Aerospace Design Project - Initial Concept Design

Team 13

[redacted]

1 Part I - Detailed Solutions

1.1 Question 1 - Analysis of the Ariane 5 - ECA

a, b)

The first stage of the Ariane 5 ECA launcher consists of two boosters coupled to the main core. After the boosters are depleted of fuel, the second stage begins once they are decoupled, with the main core continuing to provide thrust. Finally, the third stage is the upper cryogenic stage. The total trust for the Ariane 5 ECA launcher is calculated below:

$$T = (6470 \times 2) + 960 = 13900\ kN$$

We can calculate the start masses for LEO and GTO missions:

$$M_{LEO} = (184700 + (2 \times 268000) + 19440 + 21000 + 500 + 2000 = 763640\ kg$$
$$M_{GTO} = (184700 + (2 \times 268000) + 19440 + 10500 + 500 + 2000 = 753140\ kg$$

Given that the trust-to-weight ratio is $\frac{T}{(W \times 9.81)}$, for LEO and GTO missions,

$$\frac{T}{W}_{LEO} = \frac{(13900 \times 10^3)}{(763640 \times 9.81)} = 1.855$$
$$\frac{T}{W}_{GTO} = \frac{(13900 \times 10^3)}{(763640 \times 9.81)} = 1.881$$

c, d, e)

The equation for structural efficiency, σ is defined below:

$$\sigma = (\frac{M_s}{M_s + M_f}) \tag{1}$$

where M_s is the structural mass and M_f is the mass of fuel and propellant. Based on the table of data for the Ariane 5 ECA launcher, the structural efficiency of the core, upper stage, and the solid-propellant boosters are calculated:

	Core	**SRB**	**Upper**
M$_s/kg$	14,700	30,200	4,540
M$_f/kg$	17,000	237,800	14,900
σ	0.0796	0.1127	0.2335

Table 1: Structural and propellant masses of the Core, SRB and Upper stages, with the calculated structural efficiency for each stage.

f)

Stage 1:

$$Ve = \frac{T}{m} \tag{2}$$

m. booster =

$$\frac{(2 \times 237800)}{130}$$

= 2658.5 kg/s

m. core =

$$\frac{170000}{540}$$

= 314.81 kg/s

$$m.total = 314.81 + 3658.5 = 3973.31 kg/s$$

$$V_{e1} = \frac{(13900 \times 10^3)}{3973.31} = 3498.34 m/s$$

$$\mu = \frac{763640}{(763640–(2 \times 237800)–(130 \times 314.81)} = 3.09$$

$$V_1 = 3498.34 \times ln(3.09) = 3946.98 m/s$$

Stage 2:

$$V_e = Ispvac \times 9.81$$

$$V_e = 432 \times 9.81 = 4237.92 m/s$$

$$M_2 = 184700 - (130 \times 314.81) + 19440 + 21000 + 2000 + 500 = 186714.7 kg$$

$$M_{2Bo} = 147000 + 19440 + 21000 + 500 + 2000 = 57640 kg$$

$$\Delta V_2 = 4237.92 \times ln(\frac{186714.7}{57640}) = 4981.1 m/s$$

Stage 3:

$$V_e = 446 \times 9.81 = 4375.26 m/s$$

$$M_3 = 19440 + 21000 + 500 = 40940 kg$$

$$\Delta V_3 = 4375.26 \times (ln(\frac{40940}{(40940 - 14900)}) = 1979.69 m/s$$

Final Velocity:

$$Total \Delta V = 3946.98 + 4981.1 + 1979.69 = 10907.77 m/s$$

g)

Stage 1:

GTO Payload;

$$M_1 = 753140kg$$

$$T = 13900kN$$

$$V_e = 3498.34m/s$$

$$\mu = \frac{753140}{(753140 - (2 \times 237800) – (130 \times 314.81)} = 3.183$$

$$V_1 = 3498.34 \times ln(3.183) = 4050.44m/s$$

Stage 2:

$$V_{e2} = 4237.92m/s$$

$$M_2 = 184700 – (130 \times 314.81) + 19440 + 200 + 500 + 10500 = 176214.7kg$$

$$M_{Bo2} = 14700 + 19440 + 2000 + 500 + 10500 = 47140kg$$

$$V_2 = 4237.92 \times ln(\frac{176214.7}{47140}) = 5588.04m/s$$

Stage 3:

$$V_e = 440 \times 9.81 = 4375.26m/s$$

$$M_3 = 19440 + 500 + 10500 = 30440kg$$

$$M_{3Bo} = 4540 + 500 + 10500 = 15540kg$$

$$V_3 = 4375.26 \times ln(\frac{30440}{15540}) = 2941.66m/s$$

Final Velocity:

$$Total\Delta V = 4050.44 + 5588.04 + 2941.66 = 12580.14m/s$$

Question 2 - Analysis of new Ariane 6 system:

a)

$$M_s = \frac{M_f}{((1/efficiency) - 1)}$$

$$EfficiencyCore = 0.0796$$

$$M_s = \frac{140000}{((1/0.0796) - 1)} = 12107.78kg$$

$$LaunchMass = 12107.78 + 140000 = 152107.78kg$$

b)

$$BoosterEfficiency = 0.113$$

$$M_s = 142000/((1/0.113) - 1) = 18090.2kg$$

$$LaunchM = 160090.19kg$$

c)

Booster With Extra Fuel

$$M_s = 156000/((1/0.113) - 1) = 19873.73kg$$

$$LaunchM = 175873.7kg$$

d)

$$UpperEfficiency = 0.2335$$

$$M_s = 31000/((1/0.2335) - 1) = 9443.57kg$$

$$LaunchM = 40443.57kg$$

e)

A62 P120C:

$$TotalM = 152107.78 + (160090.19 \times 2) + 40443.57 + 500 + 2000 + 10350 = 525581.73kg$$

f)

A62 P120C+:

$$TotalM = 152107.78 + (175873.7 \times 2) + 40443.57 + 500 + 2000 + 10350 = 557148.75kg$$

g)

$$A62T/W = \frac{((2 \times 4500 + 960) \times 10^3)}{525581.73} = 1.932$$

$$A62 + T/W = \frac{(9960x10^3)}{557148.75} = 1.822$$

h)

A64 P120C:

$$LaunchM = 152107.78 + (160090.19 \times 4) + 40443.57 + 500 + 2000 + 21650 = 857062.11kg$$

i)

A64 P120C+:

$$LaunchM = 152107.78{+}(175873.7{\times}4){+}40443.57{+}500{+}2000{+}21650 = 920196.15kg$$

j)

$$A64T/W = \frac{((4x4500 + 960)x10^3)}{857062.11} = 2.255$$

$$A64 + T/W = \frac{(18960x10^3)}{920196.15} = 2.1$$

k)

[A62 P120C] Stage 1:

$$m.core = 311.111kg/s$$

$$m.Booster = 1651.16kg/s$$

$$V_{e1} = \frac{(9960 \times 10^3)}{(1651.16 \times 2) + 311.111} = 2756.38m/s$$

$$\mu = \frac{525581.73}{(525581.73 – (142000 \times 2) - (86 \times 311.111)} = 2.447$$

$$\Delta V_1 = 2756.383 \times ln(2.447) = 2460.07m/s$$

Stage 2:

$$V_{e2} = 429 \times 9.81 = 4208.49m/s$$

$$M_2 = 152107.78 – (86 \times 311.111) + 40443.57 + 500 + 2000 + 10350 = 178645.804kg$$

$$M_{2Bo} = 152107.78 – 140000 + 40443.57 + 500 + 2000 + 10350 = 65401.35Kg$$

$$V2 = 4208.49 \times ln(\frac{178645.804}{65401.35}) = 4228.95m/s$$

Stage 3:

$$V_e = 465 \times 9.81 = 4501.65m/s$$

$$M_3 = 40443.57 + 500 + 10350 = 51293.57kg$$

$$M_{3Bo} = 51293.57–31000 = 20293.57kg$$

$$\Delta V_3 = 4501.65 \times ln(\frac{51293.57}{20293.57}) = 4174.21m/s$$

Final Velocity:

$$\Delta V total = 2460.07 + 4228.95 + 4174.21 = 10863.23m/s$$

[A62 P120C+]

Stage 1:

$$m.core = 311.111kg/s$$

$$m.boosterwithextrafuel = 156000/94.5 = 1650.79kg/s$$

$$V_{e1} = (\frac{(9960 \times 10^3)}{2} \times 1650.79) + 311.111 = 2756.95m/s$$

$$M_1 = 557148.75kg$$

$$M_{1Bo} = 557148.75–(2 \times 156000) - (94.5 \times 311.111) = 215748.76kg$$

$$\mu = \frac{557148.75}{215748.76} = 2.582$$

$$\Delta V_1 = 2756.95 \times ln(2.582) = 2615.56m/s$$

Stage 2:

$$V_{e2} = 429 \times 9.81 = 4208.49m/s$$

$$M_2 = 152107.78–(94.5 \times 311.111) + 40443.57 + 500 + 2000 + 10350 = 176001.36kg$$

$$M_{2Bo} = 152107.78–140000 + 40443.57 + 500 + 2000 + 10350 = 65401.35kg$$

$$\Delta V_2 = 4208.49 \times ln(\frac{176001.36}{}65401.35) = 4166.19m/s$$

Stage 3:

$$V_{e3} = 465 \times 9.81 = 4561.65m/s$$

$$M_3 = 40443.57 + 500 + 10350 = 51293.57kg$$

$$M_{3Bo} = 51293.57–31000 = 20293.5kg$$

$$\Delta V_3 = 4561.65 \times ln(\frac{51293.57}{20293.5}) = 4229.84m/s$$

Final Velocity:

$$\Delta V total = 2615.56 + 4166.19 + 4229.84 = 11011.59m/s$$

l)

[A64 P120C]

Stage 1:

$$m.core = 311.111kg/s$$

$$m.Boost = 1651.163kg/s$$

$$V_{e1} = \frac{(18960x10^3)}{((4 \times 1651.163) + 311.111)} / = 2741.56m/s$$

$$M_1 = 857062.11kg$$

$$M_{1Bo} = 857062.11 = (4 \times 142000) – (86 \times 311.111) = 262306.56kg$$

$$\Delta V_1 = 2741.56 \times ln(\frac{857062.11}{262306.56}) = 3245.997m/s$$

Stage 2:

$$M_2 = 152107.78 – (86 \times 311.111) + 40443.57 + 2000 + 500 + 21650 = 189945.804kg$$

$$M_{2Bo} = 152107.78 – (140000 + 40443.57 + 2000 + 500 + 21650) = 76701.35kg$$

$$V_{e2} = 429 \times 9.81 = 4208.49m/s$$

$$\Delta V_2 = 4208.49 \times ln(\frac{189945.804}{76701.35}) = 3816.34m/s$$

Stage 3:

$$V_{e3} = 4561.65m/s$$

$$M_3 = 40443.57 + 500 + 21650 = 62593.57kg$$

$$M_{3Bo} = 62593.57 – 31000 = 31593.57kg$$

$$\Delta V_3 = 4561.65 \times ln(\frac{62593.57}{}31593.57) = 3118.84m/s$$

Final Velocity:

$$\Delta V_total = 3245.997 + 3816.34 + 3118.84 = 10181.18m/s$$

A64 P120C+
Stage 1:

$$m.core = 311.111kg/s$$

$$m.booster+ = 156000/94.5 = 1650.794kg/s$$

$$V_{e1} = \frac{(18960 \times 10^3)}{((1650.794 \times 4) + 311.111)} = 2742.15m/s$$

$$M_1 = 920196.15kg$$

$$M_{1Bo} = 920196.15–(4 \times 156000)–(94.5 \times 311.111) = 266796.16kg$$

$$\Delta V_1 = 2742.15 \times ln(\frac{920196.15}{266796.16}) = 3395.06m/s$$

Stage 2:

$$V_{e2} = 4208.49m/s$$

$$M_2 = 152107.78-(94.5\times311.111)+40443.57+500+2000+21650 = 187301.36kg$$

$$M_{2Bo} = 152107.78–(140000 + 40443.57 + 500 + 2000 + 21650) = 76701.35kg$$

$$\Delta V_2 = 4208.49 \times ln(\frac{187301.36}{76701.35}) = 3757.34m/s$$

Stage 3:

$$V_{e3} = 4561.65 m/s$$

$$M_3 = 40443.57 + 500 + 21650 = 62593.57 kg$$

$$M_{3Bo} = 62593.57 – 31000 = 31593.57 kg$$

$$\Delta V_3 = 4561.65 \times ln(\frac{62593.57}{31593.57}) = 3118.84 m/s$$

Final Velocity:

$$\Delta V total = 3395.06 + 3757.34 + 3118.84 = 10271.24 m/s$$

m)

Total ΔV :

$$A62P120C = 10863.23 m/s$$

$$A62P120C+ = 11011.59 m/s$$

$$A64P120C = 10181.18 m/s$$

$$A64P120C+ = 10271.24 m/s$$

When comparing the total ΔV between them all. We can see that with the added extra fuel in the booster increases the ΔV of the A62's by 148.36 m/s. while it increased the ΔV of the A64's by 90.06 m/s. the increase is greater in the A62's due to less of an inert mass from the Payload.

Question 3 - Analysis of the Pegasus XL system

The Pegasus XL launcher consists of three stages. The data for the entire system is tabulated below:

In addition to the inert mass, the first stage consists of a deltawing with a mass of about 1000 kg. For a LEO mission, the payload has mass of 443 kg, with the payload adapter mass included.

	Stage 1	Stage 2	Stage 3
Launch mass (kg)	16,383	4,306	872.3
Inert mass (kg)	1,369	391	102.1
Propellant mass (kg)	15,014	3,915	770.2
Mf/Ms	10.97	10.0	7.5
Isp (s)	295 (Vac.)	289	287
T (kN)	726 (Vac.)	158	32.7
Burn time (s)	68.6	71	67

Figure 1: Table of data for the Pegasus XL launcher

a)

Given $\frac{M_f}{M_s}$ in the table, we can use another method to calculate the structural efficiency:

$$\sigma = \left(\frac{1}{1 + \frac{M_f}{M_s}}\right) \tag{3}$$

By using this new equation (3), the structural efficiency for each stage is calculated:

$$\sigma_1 = \left(\frac{1}{1 + 10.97}\right) = 0.0835$$

$$\sigma_2 = \left(\frac{1}{1 + 10.00}\right) = 0.0909$$

$$\sigma_3 = \left(\frac{1}{1 + 7.500}\right) = 0.1180$$

Another parameter to calculate is the payload fraction, which has equation:

$$L = \left(\frac{M_p}{M_s + M_f}\right) \tag{4}$$

where M_p is the mass that each particular stage is carrying. Hence, by using this equation (4), the payload fraction for each stage can be calculated, as shown below:

$$L_1 = \left(\frac{4306 + 872.3 + 1000 + 443}{17383}\right) = 0.381$$

$$L_2 = \left(\frac{872.3 + 443}{4306}\right) = 0.305$$

$$L_3 = \left(\frac{443}{872.3}\right) = 0.508$$

	Structural Efficiencies	Payload Fraction
Stage 1	0.0835	0.404
Stage 2	0.0909	0.305
Stage 3	0.118	0.508

Table 2: Structural efficiencies and payload fractions (Question 3(a))

b)

The average propellant mass-flow rate can be found by applying the following equation:

$$\Delta m = \frac{M_f}{t} \tag{5}$$

where t is the burn time for each stage, measured in seconds. For each stage, the average propellant mass-flow rate can be found:

$$\Delta m_1 = \frac{15014}{68.6} = 218.9\ kg/s$$

$$\Delta m_2 = \frac{3915}{71} = 55.1\ kg/s$$

$$\Delta m_3 = \frac{770.2}{67} = 11.5\ kg/s$$

Furthermore, the exhaust gas velocity (2) for each stage can be calculated:

$$Ve_1 = \frac{726000}{218.9} = 3316.6\ m/s$$

$$Ve_2 = \frac{158000}{55.1} = 2867.5\ m/s$$

$$Ve_3 = \frac{32700}{11.5} = 2843.5\ m/s$$

	Average mass-flow rates (kg/s)	Estimate of exhaust gas velocity (m/s)
Stage 1	218.9	3316.6
Stage 2	55.1	2867.5
Stage 3	11.5	2843.5

Table 3: Average mass-flow rates and Estimate of exhaust gas velocity (Question 3(b))

c)

For maximum LEO payload, the speed increase for each stage can be computed. Although Ve data has been obtained above, Ve can also be calculated by multiplying the Isp by g (9.81 m/s^2). Using this value for Ve:

$$\Delta V = Ve \times ln(\frac{M_0}{M}) \tag{6}$$

where M_0 is the initial mass of the stage and M is the current mass, both in kg. Hence, for each stage:

$$\Delta V_1 = 2893.95 \times ln(\frac{23004.3}{23004.3 - 15014}) = 3060.2\ m/s$$
$$\Delta V_2 = 2835.09 \times ln(\frac{5621.3}{5621.3 - 3915}) = 3380.1\ m/s$$
$$\Delta V_3 = 2815.47 \times ln(\frac{1315.3}{1315.3 - 770.2}) = 2480.0\ m/s$$

From this data, we can obtain the total change in velocity:

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 = 3060.2 + 3380.1 + 2480.0 = 8920.3\ m/s$$

	Delta V
Stage 1	3060.2
Stage 2	3380.1
Stage 3	2480.0
Total	8920.3

Table 4: Delta V for Stages 1-3 (Question 3(c))

In calculating the velocity increase, the effect of decoupling the stages are neglected, and Isp values are used considering the launcher is already flying in a high enough altitude.

Question 4 - Analysis of Stratolaunch-based launchers:

a)

	Structural mass (kg)	Propellant mass (kg)
Stage 1	10888.13	170580.71
Stage 2	1008.85	15805.29

Table 5: Structural and Propellant masses for Stages 1 and 2 (Question 4(a))

Maximum payload mass =

$$1717.02kg$$

New V$_e increase =$

$$3.27ms^{-1}$$

b)

	Structural mass (kg)	Propellant mass (kg)
Stage 1	12839.41	170580.71
Stage 2	1064.38	14141.04

Table 6: Structural and Propellant masses for Stages 1 and 2 (Question 4(b))

Maximum payload mass =

$$1374.46kg$$

New V$_e increase =$

$$3.67ms^{-1}$$

c)

	Structural mass (kg)	Propellant mass (kg)
Stage 1	10858.94	144268.83
Stage 2	2436.32	32368.32
Stage 3	546.62	7262.19

Table 7: Structural and Propellant masses for Stages 1-3 (Question 4(c))

Maximum payload mass = $2258.77kg$

New V$_e increase =$

$$2.44ms^{-1}$$

d)

Question 4 has been answered using Python code - Jupyter Notebook.
Full code source can be found in the Appendix.

	Structural mass (kg)	Propellant mass (kg)
Stage 1	9280.17	123293.69
Stage 2	3128.63	41566.09
Stage 3	1054.76	14013.21
Stage 4	355.59	4724.28

Table 8: Structural and Propellant masses for Stages 1-4 (Question 4(d))

2 Part II - Initial Design Concept

2.1 Introduction

The evolving market of space exploration has resulted in increased competition from alternative providers like SpaceX. Acknowledging the evolving needs of the industry and the requirement for more better alternatives; the foundational pillars of our Reusable Launch Vehicle design consist of reusability, flexibility and cost-effectiveness. Successful innovation meeting these criteria will revolutionize future space access.

A thorough analysis of existing launch systems is key to planning initial concepts. Recognizing the success of Ariane 5, we can strategically utilise its proven technologies while incorporating advancements from Ariane 6. To add to this, our understanding can be further enriched by using the insights from *Space X*'s Super Heavy lower stage. Though the specifics of our Reusable Launch Vehicle design remain open, drawing inspiration from successful proven technologies of Ariane 5, Ariane 6 and *Space X*'s Super Heavy can be used effectively to further refine our final design concept.

We are focusing on adaptability by mirroring the 'family' concept mentioned in the design brief. This family will include a primary configuration optimised for delivering payloads to both Geostationary Transfer Orbit, GTO, and Low Earth Orbit, LEO. With this primary configuration is a secondary launcher which will be designed for efficient handling of smaller payloads, all while achieving cost reduction. Emphasising reusability as one of our key design foundational pillars, we can directly confront the economic challenges inherent within space exploration. Shared components are equally vital to the future of space access, due to the fact that cross compatibility can allow for a more universal approach to launchers. Again, this will assist with meeting the design brief's requirements for a 'family' concept. This will reduce manufacturing time/costs, while increasing potential operational efficiency. The engine selection process will involve a decisive balance between reliability and creativity. Two engines are required, one for each of the stages - we are considering a 2 stage primary launcher and a 2 stage secondary launcher. Factors to consider include, (but are not limited to): optimal performance, adaptability, maximum thrust, and efficiency. Performance estimates, a crucial aspect outlined in the design brief, will indicate that our proposed design met and exceeded the payload capabilities of Ariane

5.

Based upon the propellants used by each engine selection, we will be estimating structural efficiency for each of the stages while accounting for the additional mass introduced by features necessary for reusability. While reusability brings economic advantages, the associated structural mass introduces complexities to maintain the ideal equilibrium between performance and sustainability. Delta V performance estimates will support our design ensuring we can reach LEO and GTO. Calculating propellant properties will help to further optimize our design.

Our secondary launcher will be similar to our primary launcher but will have a smaller first stage due to the payload demand being lowered it will also be fully reusable but will have the advantage of being cheaper to run due to is reduced size. The secondary configuration will also be able to add in a second much smaller payload for smaller companies to use, saving both time and money.

2.2 Background Research

In the aerospace industry reusability has been a major factor when designing rockets as of late, this is due to the increasing cost of fully remaking launchers and the lack of any reusable parts.

The first reusable launcher was the Space Shuttle. Its unique design set it apart from rockets of its time; it resembled a plane more than an actual rocket. This design, however, was precisely what made it reusable. The plane-like structure allowed it to re-enter the atmosphere after completing its designated missions and land on a runway. But that wasn't the only reusable component. The side boosters used during its initial staging were also reusable. Once they had fully expended their propellant, they would parachute back down to the ground, where they could be collected and used again. This factor of reusability paved way for new companies and ideas to emerge to change the future of space travel.

Space X are the current industry leaders for reusable rockets. Their latest achievement is the Falcon 9, a partially reusable multi-stage rocket. The Falcon 9 is quintessentially and fundamentally shaped and designed like an ordinary rocket, however it has a major difference in that its first stage can re-centre itself upright, thereafter expending most of its fuel in a burn to reduce the descent velocity and eventually lands using retractable landing legs. This groundbreaking innovation has revolutionized the industry, enabling more cost-effective and swifter space missions.

Many other companies have been designing and testing new engines and types of reusability within the industry. For instance, Blue Origin have developed, created and tested a new sub-orbital launcher that is entirely reusable and suitable for space tourism. This type of launcher allows for a new type of industry to be created due to its reusability. Other companies such as '*China Aerospace*

Science and Technology Corporation' and '*Space X*' have been experimenting in making fully reusable multi-staged rockets. Each of these rocket have been made to allow for payloads to go to Low-Earth Orbit and have their stages come back to Earth and be reused for much cheaper than conventional payload delivery.

Our design takes inspiration from 2 current launchers, The Starship by *Space X* and The Ariane 6 by *Ariane Group*:

The Starships first stage uses an incredibly powerful rocket engine named '*Raptor*', we realised that if we wanted to make our launcher full reusable we had to use relatively new and extremely powerful engines otherwise we would have had no fuel left to allow for a reusable first stage. These engines also allow for a Δ V of around 9000 due to these engines producting an excess of thrust therefore allowing for a conservation of fuel, in turn allowing for a a re-landing. The Starship itself is a 2 stage fully reusable launcher that is powered by 33 *Raptor* engines in its first stage and 3 *Raptor* engines in its second stage. However the actual Starship is too immense for the project at hand so we have scaled down our version of needing to use that many *Raptor* engines by nearly half and also using less weight in general since our payload requirements are nowhere near the size of the Starships.

Ariane 5:

Ariane 5 was a heavy-lift space launch vehicle which retired in July 2023. Ariane 5 ECA was the most recent iteration of the launcher as specifically engineered to handle payloads weighing up to 9.6 tonnes into GTO. This helped maintain competitiveness in the commercial space transport sector - by offering customers the flexibility to launch a broader range of heavier satellites, Ariane 5 ECA addressed the evolving demands of the industry while contributing to the reduction of production costs. The successes of this space launch vehicle inspire our design to achieve simpler, more flexible and cost-effective solution to space access.

Ariane 5 ECA's upper stage comprises of the fairing protecting the payload at liftoff/atmospheric flight and Sylda 5 - the structure accommodating the upper/lower satellites. As the structural mass ratio for the Araine 6 is unavailable, we will be including the study of the Ariane 5 ECA in our analysis. Ariane 5 is a launcher that uses the same propellants as the Vinci engine for its upper stage. Therefore, the structural efficiency value of the Araine 5 will be a suitable alternative.

Specifications of Ariane 5 ECA	
Max. Height	53m
Max. Diameter	5.4m
Liftoff mass (double launch)	780 tonnes
Payload mass (into GTO)	10 tonnes
Number of Stages	2

Table 9: General Specifications of Ariane 5 ECA

Ariane 6:

Ariane 6, developed as a cost-effective successor to Ariane 5, aimed to build on the achievements of its predecessor while meeting the changing demands of the commercial space launch market. The objective was to cut launch costs in half compared to Ariane 5, increase annual launch capacity, and compete with lower-cost options like SpaceX Falcon 9. Ariane 6 had an estimated cost per launch of around US95$million in 2014, compared to US$57 million for smaller payloads on SpaceX Falcon 9. The rocket features hydro-lox engines in two of its stages, with an upgraded Vulcain engine in the first stage and a newly designed Vinci engine in the second stage. Originally designed for single payloads, the rocket now offers the capability to launch both heavy and light payloads to various orbits. A ride-share multi-launch service facilitates cost-effective access to space for smaller satellites, benefiting applications such as telecommunications and navigation.

General Specifications Of The Ariane 6	
Height	63m
Diameter	5.4m
Mass	530–860 tonnes
Number of stages	2

Table 10: General Specifications of Ariane 6

After carefully researching the Ariane 6 we were particularly interested in the upper/second stage of Ariane 6 – called the Upper Liquid Propulsion Module. We determined the Vinci engine to be a suitable engine for the second stage of our design.

Specifications of Upper Liquid Propulsion Module	
Diameter	5.4m
Engine	Vinci
Propellant	Liquid Hydrogen/Oxygen
Propellant mass	31000kg
Max thrust	180kN

Table 11: Specifications of ULPM

The Vinci Engine is the latest generation cryogenic upper stage engine designed for space launchers. It operates on liquid hydrogen and liquid oxygen and utilizes an expanded cycle, eliminating the requirement for a gas generator to drive oxidizer/fuel pumps. In contrast to the gas generator cycle, where exhaust products are expelled overboard, the expanded cycle ensures all propellants contribute to the combustion chamber, optimizing thrust efficiency. A notable feature is the retracted nozzle, extending by 1.9m only after the upper stage separates from the rocket, increasing the engine's length during specific launch stages.

Dimensions of Vinci Engine	
Length	3.22m
Diameter	1.84m
Dry weight	≈ 550kg 160kg, excluding nozzle
Performance of Vinci Engine	
Thrust, vacuum	180kN
Chamber Pressure	6.08MPa
Specific Impulse, vacuum	457.2s
Propellant flow rate (Liquid Oxygen)	34.11kg/s
Propellant flow rate (Liquid Hydrogen)	5.59kg/s

Table 12: Dimensions and Performance Specifications of Vinci Engine

2.3 The Launcher Design

The launcher family will consist of a primary and secondary launcher in-order to fulfill different mission requirements.

	Primary Launcher	Secondary Launcher
LEO Payload (kg)	21000	10500
GTO Payload (kg)	10500	5250

Table 13: Table to show required payloads for both launchers

2.3.1 Primary Launcher

The objectives of the primary launcher comprises of being able to launch a maximum payload of 21000 kg for a LEO mission and 10500 kg for a GTO mission, similar parameters to that of the Ariane 5 ECA. To accomplish this, the primary launcher will consist of two stages: a core and an upper stage that is coupled with the payload.

2.3.2 Secondary Launcher

Similarly, the secondary launcher must be able to launch both a LEO mission payload and a GTO mission payload. However, in-contrast to the primary launcher, this must be approximately half of that required by the Ariane 5 ECA. As with the primary launcher, a two stage setup will be used, however, in-order to reduce unnecessary weight, the size of the second stage will be halved. Sharing a common core stage with the same engines would result in reduced launcher complexity and reduced costs in terms of research and development and relaunchability.

2.4 Engine Choice

The initial core stage will consist Raptor engines similar to that used in the SpaceX Starship. Using a combination of liquid oxygen and methane, sixteen of these engines will be used. The upper core will utilise Vinci engines as found in the the upcoming Ariane 6. For this stage, the propellants used would be a combination of liquid hydrogen and liquid oxygen.

Given that the maximum thrust of one Raptor engine is 2.26 MN at sea-level, the total thrust at launch can be found:

$$2.26 \times 16 = 36.16\ MN$$

A further calculation of the thrust-to-weight ratio is done later in the report to prove the design's ability to lift-off successfully.

2.5 Analysis of the Concept Design

2.5.1 Structural Efficiency

Given that the first stage of the concept design uses Raptor engines, we can estimate the structural efficiency based on data available for the SpaceX Starship. The first stage of the Starship has a structural mass of 200 tonnes, with a propellant mass of 3400 tonnes. From this we can find the ratio of propellant mass to structural mass:

$$\frac{M_{f1}}{M_{s1}} = \frac{3400}{200} = 17$$

With this, we can estimate the structural efficiency of the Starship's first stage. From the equation for structural efficiency (3):

$$\sigma_1 = (\frac{1}{1+17}) = 0.0556$$

The Starship is designed to be a fully-reusable rocket, therefore has reusability features housed in the design. Nevertheless, we can estimate a structural efficiency for our launcher design to be ≈ 0.07 to account for differences in the launcher design.

The propellant mass to structural mass ratio for the Ariane 6 is unavailable, so in-order to estimate a reasonable structural efficiency for the second stage, we can study a launcher that uses the same propellants as the Vinci engine. The upper stage of the Ariane 5 ECA houses similar propellants as that required for the Vinci engine, therefore the structural efficiency value of the Ariane 5 ECA will be used. Given that this value is 0.2335, a good estimate for the structural efficiency of the concept design's second stage would be ≈ 0.3, given the added mass due to reusability features.

2.5.2 Exhaust Gas Velocities

In light of the structural efficiencies of both the core and upper stages, we can conduct performance analysis on our launcher design. An important parameter to consider is the exhaust gas velocities for each stage. Given the data available for both the Raptor and Vinci engines, we know that the specific impulses for the Raptor engines are $363s$ in vacuum and $327s$ in sea-level. For the Vinci engine, a specific impulse of $457s$ in vacuum is provided. From this, the exhaust gas velocities are calculated as follows:

$$Ve_1 = \frac{1}{2}(363 + 327) \times 9.81 = 3384.45 \ ms^{-1}$$
$$Ve_2 = 457 \times 9.81 = 4483.17 \ ms^{-1}$$

The average specific impulse is used for the first stage in-order to calculate the exhaust gas velocity given that the values in vacuum and sea-level are provided.

2.5.3 LEO Mission Capability - Primary Launcher

As stated above, the primary launcher must be able to carry a payload of 21000 kg for a LEO mission. Provided that the ΔV required for a LEO mission is about 7.9 kms^{-1}, by simple algebraic manipulation of the Tsiolkovsky Equation (6), we obtain the mass ratio:

$$\mu = exp(\frac{\Delta V}{Ve}) \tag{7}$$

Additional derivations of the mass ratio will be used:

$$\mu = \frac{M_s + M_f + M_p}{M_s + M_p} \tag{8}$$

$$\mu = \frac{M_s(1 + \frac{M_f}{M_s}) + M_p}{M_s + M_p} \tag{9}$$

The total ΔV that includes both values for the core and upper stage must be at least sufficient enough for a LEO mission. Firstly, we calculate the ΔV_2, the upper stage. As our structural efficiency estimates stems from existing data of launchers, we assume the structural mass based from stages that use the same engines in our design, for now.

The structural mass of the upper stage of the Ariane 5 ECA is 4540 kg. We assume that the structural mass of our launcher design's second stage will be about 5000 kg to account for new features. Given this fact, the mass ratio of the second stage can be found:

$$\mu_2 = \frac{5000(1 + 3.28) + 21000}{5000 + 21000} = 1.63$$

Hence, from this value, the expected ΔV_2:

$$\Delta V_2 = 4483.17 \times ln(1.63) = 2190.4 \ ms^{-1}$$

Therefore, we know that the velocity increase for the core stage must be about:

$$7900 - 2190.4 = 5709.6 \ ms^{-1}.$$

It is important to realise that this estimate negates the effects of losses due to gravity and aerodynamics, hence it would be ideal to achieve a total ΔV higher than this. For the purpose of our analysis, we will try and achieve a total ΔV value of at least 8500 ms^{-1}.

In-order to calculate the mass ratio of the core stage, we need to estimate the propellant mass of the second stage. Given that for the second stage. $\frac{M_f}{M_s} = 3.28$, and the estimate for the structural mass being 5000 kg, then:

$$M_{f2} = 3.28 \times 5000 = 16400 \ kg$$

Bearing this in mind, we can calculate the mass ratio of the core stage. The structural mass of the Starship's core stage is 200000 kg, however 33 Raptor engines are being used. To reduce weight and complexity, we will half the structural mass assuming that we would use at most half the number of engines on the Starship's first stage. By assuming our design will have a structural mass of 100 tonnes,

$$\mu_1 = \frac{100000(1+17)+5000+16400+21000}{100000+5000+16400+21000} = 12.94$$

With the value of Ve_1 above, we can estimate the increase in velocity for the core stage:

$$\Delta V_1 = 3384.455 \times ln(12.94) = 8665.286\ ms^{-1}$$

Hence, the total ΔV for the primary launcher with a LEO payload will be:

$$\Delta V = \Delta V_1 + \Delta V_2 = 8665.286 + 2190.4 = 10855.686 ms^{-1}$$

This value is greater than what is required, however reusability is key feature in our launcher design. The propellant mass estimates are bigger than what is required for the payload to reach orbit itself, however with the plan for each stage to return back to Earth, the extra propellants would be of use. Given $\frac{M_f}{M_s} = 17$ for the core stage, the mass of propellants would be:

$$M_{f1} = 17 \times 100000 = 1700000\ kg$$

With the estimated values of masses calculated, we can do a calculation to ensure that the design is capable of launching in the first place. A fundamental requirement for this to be met is that the thrust-to-weight ratio of the launcher must exceed 1. This is the case with the primary launcher in LEO mission configuration, as shown below:

$$\frac{T}{W} = \frac{36\ 160\ 000}{17\ 142\ 400} = 2.11$$

2.5.4 GTO Mission Capability - Primary Launcher

Along with LEO missions, the objective of the primary launcher is to be able to carry a payload size of 10500 kg for GTO missions. Provided that the ΔV required for a GTO mission is about 10.3 kms^{-1}, we can estimate the total ΔVs required in the same way (7) (8) (9) that was done for the LEO missions.

For the second stage:

$$\mu_2 = \frac{5000(1+3.28)+10500}{5000+10500} = 2.06$$

$$\Delta V_2 = 4483.17 \times ln(2.06) = 3240\ ms^{-1}$$

For the first stage:

$$\mu_1 = \frac{100000(1+17)+5000+16400+10500}{100000+5000+16400+10500} = 13.89$$

$$\Delta V_1 = 3384.455 \times ln(13.89) = 8905.07\ ms^{-1}$$

Hence, the total ΔV for a GTO mission would be:

$$\Delta V = \Delta V_1 + \Delta V_2 = 8905.07 + 3240 = 12145.07\ ms^{-1}$$

As with the LEO mission, the total ΔV achieved for the GTO mission far surpasses than that required.

2.5.5 LEO Mission Capability - Secondary Launcher

As seen earlier, the requirements for the secondary launcher is to be able to carry a payload of at least 10500 kg for a LEO mission. With the second stage being half the size of that in the primary launcher, the mass ratio of the second stage will be different:

$$\mu_2 = \frac{2500(1+3.28)+10500}{2500+10500} = 1.63$$

Similar to the method seen above, we can now find the ΔV of this new second stage:

$$\Delta V_2 = 4483.17 \times ln(1.63) = 2190.39\ ms^{-1}$$

As the secondary stage has the same structural efficiency, the propellant mass of this new second stage can be estimated:

$$M_{f2} = 3.28 \times 2500 = 8200\ kg$$

As the core stage has the same structural mass and structural efficiency, the propellant mass will be the same (1700000 kg). From this, we can calculate the mass ratio of the first stage, and thereafter ΔV_1:

$$\mu_1 = \frac{100000(1+17)+8200+2500+10500}{100000+8200+2500+10500} = 15.02$$

$$\Delta V_1 = 3384.45 \times ln(15.02) = 9169.77\ ms^{-1}$$

Given the velocity increase of both stages, the total ΔV of the secondary launcher for a LEO mission can be found, which, as seen from the primary launcher, will exceed the ΔV required:

$$\Delta V = \Delta V_1 + \Delta V_2 = 9169.77 + 2190.39 = 11360.16\ ms^{-1}$$

2.5.6 GTO Mission Capability - Secondary Launcher

A GTO mission is also feasible with this secondary design. Given that the requirements for the launcher is to carry a payload of about 5250 kg, the mass ratio and subsequently ΔV_2 is calculated:

$$\mu_2 = \frac{2500(1 + 3.28) + 5250}{2500 + 5250} = 2.06$$

$$\Delta V_2 = 4483.17 \times ln(2.06) = 3240.01\ ms^{-1}$$

And with this, the mass ratio of the first stage and thereafter ΔV_1 can be found:

$$\mu_1 = \frac{100000(1 + 17) + 8200 + 2500 + 5250}{100000 + 8200 + 2500 + 5250} = 15.66$$

$$\Delta V_1 = 3384.45 \times ln(15.66) = 9310.99\ ms^{-1}$$

Finally, for the total ΔV:

$$\Delta V = \Delta V_{1+} \Delta V_2 = 9310.99 + 3240.01 = 12551\ ms^{-1}$$

2.5.7 Propellant Tank Volumes

By estimating the mass of propellant for the primary and secondary launchers, the dimensions of the tanks housing the fuel and oxidizers can be found, and from that the dimensions of the stages themselves.

The following data of the Raptor engine to be used in the core stage is given. From this we can approximate the dimensions of the propellant tanks and the core stage itself.

Raptor Engine Propellant Data	
Fuel	Liquid Methane
Oxidizer	Liquid Oxygen
Dry weight	1600 kg
Propellant Mass Estimate	1 700 000 kg
Oxidizer/Fuel Ratio	3.8

Table 14: Propellant data for the Raptor Engine

Stage 1:

Determine masses:

mass of propellants required = 1700000kg
Fuel = Liquid Methane
Oxidiser = Liquid Oxygen
O/F = 3.8

$$M_f = \frac{M_p}{1 + O/F}$$

$$M_(oxidiser) = 3.8 \times M_f$$

$$M_f = \frac{1700000}{4.8} = 354166.67kg$$

$$M_(oxidiser) = 3.8 \times 354166.667 = 1345833.33kg$$

Determine Volumes:

$$V_f = \frac{M_f}{\rho_f} = \frac{354166.6667}{423} = 837.27m^3$$

$$V_f = \frac{M_(oxidiser)}{\rho_(oxidiser)} = \frac{1345833.333}{1141} = 1179.52m^3$$

Total Tank Volume:

$$V_(total) = V_(fuel) + V_(oxidiser)$$

$$V_(total) = 837.2734437 + 1179.520888 = 2016.79m^3$$

Similar to the core stage, the dimensions of the upper stage can also be approximated. The relevant data for the Vinci engine are provided, and from this the estimation of the upper stage's oxidiser and fuel dimensions can be carried out.

Vinci Engine Propellant Data	
Fuel	Liquid Hydrogen
Oxidizer	Liquid Oxygen
Dry weight	≈ 550kg 160kg, excluding nozzle
Propellant Mass Estimate (Primary Launcher)	16400 kg
Propellant Mass Estimate (Secondary Launcher)	8200 kg
Oxidizer/Fuel Ratio	6.1

Table 15: Propellant data for the Vinci Engine

Stage 2:
Determine masses:

mass of propellants required = 16400kg
Fuel = Liquid Hydrogen
Oxidiser = Liquid Oxygen
O/F = 6.1

$$M_f = \frac{M_p}{1 + O/F}$$

$$M_(oxidiser) = 6.1 \times M_f$$

$$M_f = \frac{16400}{7.1} = 2309.86kg$$

$$M_(oxidiser) = 6.1 \times 2309.86 = 14090.14085kg$$

Determine Volumes:

$$V_f = \frac{M_f}{\rho_f} = \frac{2309.89}{71} = 32.53m^3$$

$$V_f = \frac{M_(oxidiser)}{\rho_(oxidiser)} = \frac{14103.98}{1141} = 12.35m^3$$

Total Tank Volume:

$$V_(total) = V_(fuel) + V_(oxidiser)$$

$$V_(total) = 32.53 + 12.35 = 44.88m^3$$

Secondary launcher:

Stage 1:

Determine masses:

Mass of propellants required = 1700000kg
Fuel = Liquid Methane
Oxidiser = Liquid Oxygen
O/F = 3.8

$$M_f = \frac{M_p}{1 + O/F}$$

$$M_(oxidiser) = 3.8 \times M_f$$

$$M_f = \frac{1700000}{4.8} = 354166.67kg$$

$$M_(oxidiser) = 3.8 \times 354166.667 = 1345833.33kg$$

Determine Volumes:

$$V_f = \frac{M_f}{\rho_f} = \frac{354166.6667}{423} = 837.27m^3$$

$$V_(oxidiser) = \frac{M_(oxidiser)}{\rho_(oxidiser)} = \frac{1345833.333}{1141} = 1179.52m^3$$

Total Tank Volume:

$$V_(total) = V_(fuel) + V_(oxidiser)$$

$$V_(total) = 837.2734437 + 1179.520888 = 2016.79m^3$$

Stage 2:

Determine masses:

Mass of propellants required = 8200kg
Fuel = Liquid Hydrogen
Oxidiser = Liquid Oxygen
O/F = 6.1

$$M_f = \frac{M_p}{1 + O/F}$$

$$M_(oxidiser) = 6.1 \times M_f$$

$$M_f = \frac{8200}{7.1} = 1154.93kg$$

$$M_(oxidiser) = 6.1 \times 1154.93 = 7045.07kg$$

Determine Volumes:

$$V_f = \frac{M_f}{\rho_f} = \frac{1154.93}{71} = 16.27m^3$$

$$V_f = \frac{M_(oxidiser)}{\rho_(oxidiser)} = \frac{7045.07}{1141} = 6.17m^3$$

Total Tank Volume:

$$V_(total) = V_(fuel) + V_(oxidiser)$$

$$V_(total) = 16.27 + 6.17 = 22.44m^3$$

By comparing required tank volumes for each stage with the available space in each stage, we can confirm whether or not our propellant can fit.

Primary Launcher Stage 1:

$$V_(max.available) = \pi \times (diameter/2)^2 \times height$$

$$V_(max.available) = \pi \times (9/2)^2 \times 35.5 = 2258.41m^3$$

2258.41 m^3 > 2016.79 m^3.
Therefore propellant for Stage 1 fits.

Stage 2:

$$V_(max.available) = \pi \times (diameter/2)^2 \times height$$

$$V_(max.available) = \pi \times (5.4/2)^2 \times 20 = 458.04 m^3$$

458.04 m^3 > 44.88 m^3.
Therefore propellant for stage 2 fits.

Stage 1 Liquid Oxygen tank:

$$V = 4/3 \times \pi \times (r)^3 + L_(cyl) \times \pi \times r^2 = V_(oxidation)$$

$$V = 4/3 \times \pi \times (8.8/2)^3 + L_(cyl) \times \pi \times (8.8/2)^2 = 1179.520888$$

By rearranging this expression, we can find $L_(cyl)$:

$$L_(cyl) = 13.53 m$$

$$L_(total) = L_(cyl) + diameter$$

$$L_(total) = 13.53 + 8.8 = 22.33 m$$

Stage 1 Liquid Methane tank:

$$V = 4/3 \times \pi \times (r)^3 + L_(cyl) \times \pi \times r^2 = V_(fuel)$$

$$V = 4/3 \times \pi \times (8.8/2)^3 + L_(cyl) \times \pi \times (8.8/2)^2 = 837.27$$

By rearranging this expression, we can find $L_(cyl)$:

$$L_(cyl) = 7.90 m$$

$$L_(total) = L_(cyl) + diameter$$

$$L_(total) = 7.90 + 8.8 = 16.70m$$

For the second stages of both the primary and secondary launcher, the shape of the oxidizer and fuel tanks are spherical given that the volume of propellants are small enough that a cylindrical shape is not required. The volumes of these spherical tanks are calculated below:

Primary Stage 2 Liquid Oxygen tank:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r_{(LOX)} = \sqrt[3]{\frac{3 \times 32.53}{4\pi}} = 1.98m$$

$$L_{(Total)} = 2r = 1.98 \times 2 = 3.96m$$

Primary Stage 2 Liquid Hydrogen tank:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r_{(LH2)} = \sqrt[3]{\frac{3 \times 12.35}{4\pi}} = 1.43m$$

$$L_{(Total)} = 2r = 1.43 \times 2 = 2.86m$$

Secondary Stage 2 Liquid Hydrogen tank:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r_{(LH2)} = \sqrt[3]{\frac{3 \times 16.27}{4\pi}} = 1.57m$$

$$L_{(Total)} = 2r = 1.57 \times 2 = 3.14m$$

Secondary Stage 2 Liquid Oxygen tank:

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r_{(LOX)} = \sqrt[3]{\frac{3 \times 6.17}{4\pi}} = 1.14m$$

$$L_{(Total)} = 2r = 1.43 \times 2 = 2.28m$$

2.6 Sketches:

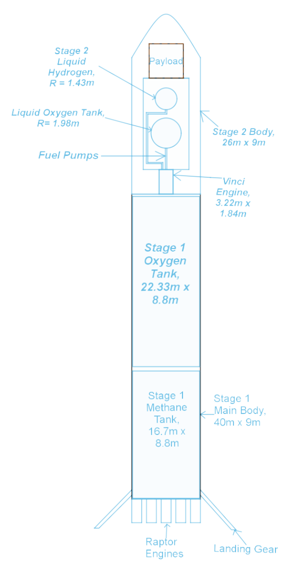

Figure 2: Side View Sketch of Primary Launcher

Our secondary rocket sketches would be very similar due to the fact that we are only changing the upper stage by halving its length. This means only one of the

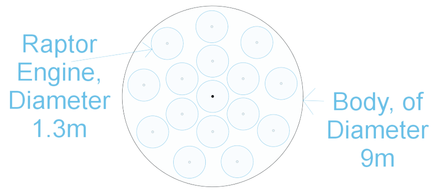

Figure 3: Base View to Show Engine Layout

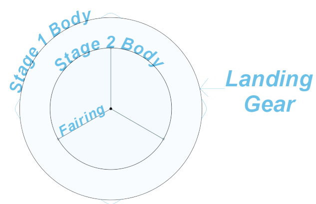

Figure 4: Top Down View of Primary Rocket

sketches truly show the difference between our primary and secondary rocket.

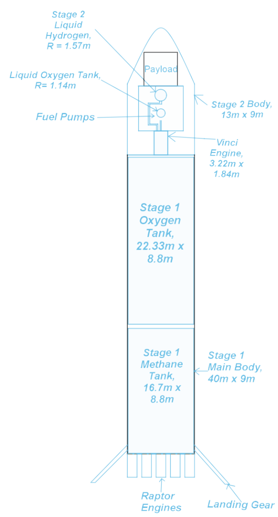

Figure 5: Side View Sketch of Secondary Launcher

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4 Appendix

Source Code (Part 1, Question 4):

```
import math

def stratolauncher(n, m_max, o, deltaV, Ve, m_p):
# define mass ratio
u = math.exp((1/n)*(deltaV/Ve))

# define payload ratio
L = (1 - o * u)/(u - 1)
```

```
m_ratio = (1 - o)/o

# Initialize the payload mass
m_pn = m_max

# Initialize a dictionary to store the results
results = {}

for i in range(1, n+1):
# Calculate the payload, structural, and propellant masses for each stage
m_pn = m_pn/((1/L) + 1)
m_sn = (m_pn * (1 - u))/(u - (1 + m_ratio))
m_fn = m_ratio * m_sn

# Store the results in the dictionary
results[i] = {'payload_mass': m_pn, 'structural_mass': m_sn, 'propellant_mass':
m_fn }

# Calculate new Ve
u2= (results[2]['structural_mass'] + results[2]['propellant_mass'] + m_p)/(results[2]['structural_mass']
+ m_p)
Ve_new = deltaV/math.log(u2)

# Print the results
print(f'Q4{chr(96+n)}')
print('Maximum payload mass: %.2f kg' % results[i]['payload_mass'])
print('Structural mass for stage 1: %.2f kg' % results[1]['structural_mass'])
print('Propellant mass for stage 1: %.2f kg' % results[1]['propellant_mass'])

for i in range(2, n+1):
print(f'Structural mass of stage {i}: %.2f kg' % results[i]['structural_mass'])
print(f'Propellant mass for stage {i}: %.2f kg' % results[i]['propellant_mass'])
print('New Ve increase: %.2f m/s\n' % (Ve_new - 3))
```

stratolauncher(2, 200000, 0.06, 11.5, 3, 2000)

stratolauncher(2, 200000, 0.07, 11.5, 3, 2000)

stratolauncher(3, 200000, 0.07, 11.5, 3, 2000)

stratolauncher(4, 200000, 0.07, 11.5, 3, 2000)