Homework 7 Solutions

ESE 402/542

Due Dec 15th, 2020 11:59 pm

Problem 1. *Solution:*

$$E = \sum_{i=1}^{n} ||X_i||^2$$

$$E = \sum_{i=1}^{n} \sigma_j^2 = 385$$

We want to find a compression algorithm that retains at least 50% of the energy. Thus, the sum of the singular values must add up to at least $\frac{385}{2} = 192.5$.

If we select the first 3 components:

$$E = 10^2 + 9^2 + 8^2 = 245$$

Since $245 > 192.5$ the minimum value of the dimension is 3.

Problem 2. *Solution:*

1. For $c = \{1, 2\}$:
$$\mathcal{H}_c = \{(0,0), (0,1), (1,1), (1,0)\}$$
For $c = \{1, 2, 3\}$:
$$\mathcal{H}_c = \{(0,0,0), (1,0,0), (1,1,0), (1,1,1), (0,0,1), (0,1,1), (0,1,0)\}$$
$(1, 0, 1)$ is not possible.
2. Let $C_1 = \{1, 2\}$. $|\mathcal{H}_c| = 4$.
Let $C_2 = \{1, 2, 3\}$. $|\mathcal{H}_c| = 7 < 8$,
3. For $|C| = 2, \mathcal{H}_\mathcal{C}$ shatters C since $|\mathcal{H}_\mathcal{C}| = 4 = 2^{|C|}$.
For $|C| = 3, \mathcal{H}_\mathcal{C}$ does not shatter C since $|\mathcal{H}_\mathcal{C}| < 2^{|C|}$.
4. $VCdim(\mathcal{H}) = 2$

Problem 3. *Solution:*

(a) If a hypothesis class $\mathcal{H}$ is PAC learnable then it means there is a hypothesis h that yields a true error lower than ϵ with probability $1 - \delta$.

(b) $VCdim(\mathcal{H}) = \infty$.

(c) We show that $\text{VCdim}(\mathcal{H}) \leq \log_2 m$ as follows:

Let $\text{VCdim}(\mathcal{H}) = c$.

We know that $|\mathcal{H}| = m \geq |\mathcal{H}_\mathcal{C}|$ and that $|\mathcal{H}_\mathcal{C}| = 2^c$.

Thus:

$$m \geq |\mathcal{H}_\mathcal{C}|$$
$$m \geq 2^c$$
$$m \geq 2^{VCdim(\mathcal{H})}$$

And after taking log of both sides:

$$\log_2 m \geq VCdim(\mathcal{H})$$