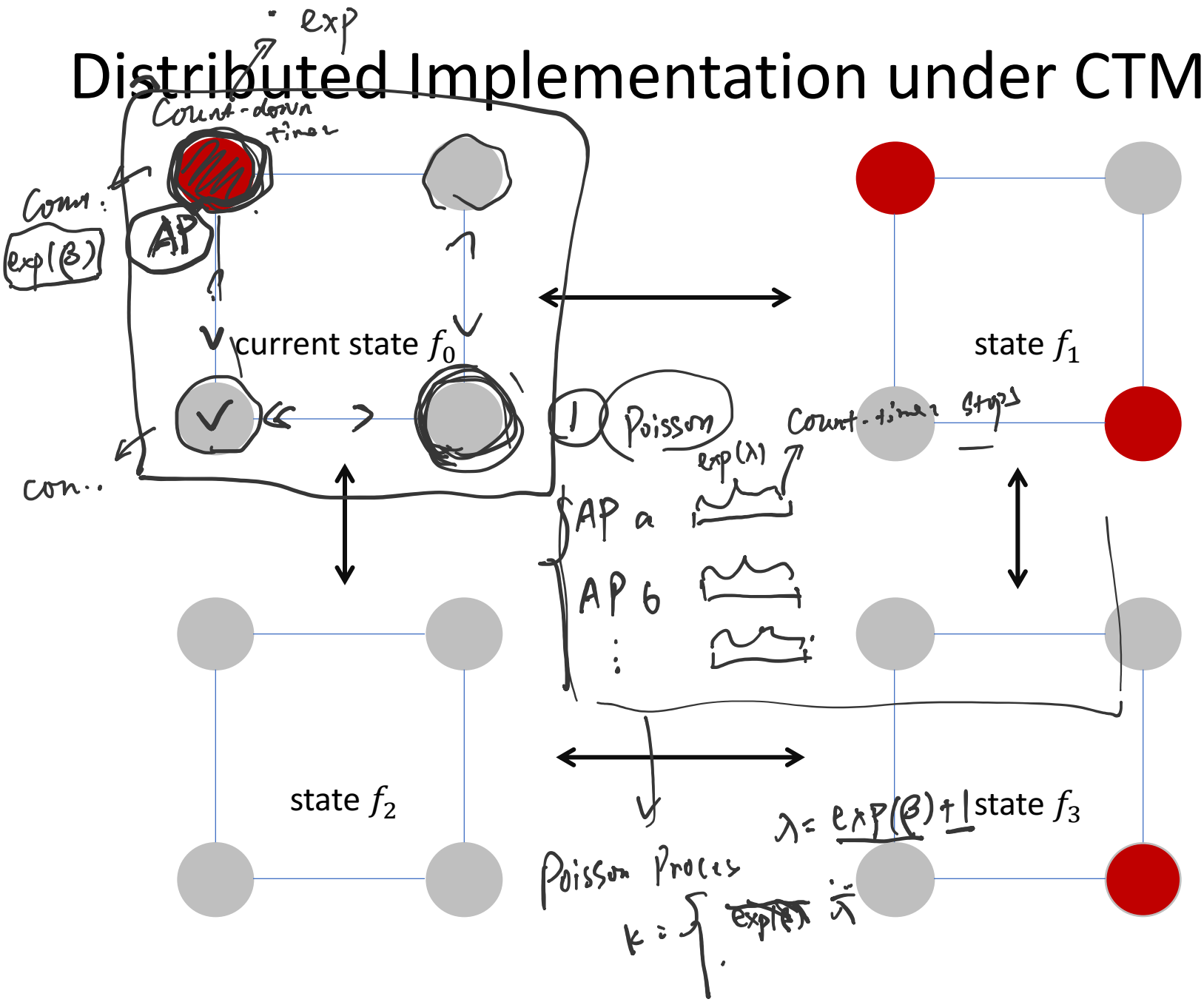
Transition rate:

$$q_{f,f'} = \min\{1, \exp\big(\beta(|f'| - |f|)\big)\}$$

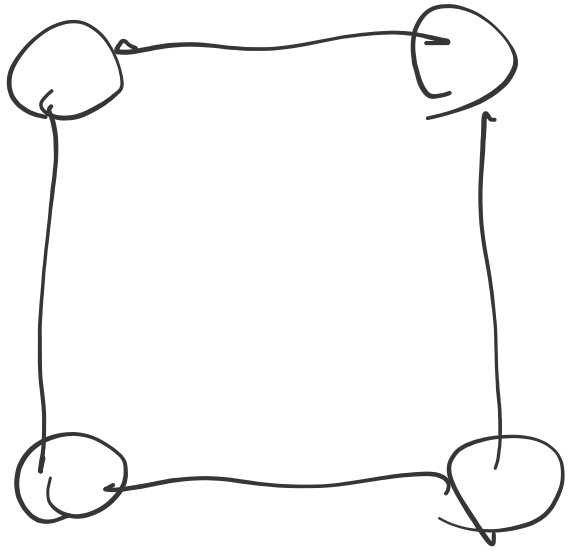
Chain Design:



Distributed Implementation under CTMC

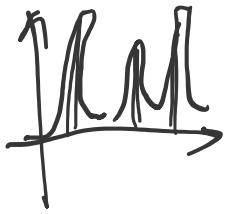


Design as a Continuous-Time Markov Chain

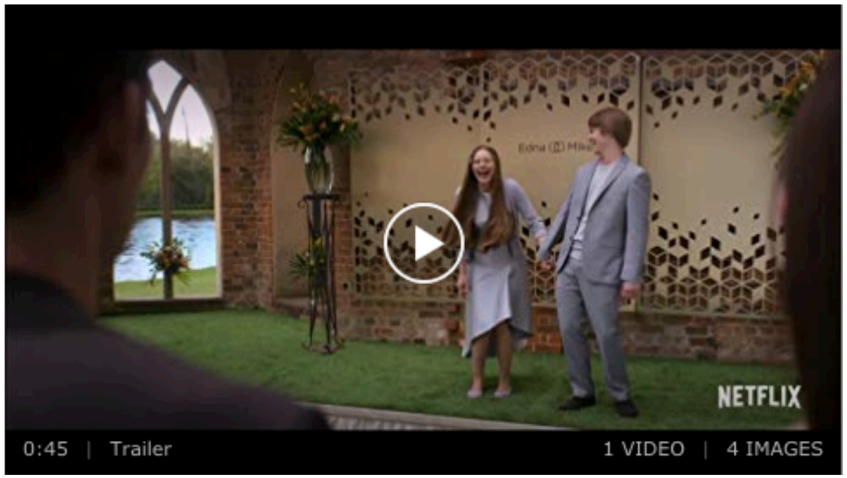


Comparison of MCMC & MCTS

	MCMC	MCTS
Performance Guarantee	✓	—
Degrees of Freedom		
Anytime Property	✗	✓



American Drama w/ Similar Idea to MCTS



Black Mirror Season 4, Ep. 4


Thank you!
Any questions?

This lecture only gives you a *taste* of

Reinforcement Learning.

Omnipresent Idea of *Approximation* in RL

Monte Carlo Methods

Temporal Difference (TD)

Bootstrapping

Learning-aided Planning