MATLAB Project - Wind Tunnel Data Processing

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Abstract

This report presents a method for processing data from a wind tunnel experiment on a model aeroplane, conducted at various pitch and yaw angles, to produce plots of the lift coefficient, drag coefficient, and slip force coefficient. The report concludes that using block matrices to process the data is an efficient approach to generating these plots in MATLAB. It also finds that, while the relationship between the slip force coefficient and the pitch angle is fairly linear, the relationship between the lift coefficient and pitch angle is quadratic.

1 Introduction

Wind tunnel experiments are a fundamental tool in aerodynamics, allowing researchers to analyse the forces acting on a model aircraft under controlled conditions. The wind tunnel experiment can give results for forces in x, y, and z axis, relative to the model at different yaw, pitch and roll angles. It is important to convert these forces to lift, drag, and side forces in the inertial frame and then compute their dimensionless coefficients.

This report explores a method for processing wind tunnel data collected at various pitch and yaw angles, with the aim of computing the lift coefficient, drag coefficient, and slip force coefficient. By employing block matrix operations in MATLAB, the study demonstrates an efficient approach to handling large data sets and generating accurate aerodynamic force plots.

This report is structured as follows. Section 2.1 discusses the theory behind obtaining aerodynamic force coefficients from relative body-frame forces. Section 2.2 expands on this by presenting a computationally efficient method for numerically computing these dimensionless coefficients for a large set of forces. Next, in Section 3, the results are presented, plotted, and discussed. Finally, Section 4 concludes the report.

2 Methodology and Theory

2.1 Theory

The wind tunnel experiment measures forces along the x, y, and z axes at different pitch and yaw angles in the **body frame**, represented as:

$$F_{\text{abs}} = \begin{bmatrix} F_{x,\text{abs}} \\ F_{y,\text{abs}} \\ F_{z,\text{abs}} \end{bmatrix} \tag{1}$$

where F_{abs}[N] is the absolute force vector in the **body frame**, and $F_{x,\text{abs}}$[N], $F_{y,\text{abs}}$[N], and $F_{z,\text{abs}}$[N] are its respective components. However, since the model is mounted on a stand, the stand also experiences some tare forces. To measure these forces, a wind tunnel experiment is first conducted without the model to determine the tare forces [1], F_{tare} [N], which are computed as:

$$F_{\text{tare}} = \begin{bmatrix} F_{x,\text{tare}} \\ F_{y,\text{tare}} \\ F_{z,\text{tare}} \end{bmatrix} \tag{2}$$

Therefore, the corrected body force vector F can be given as $F = F_{abs} - F_{tare}$ [1].

Since drag D[N], side force S[N], and lift L[N] are the forces along the x', y', and z' axes in the **inertial frame**, a rotation matrix R is required to transform vectors from the **body frame** to the **inertial frame** [2]. R can be defined using the relationship,

$$R = \begin{bmatrix} \cos(\beta)\cos(\alpha) & -\sin(\beta) & \cos(\beta)\sin(\alpha) \\ \sin(\beta)\cos(\alpha) & -\cos(\beta) & \sin(\beta)\sin(\alpha) \\ \sin(\alpha) & 0 & -\cos(\alpha) \end{bmatrix} \tag{3}$$

given that, α [rad] (angle of attack) $= \theta\ [rad]\ (pitch)$, and $\beta\ [rad] = -\psi\ [rad]\ (yaw)$. Therefore, the aerodynamic force vector can be computed as,

$$\underbrace{\begin{bmatrix} D \\ S \\ L \end{bmatrix}}_{F_{Aero}} = \underbrace{\begin{bmatrix} \cos(\beta)\cos(\alpha) & -\sin(\beta) & \cos(\beta)\sin(\alpha) \\ \sin(\beta)\cos(\alpha) & -\cos(\beta) & \sin(\beta)\sin(\alpha) \\ \sin(\alpha) & 0 & -\cos(\alpha) \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}}_{F_{body}} \tag{4}$$

where, $D[N]$, $S[N]$, $L[N]$ are the lift, side and drag forces, respectively.

The aerodynamic force vector F_{Aero} can be converted into a dimensionless force coefficient vector (C_F) using the dynamic pressure [3]

$$q_\infty = \frac{1}{2}\rho V^2 \text{ [Pa]} \tag{5}$$

and the wing area S [m^2], where ρ [$\text{kg}\cdot\text{m}^{-3}$] is the dry air density, and V [$\text{m}\cdot\text{s}^{-1}$] is the velocity of the incident air. Then, the coefficient vector can be given as,

$$C_F = \begin{bmatrix} C_D \\ C_S \\ C_L \end{bmatrix} = \frac{1}{q_\infty S} F_{Aero} \tag{6}$$

where C_D, C_S, and C_L are the coefficients of drag, side force and lift respectively.

2.2 Methodology

Section 2.1 introduced the computation of the C_F vector given θ and ψ. This section explains how C_F can be computed for all pitch angles, given F_{body} over a range of θ values while keeping ψ constant.

For a set of N pitch angles,

$$\vartheta = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_N \end{bmatrix} \tag{7}$$

all at a constant yaw angle ψ, the coefficient vectors are given by:

$$\begin{aligned} C_{F1} &= R_{\theta_1} F_1, \\ C_{F2} &= R_{\theta_2} F_2, \\ &\vdots \\ C_{FN} &= R_{\theta_N} F_N. \end{aligned} \tag{8}$$

This means that its is possible to compute all of these vectors in a single block matrix equation as,

$$\underbrace{\begin{bmatrix} C_{F1} \\ C_{F2} \\ \vdots \\ C_{FN} \end{bmatrix}}_{\mathcal{C}} = \underbrace{\begin{bmatrix} R_{\theta_1} & 0 & \dots & 0 \\ 0 & R_{\theta_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & R_{\theta_N} \end{bmatrix}}_{\Lambda} \underbrace{\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{bmatrix}}_{\mathcal{F}} \tag{9}$$

where, $\mathcal{C} \in \Re^{3N\times 1}$, $\Lambda \in \Re^{3N\times 3N}$, and $\mathcal{F} \in \Re^{3N\times 1}$. It should be noted that each $R_{\theta_i} \in \Re^{3\times 3}$, where $i = 1, \dots, N$. Also note that $\mathcal{F}$ is in the form of

$$\mathcal{F} = \begin{bmatrix} F_{x,1} \\ F_{y,1} \\ F_{z,1} \\ F_{x,2} \\ F_{y,2} \\ F_{z,2} \\ \vdots \\ F_{x,N} \\ F_{y,N} \\ F_{z,N} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} F_1 \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} F_2 \\ \vdots \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} F_N \end{matrix} \tag{10}$$

Besides, we can take the block vector transpose of $\mathcal{C}$, which results in each row representing the values of aerodynamic coefficients as the pitch angle varies, such as:

$$\mathcal{C}^\top = \begin{bmatrix} C_{D,1} & C_{D,2} & C_{D,3} & \dots & C_{D,N} \\ C_{S,1} & C_{S,2} & C_{S,3} & \dots & C_{S,N} \\ C_{L,1} & C_{L,2} & C_{L,3} & \dots & C_{L,N} \end{bmatrix}. \tag{11}$$

Finally, using MATLAB, we can plot the rows of $\mathcal{C}^\top$ against the pitch values in the ϑ vector. This *algorithm* can then be applied to the data for each yaw angle ψ, producing three graphs for each ψ.

3 Results and Discussion

All data have been plotted using MATLAB which can be found in my GitHub repository. It is always important to ensure that the data presented is accurate and well-structured. The dataset analysed in this report had certain issues that needed to be addressed. At the final pitch values, the wind tunnel experiment was potentially turned off, causing the airflow velocity to drop significantly. However, since the experiment assumes a constant velocity V, the final row of all data tables, including the tare data, was excluded. Additionally, the tare data table contained a repeated row, which caused misalignment in the number of values. Therefore, this row was also removed.

Figures (1), (2), (3) and (4) shows the plots of C_D vs θ (a), C_S vs θ (b), and C_L vs θ (c) respectively at yaw angles of 0°, 5°, 10° and 15° respectively.

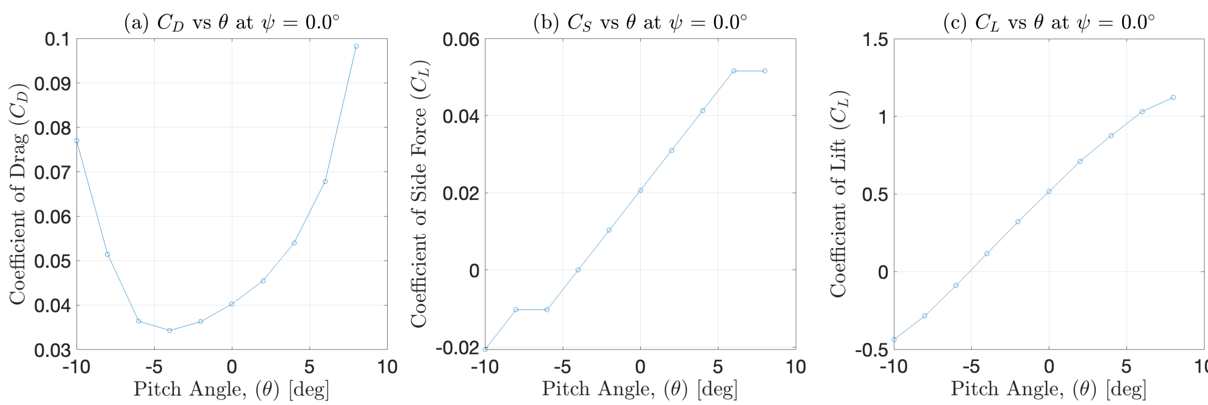

Figure 1: C_D, C_S, and C_L against pitch angle θ, at $\psi = 0°$.

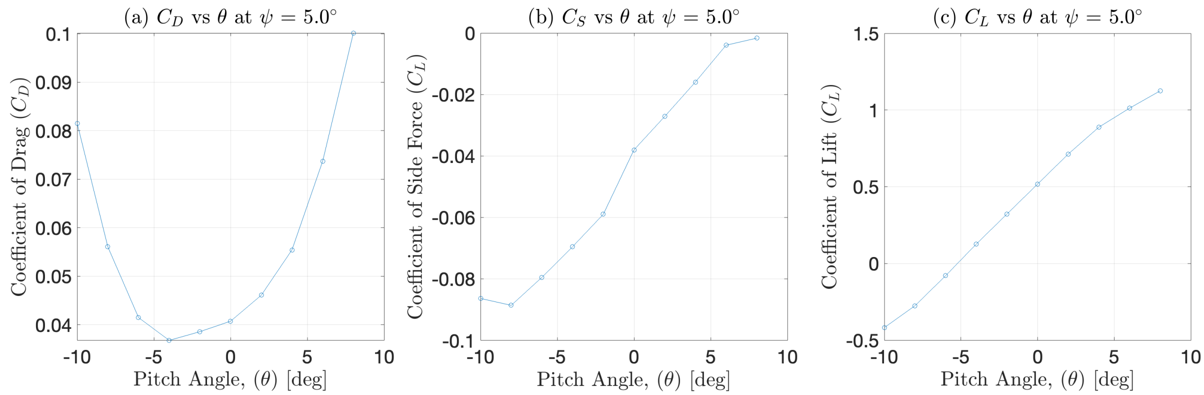

Figure 2: C_D, C_S, and C_L against pitch angle θ, at $\psi = 5°$.

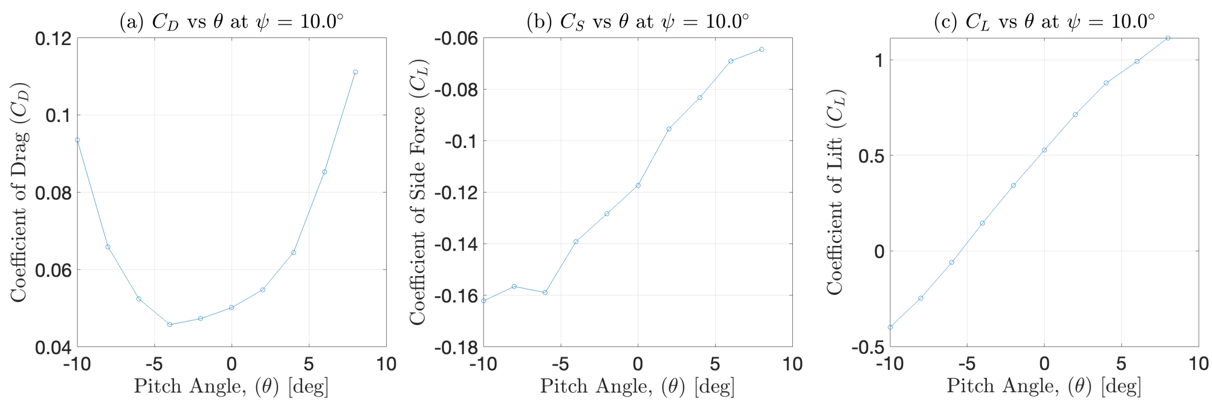

Figure 3: C_D, C_S, and C_L against pitch angle θ, at $\psi = 10°$.

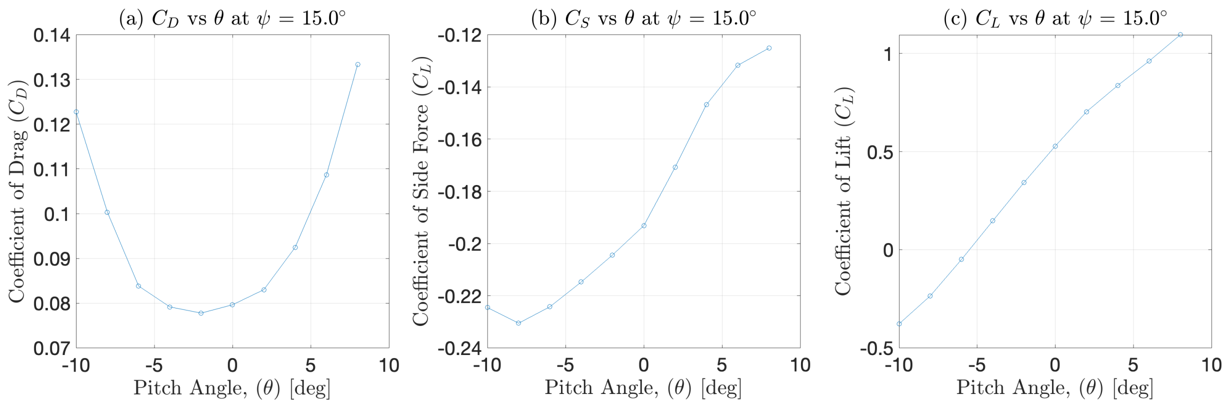

Figure 4: C_D, C_S, and C_L against pitch angle θ, at $\psi = 15°$.

Figure 5 presents the 3D plots of: (a) C_D against ψ and θ, (b) C_S against ψ and θ, (c) C_L against ψ and θ.

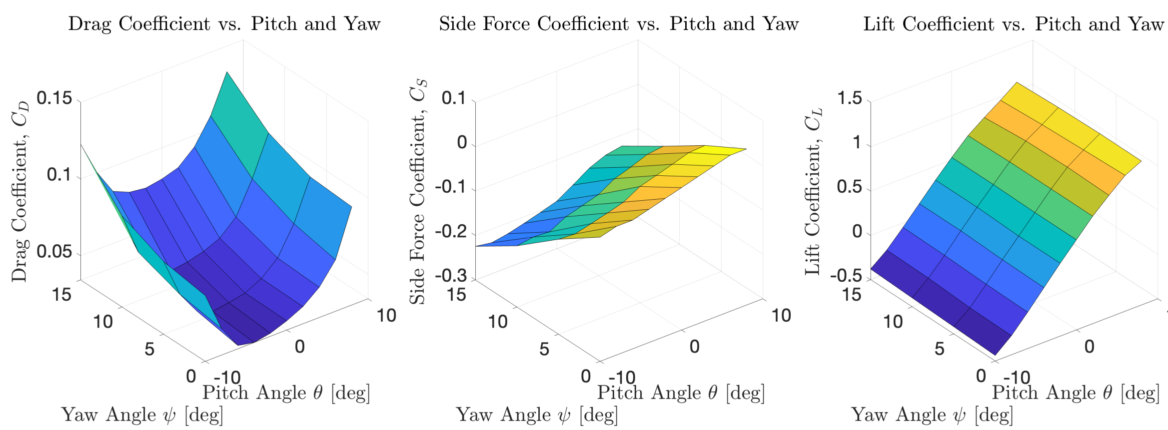

Figure 5: C_D, C_S, and C_L against pitch angle θ against yaw angle ψ.

As seen from the graphs, the drag coefficient C_D forms a saddle point, where it is convex as the pitch angle increases linearly with respect to the yaw angle. It reaches a minimum at $\theta = -2°$ and $\psi = 0°$.

The side force coefficient increases linearly with pitch and decreases linearly with yaw. This means that to achieve a side force of 0 N, the pitch angle must be $\theta = -3°$ and the yaw angle must be $\psi = 0°$.

Lastly, the lift coefficient increases linearly with pitch angle but remains unchanged with respect to yaw angle. For maximum lift, the pitch angle should be $\theta = 8°$ at $\psi = 0°$. This is expected as the flow above and below the wing does not change too much with yaw, but changes linearly with pitch [4].

4 Conclusion

In conclusion, this report successfully presents a method for determining the coefficients of aerodynamic forces C_D, C_S, and C_L at different pitch and yaw angles using block matrices in MATLAB. It demonstrates the variations in these coefficients and analyses the relationship between pitch, yaw angles, and aerodynamic forces.

Furthermore, the report concludes that the optimal angles for minimising drag are $\theta = -2^\circ$ and $\psi = 0^\circ$. To minimise side forces, the optimal angles are $\theta = -3^\circ$ and $\psi = 0^\circ$. Finally, for maximum lift, the recommended angles are $\theta = 8^\circ$ and $\psi = 0^\circ$.

References

[1] J. B. Barlow, W. H. R. Jr., and A. Pope, *Low-Speed Wind Tunnel Testing*. John Wiley & Sons, 3rd ed., 1999.

[2] T. Chow, *Classical Mechanics*. CRC Press, 2013.

[3] J. D. Anderson, "Introduction to flight," *New york: McGraw-Hill Education*, 2012.

[4] J. D. Anderson, "Fundementals of aerodynamics," *New york: McGraw-Hill Education*, 2017.