Astro 1 — Basic Astronomy
Spring 2025
Instructor: Prof. Bowler

Night Sky Activity: Our Moon

(Deadline: June 5, 5pm)

The goals of this activity are to connect what you have been learning in class about the Moon, the night sky, and Kepler's laws to the actual night (or day-time) sky using observations you will carry out yourself.

Deadline: you must upload your results as a scanned PDF to Canvas by *June 5, 5pm Pacific* (last day of class). This activity will contribute to 10% of your final grade. This work should be carried out during the quarter and cannot be left to the last minute! You may get thwarted by the weather so it is important that you start it early. You are welcome to meet up with others in the class and observe the moon together but the measurements you make and the analysis must be your own.

Your task will be to track the Moon in its orbit around Earth, sketch the ecliptic plane in your local sky, monitor the Moon's phases over time, measure how fast it's moving in the sky over the course of 1 week, find its orbital period using these observations, and use this to determine the mass of Earth.

You may print out the Observing Log (starting on Page 7 of this tutorial), or you can create your own. Just make sure you make the sketches and address all of the questions if you do so.

Part 1: Planning your observing campaign

The first step is to figure out when the Moon is up and in an appropriate location in the sky to carry out this activity. You will be making estimates of the angular distance of the Moon from a point on the horizon using the methods we discussed in class to determine angular sizes with your hand held at arms length. These measurements are more accurate when the Moon is close to the horizon instead of high up in the sky near the meridian. The apparent motion of the Moon during its orbit around Earth over the course of several weeks is Eastward in the sky, although Earth's fast rotation makes it rise in the East and set in the West every 24 hours. So here are the best times to track the moon for this activity over one week:

(1) At sunset from new phase transitioning to first quarter.
(2) In the evening (~9pm) from waxing crescent to waxing gibbous.
(3) At midnight from first quarter transitioning to the full.
(4) At mid-morning (~9am) transitioning from waning gibbous to waning crescent.
(5) At noon transitioning from third quarter to new moon.

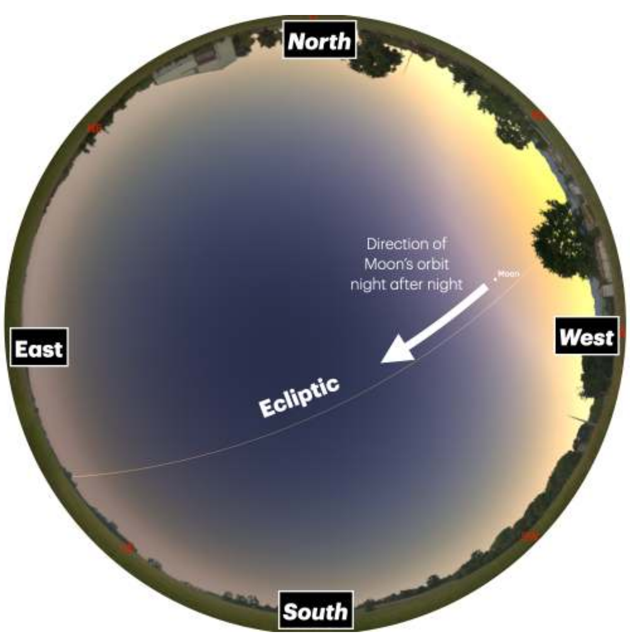
In each of these cases, the Moon will start off near the horizon in the west and ***at the exact same time*** each subsequent night (or day), it will appear rise higher in the sky. This is what you will track. Timing will be important here. ***Select one of these times and plan for your first observation using the Moon calendar below.***

2025 Phases of the Moon Calendar

New Moon				First Quarter				Full Moon				Third Quarter			
				Jan	6	15:56	PST	Jan	13	14:27	PST	Jan	21	12:31	PST
Jan	29	4:36	PST	Feb	5	0:02	PST	Feb	12	5:53	PST	Feb	20	9:32	PST
Feb	27	16:45	PST	Mar	6	8:32	PST	Mar	13	23:55	PDT	Mar	22	4:29	PDT
Mar	29	3:58	PDT	Apr	4	19:15	PDT	Apr	12	17:22	PDT	Apr	20	18:36	PDT
Apr	27	12:31	PDT	May	4	6:52	PDT	May	12	9:56	PDT	May	20	4:59	PDT
May	26	20:02	PDT	Jun	2	20:41	PDT	Jun	11	0:44	PDT	Jun	18	12:19	PDT
Jun	25	3:32	PDT	Jul	2	12:30	PDT	Jul	10	13:37	PDT	Jul	17	17:38	PDT
Jul	24	12:11	PDT	Aug	1	5:41	PDT	Aug	9	0:55	PDT	Aug	15	22:12	PDT
Aug	22	23:06	PDT	Aug	30	23:25	PDT	Sep	7	11:09	PDT	Sep	14	3:33	PDT
Sep	21	12:54	PDT	Sep	29	16:54	PDT	Oct	6	20:48	PDT	Oct	13	11:13	PDT
Oct	21	5:25	PDT	Oct	29	9:21	PDT	Nov	5	5:19	PST	Nov	11	21:28	PST
Nov	19	22:47	PST	Nov	27	22:59	PST	Dec	4	15:14	PST	Dec	11	12:52	PST
Dec	19	17:43	PST	Dec	27	11:10	PST								

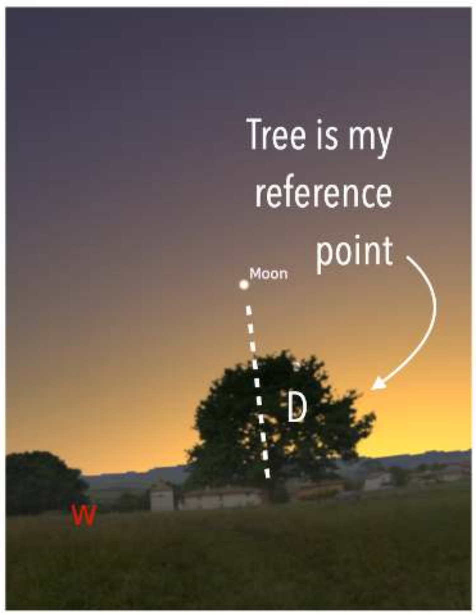
<u>Part 2: Choosing a Reference Point</u>

Once you've found the Moon in the sky, your task is to record its position along the ecliptic plane (see below for details). The ecliptic spans an arc in the sky and is the plane of the Solar System. The moon orbits close to this plane and travels Eastward in its journey around Earth. Here is an example of the ecliptic plane from Santa Barbara in April 2025.



After you find the Moon, you will need to **choose a reference point** on the horizon that you can compare with night after night (or day after day, if you are observing during daytime hours). You are going to watch how far the Moon moves with respect to this reference point. This could be a tree, Broida Hall, your favorite coffee shop, a telephone pole, etc. Your reference point should be close to where the moon is located on the first night of your observations when the Moon is near the horizon. It should be as close to the path of the Moon along the ecliptic as possible; that is, don't choose a reference point far to the South, or toward the North. Select one near the horizon ideally facing West along the ecliptic.

In this example, the reference point is this big tree:

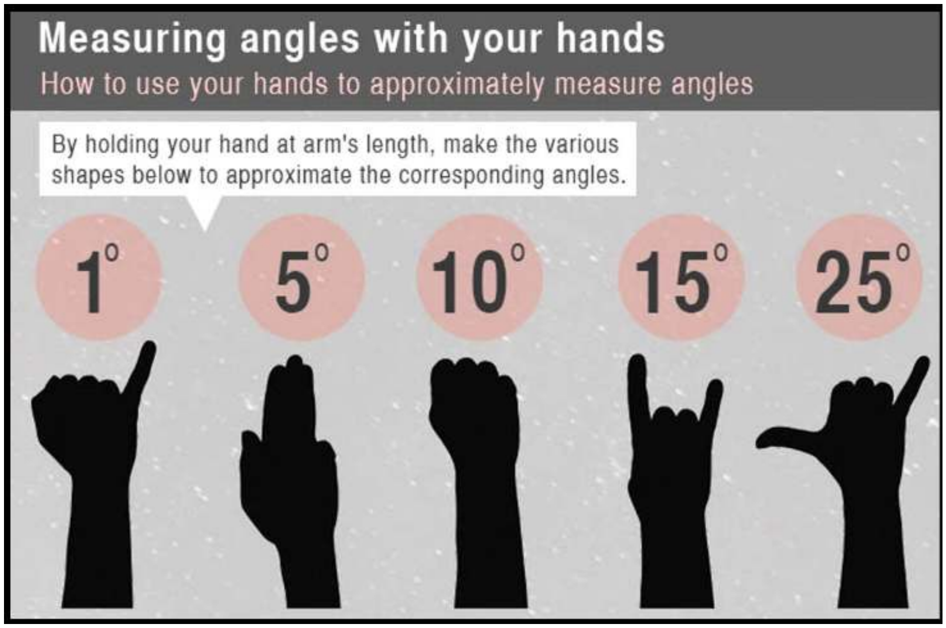


Part 3: Tracking the Moon

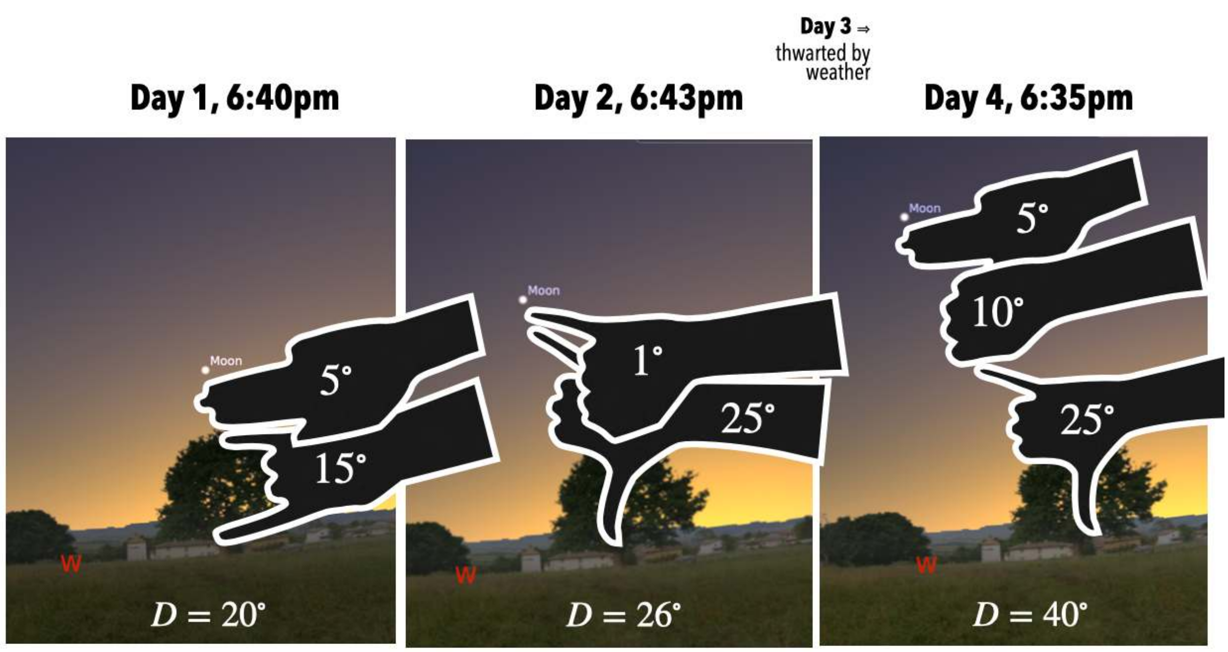
Once you've chosen your reference point, you will need to determine how far it is from the Moon in degrees. You will make this measurement for every nightly (or daily) observation you carry out ***at a fixed time and at the same location for five days in a row.*** Try to choose a window when the forecast is clear. If the weather thwarts an attempt, record that and try again. **At least 3 of your 5 attempts must be successful (clear).**

Because of Earth's rotation, the Moon will of course move in your local sky over time (360 degrees in 24 hours, or 15 deg in 1 hour, or 2.5 deg every 10 min). Since you want to track how the Moon moves in its orbit from West to East in the sky—which is in the oppose sense of the way the sky rotates from the vantage point of an observer on Earth (East to West)—this means you should be consistent to within 10 minutes or so from your first observation on every subsequent night. ***I suggest setting an alarm!***

Use this guide to measure the angular distance *D* between the reference point and the Moon using your hand held at arms length:



Aim to be as accurate as possible. Here are mock examples of how this might look for a series of 3 observations over 4 nights with this large tree as a reference point:



On Day 1, *D* was found to be 15° + 5° = 20°. On Day 2, coming back to the same location at the same time, the Moon appears to have moved up and to the left relative to the tree. Now I find *D* = 26°. Day 3 was thwarted by clouds. On Day 4, *D* was found to be 25° + 10° + 5° = 40°

<u>Part 4: The Moon's rate of change and orbital period</u>

You now should have at least 3 successful observations of the Moon. For each of these, and starting on Day 2, compute the difference between the angular distance of the Moon relative to the reference point (*D*) and the value you obtained from the previous observation. Then divide this by the amount of time that elapsed between these observations, in units of days:

$$\text{Rate of change} = \frac{D_2 - D_1}{t_2 - t_2}.$$

Here, the *D*'s should be in degrees and the *t*'s should be in days. We'll end up with degrees per day.

The goal here is to find the rate of change (degrees per day) for each pair of observations (Day 1 and Day 2, Day 2 and Day 4, etc). If 2 days elapsed then the denominator would be 2 instead of 1. The more observations, the better.

For the examples above in Step 3, we found *D* = 20°, *D* = 26°, and *D* = 40° on Day 1, Day 2, and Day 4, respectively. The rates of change would be (26° - 20°) / 1 day = 6°/day for Days 1 and 2, and (40° - 26°) / 2 days = 7°/day for Days 2 and 4.

The final step is to average however many rates of change values you measured. Recall that an average is the sum of all of the measurements divided by the number of measurements.

For this example, we would get (6°/day + 7°/day) / 2 = 6.5°/day.

The final step is to estimate the orbital period of the Moon. The Moon undergoes 360° in one orbit. So to find the duration of one orbit, we want to divide 360° by our average rate of change:

$$\text{Orbital Period} = \frac{360^\circ}{\text{Average rate of change}}.$$

For this example, we would find an orbital period of 360° / 6.5°/day = 55.4 days. Here the numbers are made up so this value is quite far from the actual value of 27.3 days.

Note: you will not be graded on the accuracy of your orbital period. If this differs from the true value, I want you to think about why that might be the case.

<u>Part 5: The ecliptic plane in your local sky</u>

Now that you know the motion of the Moon in your local sky, you can also trace out the ecliptic plane in your local sky. This is the plane of Earth's orbit around the Sun. The Moon's orbit around earth is only a few degrees off from this plane. Nearly everything in the Solar System is located close to this plane.

Part 6: Using Kepler's Third Law

The final step is to use Kepler's Third Law to derive the mass of Earth using the distance to the Moon, 3.84×10^5 km. As a reminder, Kepler's Third Law is:

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM}$$

Where *P* is the orbital period, *a* is the semi-major axis of the orbit, *M* is the mass of the central object, and *G* is the gravitational constant—6.67×10^{-11} m^3 kg^{-1} s^{-2}. Note: as a reminder, when referring to an object orbiting the Sun, P may be expressed in years and a can be expressed in AU. In that case, $P^2 = a^3$. However, for the case of the Moon orbiting Earth, the central object is Earth, so the general form of Kepler's Third Law must be used. Remember to convert P and a into appropriate units (seconds and meters).

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Spring 2025

Observing Log

You may print this log and fill it in, or make your own version.
If you create your own, be sure to answer all of the questions.

Name: Demir Ince

Range of dates of Moon observations: May 7th - 12th

Part 1: Design your campaign.

(1) How did you plan your observations? Which time of day and phase of the Moon did you begin with?

I wrote a python program that measures the angular distance of any two points on a 360 degree image. The image is projected onto the inside of a sphere and the arc between the points is measured. I will place my camera on the same spot for every measurement and use my program to measure the angle. The distortions in the images are due to the flat projection of a spherical image. The straight yellow lines are for reference only and are not accurate. Began with Waxing Gibbous.

Part 2: Define your reference point.

(2) On the first night (or day) of your observations, make a sketch of objects on the horizon in the direction of the Moon and label what your reference point is. This might be a building, a tree, a telephone poll, and any other fixed object that you will use to when tracking the Moon.

Date and time: May 7th 12:00 am

Scene:

Location of these observations:

Corner of my street.

Part 3: Daily Moon observations.

(3) Sketch the scene, including the position of the Moon and your reference point. Measure the angular distance to the Moon from the reference point and label that length.

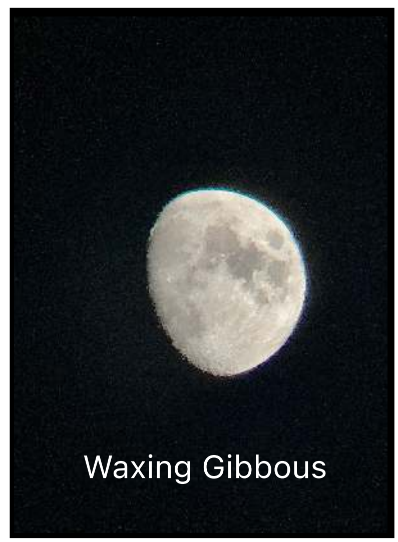
(4) Make a sketch of the Moon, including any features you see. What part of the lunar cycle is it? (For example, waxing crescent, first quarter, waning gibbous, etc)

Observation 1

Date and time: May 7th 12:00 am

Scene sketch:

Moon sketch:



List the measurements in the Table of Moon Positions at the bottom of this document.

(5) Describe what you notice in the space below. Was this measurement straightforward to carry out? Have you noticed any motion of the Moon caused by Earth's rotation while you are carrying out your observations? Any other notes or comments?

Because I just take a picture then do the measurement afterwards, there is no noticeable movement of the Moon. The Moon "sketch" is taken with a phone attached to the eye piece of binoculars, but a couple hours before the measurement. I'll do them together for the coming nights.

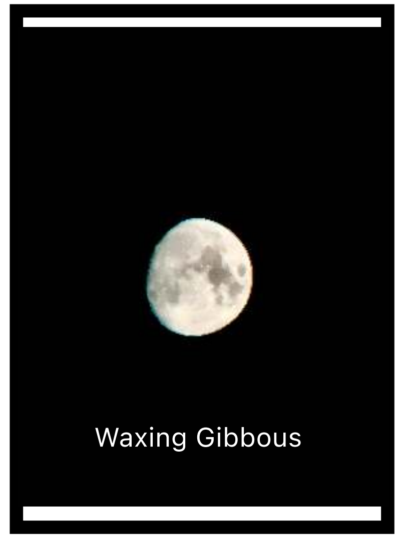
(6) Repeat this for Observations 2-5 below. (5 attempted observations are required with at least 3 measurements, but you can make more measurements if you would like!)

Observation 2

Date and time: May 8th 12:00 am

Scene sketch:

Moon sketch:



Describe what you notice in the space below.

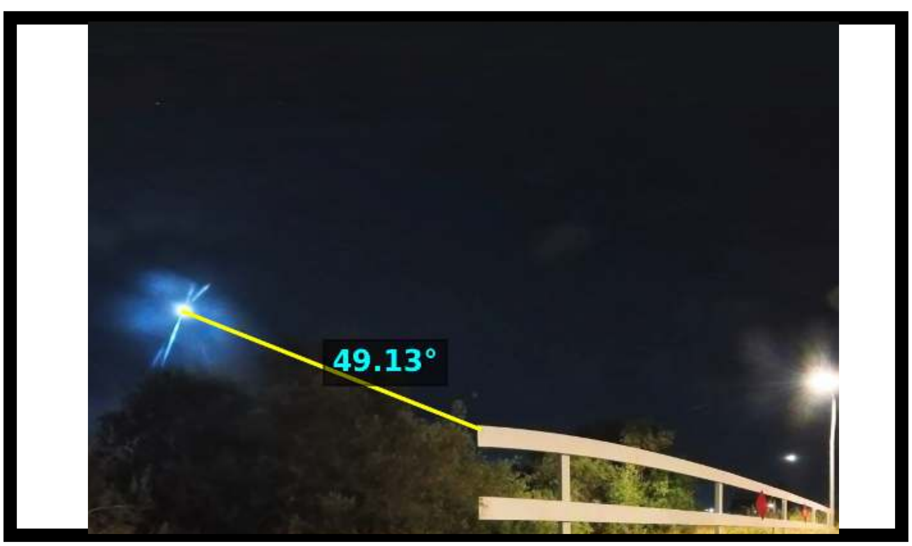
Main observation is that my method is working extremely well. I can even overlay the two images and visualize the movement of the Moon. My photo is a bit worse today, shaky hands. It looks like by the 4th day the Moon will leave this window I've been zooming into.

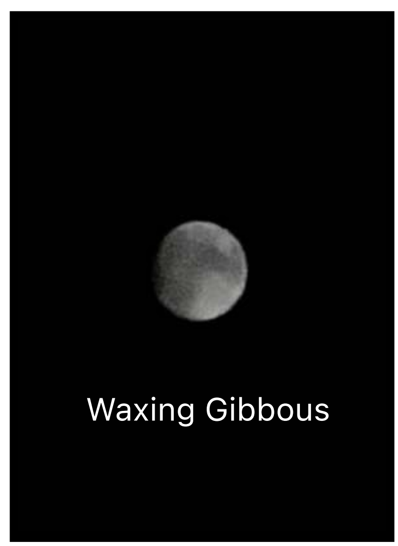
Observation 3

Date and time: May 10th 11.59 pm

Scene sketch:

Moon sketch:





Describe what you notice in the space below.

Sadly I was not able to get a measurement on the night of the 9th. This shouldn't pose an issue as we calculate time passed as well so it'll be 2 days between Obs 2 and Obs 3. I will take an extra measurement to try to reduce the error introduced by this.

Observation 4

Date and time: May 11th 11.58 pm

Scene sketch:

Moon sketch:

Describe what you notice in the space below.

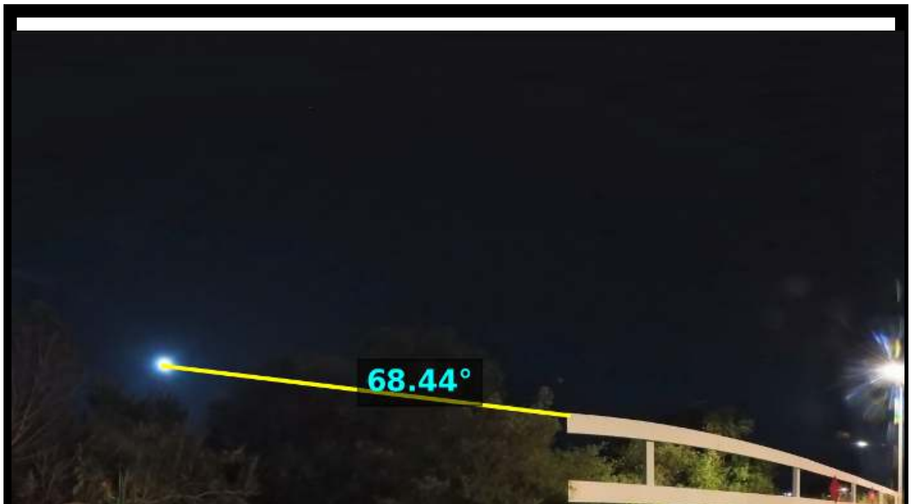
As predicted, I had to expand my window to fit the reference and the Moon into a neat frame. My main concern now is that tomorrow the Moon might be behind the tree line. Maybe I can still take a measurement with the Moon light shining through.

Observation 5

Date and time: May 12th 12.00 am

Scene sketch:

Moon sketch:



N/A

Full Moon

Describe what you notice in the space below.

Today was a rush job, I couldn't get a picture of the Moon, but it is basically identical to yesterdays. I hope by now the issue with my method has become apparent. I will adress this in full later. I've also removed the blank observation pages.

Table of Moon Positions

	Date and Time	Angular distance D in degrees from reference point to Moon	Average daily motion of moon since last observation (here t should be in days):	How many days past the last New Moon is this observation?
Obs 1 (required)	May 7th 12:00 am	$D_1 =$ 25.71	[leave blank]	10
Obs 2 (required)	May 8th 12.00 am	$D_2 =$ 31.27	$(D_2 - D_1) / (t_2 - t_1) =$ 5.56	11
Obs 3 (required)	May 10th 11.59 pm	$D_3 =$ 49.13	$(D_3 - D_2) / (t_3 - t_2) =$ 8.93	13
Obs 4 (required)	May 11th 11.58 pm	$D_4 =$ 59.17	$(D_4 - D_3) / (t_4 - t_3) =$ 10.04	14 (Full Moon)
Obs 5 (required)	May 12th 12.00 am	$D_5 =$ 68.44	$(D_5 - D_4) / (t_5 - t_4) =$ 9.27	15
Obs 6 (optional)		$D_6 =$	$(D_6 - D_5) / (t_6 - t_5) =$	
Obs 7 (optional)		$D_7 =$	$(D_7 - D_6) / (t_7 - t_6) =$	
Obs 8 (optional)		$D_8 =$	$(D_8 - D_7) / (t_8 - t_7) =$	
Obs 9 (optional)		$D_9 =$	$(D_9 - D_8) / (t_9 - t_8) =$	
Obs 10 (optional)		$D_{10} =$	$(D_{10} - D_9) / (t_{10} - t_9) =$	
Average daily motion of Moon	[leave blank]	[leave blank]	Average of the above: 8.45	[leave blank]

Part 4: The Moon's rate of change and orbital period

(7) What average daily motion do you measure in degrees per day?

8.45 degrees per day, using the reference point

(8) What is your inferred orbital period of the Moon?

360 degreees / 8.45 degrees/day = 42.6 days

(9) How does this compare to the actual value of 27.3 days? Why do you think these values differ?
(Note: you will not be graded on the accuracy of your orbital period. Do not adjust any of your measurements to conform to the actual value! Instead, think about some of the reasons why your results may differ.)

It's incredibly inaccurate, so much so that I think we can do better with the data we have gathered.
More below. (Part 6)

Part 5: Where is the ecliptic?

(1) Use your trajectory of the moon to sketch where the ecliptic is in your local sky. Include your reference point and any other relevant landmarks. Recall that the Moon roughly follows the ecliptic plane.

This compostie image is going to come in super handy later.

(11) Is the ecliptic plane fixed in your local sky throughout the year, or does it vary over time? Why?

No. The ecliptic plane varies in our local sky throughout the year because Earth's orbit and axial tilt cause the Sun's apparent path to change relative to the observers horizon.

(12) Do you see any other objects on the ecliptic plane that may be planets? Where are they located? Feel free to make a sketch or describe in words.

Not in my images. I never considered other objects and didn't look for them by eye either while taking the measurements.

Part 5: Using Kepler's Third Law

(13) Using the orbital period for the Moon you derived in Part 2 and the average distance to the Moon (3.84×10^5 km), derive the mass of Earth in kg. Compare this to the actual mass of Earth (6.0×10^{24} kg).

Since the mass of the Earth is much larger than the mass of the Moon, we can rearange Kepler's 3rd Law and simplify to remove the mass of the Earth, resulting in this formula:

M_e = (4 pi^2 r^3) / (G T^2)

Plugging in the avg distance of the Moon, G, and 42.6 days (in seconds), we get:

2.47 * 10^24 kg, once again, hilariously inaccurate.

Please read further, there is more. ↓

Part 6: Pursuit of Accuracy

As I have hinted at throughout this report, the main source of inaccuracy in my measurements is the reference point. I understand it is required if someone was using the hand method of measuring angles but we are not doing that. We can measure the movement of the moon night to night directly. But before that I want to talk about the piece of code that made all of this possible.

Code Explanation

The bulk of the code, which I've linked to below, is for processing the image, handling user input, and displaying the results. These things are not relevant to this class. I more want to talk about the geometry and math aspects of the program.

```python
def calculate_angular_distance(p1, p2):
    x1, y1 = p1
    x2, y2 = p2

    # Convert to spherical coordinates
    theta1 = (x1 / WIDTH) * 360
    phi1 = 90 - (y1 / HEIGHT) * 180

    theta2 = (x2 / WIDTH) * 360
    phi2 = 90 - (y2 / HEIGHT) * 180

    # Convert to radians
    theta1_rad = math.radians(theta1)
    phi1_rad = math.radians(phi1)
    theta2_rad = math.radians(theta2)
    phi2_rad = math.radians(phi2)

    # Spherical law of cosines
    cos_dsigma = (
        math.sin(phi1_rad) * math.sin(phi2_rad) +
        math.cos(phi1_rad) * math.cos(phi2_rad) * math.cos(theta1_rad -
            theta2_rad)
    )
    dsigma_rad = math.acos(cos_dsigma)

    # Convert to degrees
    angular_distance = math.degrees(dsigma_rad)

    return angular_distance
```

Listing 1: Calculate Angular Distance

The program accepts images with an aspect ratio of 2:1. In my case, my 360 camera produces images 6080 wide and 3040 tall. This size corresponds 360 degrees side to side and 180 degrees top to bottom, a full sphere.

The first part of this function "Convert to spherical coordinates" translates pixel coordinates from the flattened image to spherical coordinates. Note that the image is captured as a sphere and the program treats it as a sphere. The distortions we see in the flattened images are due to projecting that sphere to a flat plane, similar to the well known distortions from the Mercator projection of the Earth to a flat map. These distortions do not affect the final result.

The second part "Convert to radians" is self explanatory. To be able to do trigonometry, we need the coordinates to represent angles rather than points. The math library of Python conveniently does this for me.

Next, we employ the spherical law of cosines. The law of cosines relates the sides of a triangle projected onto a sphere, meaning one with curved sides. It states that:

$$\cos c = \sin a * \sin b * \cos C + \cos a * \cos b$$

Where a, b, c and C are:

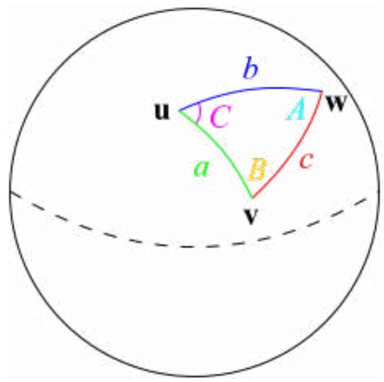


Figure 1: Spherical Law of Cosines
Wikipedia, Spherical law of Cosines

By using the elevation angles ϕ of the two points as a and b, and the difference in azimuth angles θ as angle C, we calculate the spherical arc c.

Finally, we take the arccosine of this length, and convert back to degrees to get our final angular distance.

The full code, along with all gathered data can be found here:

https://github.com/DemirInce/Angular_Distance

A Better Way of Measuring

Since we have all these images from a relatively fixed position, we can create a composite image by overlaying each nights measurement and see the entire path of the Moon over the measurement period.

Figure 2: Composite Image

We can then measure the individual angular differences between the Moon's position each night. I will skip the approximate position of the missing night for this and divide the angle there by 2 to average the movement.

Figure 3: Composite Measurements

Recalculation

We can already see that these numbers are much closer to the average daily movement of the Moon across the sky of:

$$360 \text{ degrees } /27.3 \text{ days } = 13.2 \text{ degrees}$$

- Orbital Period

Average Motion $= (13.01 + (22.66/2) + 12.16 + 10.48)/4 = 11.75$

Orbital Period $= 360/11.75 =$ **30.6 days**

This is much better. We have reached an error of 12%, not great but much better than the 56% from earlier. This remaining error is mostly due to the fact that most of my measurements were taken close to the apogee of the Moon, meaning it was traveling slower than average. We can fix this by multiplying our average motion by a correction factor based on the eccentricity of the Moon, and it's angular velocity near apogee.

- A Total Waste of Time

This works thanks to Kepler's 2nd Law: equal areas in equal times. We know that angular velocity w is inversely proportional to the square of the Moon's distance to Earth:

$$w \propto \frac{1}{r^2}$$

- At apogee the distance is $r_a = a(1+e)$, where e is the eccentricity.
- On average the distance is $r_m = a$.
- So, the ratio of angular velocity between mean and apogee is $(\frac{a}{a(1+e)})^2$.
- After we plug this into the proportion above, and the a's cancel out, we are left with this factor: $(1+e)^2$. Plugging in the known eccentricity of the Moon, we get: $(1+0.0549)^2 = 1.1138$.

$$\text{Orbital Period } = 360/(11.75 * 1.1138) = \textbf{27.5 days}$$

If only you could see the smile on my face. We have now reached an error of **0.73**%. The remaining error is down to the missing night, the slight variation in measurement time (± 2 minutes), and the slight variation in my camera placement each night. I am satisfied with this result.

- Note: This part is heavily ChatGPT aided. I think it is fair given how good the result is, and I fully understand how it works.

- Using Kepler's Third Law

Finally, let's repeat our calculations for the mass of the Earth with our *much* improved orbital period estimation. Using the formula we derived above:

$$M_E = \frac{4\pi^2 r^3}{GT^2}$$

We get:

$$M_E = \frac{4\pi^2(3.84 \times 10^8)^3}{(6.674 \times 10^{-11})(27.5 * 24 * 60 * 60)^2} = \mathbf{5.94 \times 10^{24} kg}$$

This represents an error of 1%, a staggering improvement upon the earlier 58.8%.

I think I am done.

Closing Thoughts

Admittedly, this assignment went completely off the rails. This started with me being too lazy to learn how to measure angular distance with my hands. In true engineer fashion, the solution to my problem took much more time and effort than simply dealing with the problem. I could've stopped at any point, and produced a result that satisfies the basic requirements of the assignment. However, the use of my program to measure distances opened up more and more avenues for optimization. In the end, I am glad I pursued these avenues, given the result I achieved. If anything, I hope this has produced a fun read. Please feel free to try out the program, and reach out to me if you have any questions about any of this.