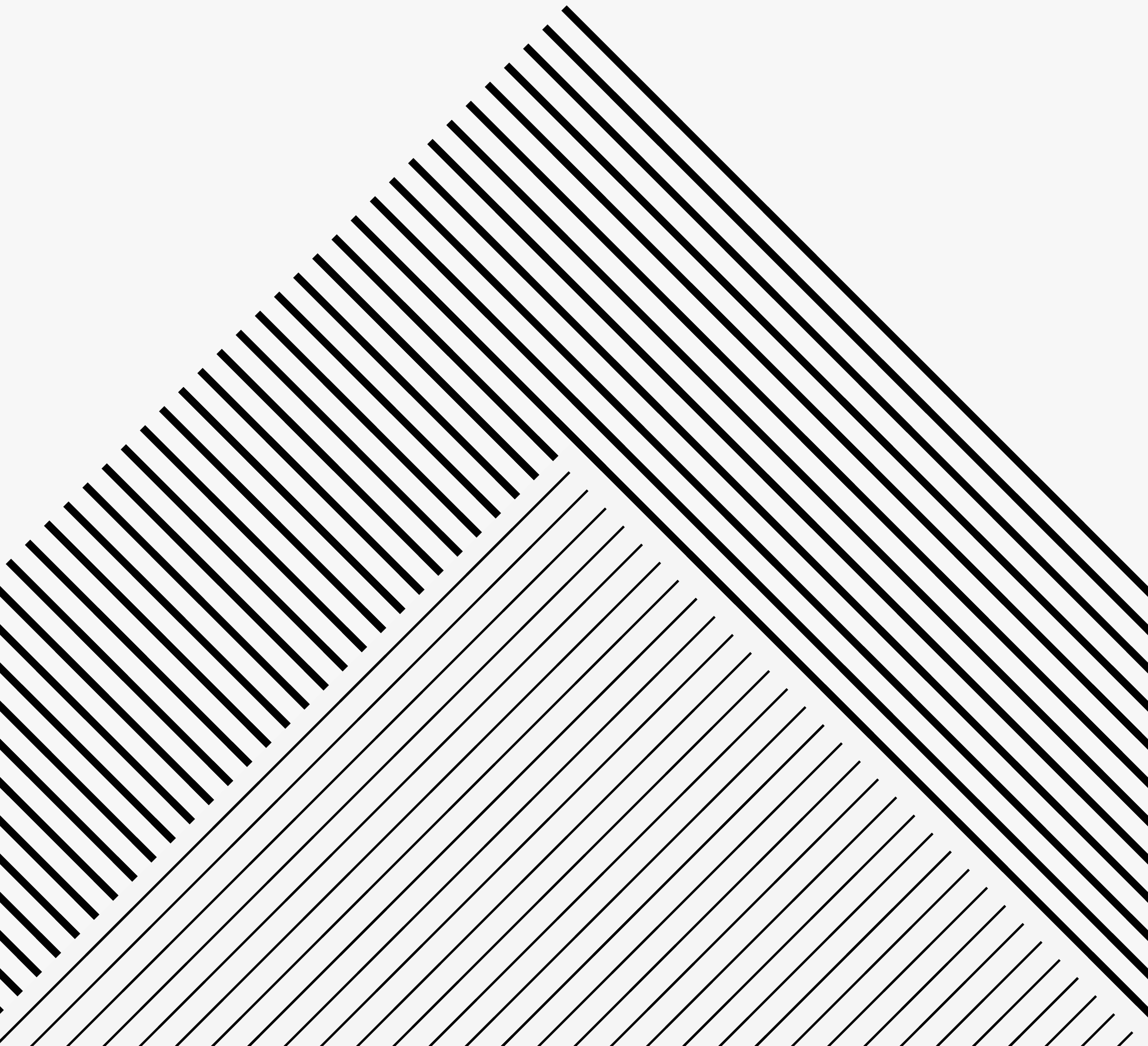


1

Ecuación para la caída libre

$$my'' + \gamma y' = mg$$

homogenea

$$e^{rt}(mr^2 + \gamma r) = 0$$

$$r(mr + \gamma) = 0$$

$$r = 0$$

$$r = -\frac{\gamma}{m}$$

$$y = C_1 + C_2 e^{-\frac{\gamma}{m}t}$$

2 particular

$$y_p = t^s A$$

3

$$s = 1$$

$$y_p = At$$

4

$$y_p = At \,,\; y'_p = A \,,\; y''_p = 0$$

$$m(0) + \gamma A = mg$$

$$A = \frac{mg}{\gamma} \rightarrow y_P = A\tau \rightarrow \frac{mg}{\gamma}\tau$$

$$y = c_1 + c_2 e^{-\frac{\gamma}{m}\tau} + \frac{mg}{\gamma}\tau$$

$$y' = \frac{-c_2 \gamma e^{-\frac{\gamma}{m}\tau}}{m} + \frac{mg}{\gamma}$$

$$y(0) = y_0 \quad , \quad y'(0) = 0$$

$$y_0 = c_1 + c_2 e^{\cancel{-\frac{\gamma}{m}\tau}} + \cancel{\frac{mg}{\gamma}\tau}$$

$$y_0 = c_1 + c_2$$

$$c_1 = y_0 - c_2 \quad , \quad 0 = \frac{-c_2\gamma}{m} + \frac{mg}{\gamma}$$

$$c_1 = y_0 - \frac{m^2 g}{\gamma^2} \qquad c_2 = \left(\frac{mg}{\gamma}\right)\cdot\frac{m}{\gamma}$$

$$c_2 = \frac{m^2 g}{\gamma^2}$$

$$y = y_0 - \frac{m^2 g}{\gamma^2} + \frac{m^2 g}{\gamma^2} e^{-\frac{\gamma}{m}\tau} + \frac{mg}{\gamma}\tau$$

---

caso inicial

$y_0 - y_c$

$$pc = y_0 - \frac{m^2 g}{\gamma^2} + \frac{m^2 g}{\gamma^2} e^{-\frac{\gamma}{m}\tau} + \frac{mg}{\gamma}\tau$$

$$\tau = \frac{g\, m^2 + a^2\, pc - a^2\, yo + g\, m^2\, \text{ProductLog}\left[-e^{-1-\frac{a^2\,(pc-yo)}{g\,m^2}}\right]}{a\, g\, m}$$

$y_0 = 200$

$y_c = 50$

$\tau = 3.32543\,s$

$y'(3.3254) = 33.8692$

$$y = c_1 + c_2 e^{-\frac{\gamma}{m}(\tau-\tau_{cp})} + \frac{mg}{\gamma}(\tau-\tau_{cp})$$

$$y' = \frac{-c_2 \gamma e^{-\frac{\gamma}{m}(\tau-\tau_{cp})}}{m} + \frac{mg}{\gamma}$$

$y(\tau_{cp}) = h$, $y'(\tau_{cp}) = v$

$h = c_1 + c_2$, $v = -\frac{c_2 \gamma}{m} + \frac{mg}{\gamma}$

$c_1 = h - c_2$

$$c_2 = \left(\frac{mg}{\gamma} - v\right)\left(\frac{m}{\gamma}\right)$$

2

Ecuación para la caida con resorte

$$my'' = mg - \gamma y' - ky \quad (y_0 - y_{cuerda})$$

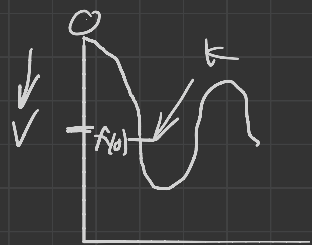
$100 - 90$

90

$W = mg$

$$my'' + \gamma y' + ky - \cancel{ky_0} + \cancel{ky_c} = W$$

$$e^{rt}(mr^2 + \gamma r + k)$$



$$\frac{-\gamma \pm \sqrt{\gamma^2 - 4(m)(k)}}{2m} = 0$$

$$r_1 = \frac{-\gamma + \sqrt{\gamma^2 - 4(m)(k)}}{2m}$$

$$r_2 = \frac{-\gamma - \sqrt{\gamma^2 - 4(m)(k)}}{2m}$$

Debido a los valores de k, m & gama la determinante siempre va a ser < 0, también se espera esto ya que el ser menor a 0 se va a tener una función con respecto a seno y coseno, lo cual se va a reflejar de una manera oscilante y es lo que queremos para representar el rebote

$$\text{si } \det < 0 \quad -\frac{\gamma}{2m} \pm \frac{\sqrt{\gamma^2-4(m)(k)}}{2m}\, i$$

$$\lambda = -\frac{\gamma}{2m}, \quad \mu = \frac{\sqrt{\gamma^2-4(m)(k)}}{2m}$$

$$y = e^{\lambda t}\left(c_1 \cos(\mu t) + c_2 \sin(\mu t)\right)$$

$$y' = \lambda e^{\lambda t}\left(c_1 \cos(\mu t) + c_2 \sin(\mu t)\right) + e^{\lambda t}\left(-\mu c_1 \sin(\mu t) + c_2 \mu \cos(\mu t)\right)$$

$$y(t_p) = p_c$$

$$y'(t_p) = V$$

Particular

2

$$y_p = t^s A$$

③ Como las raizes son $\neq 0$

$$s = 0$$

④

$$y_p = A$$

$$y'_p = 0 \qquad y''_p = 0$$

⑤

$$my'' = mg - \gamma y' - k(y - (y_0 - y_c))$$

$$my'' + \gamma y' + ky - ky_0 + ky_c = mg$$

$$+kA - ky_0 + ky_c = mg$$

$$A = \frac{mg + ky_0 - ky_c}{k}$$

$$A = \frac{mg}{k} + y_0 - y_c$$

$$tp = t - t_{pc}$$

$$y = e^{\lambda t_p}\left(c_1 \cos(\mu(t_p)) + c_2 \sin(\mu(t_p))\right) + \frac{mg}{k} + y_0 - y_c$$

$$y' = \lambda e^{\lambda(t_p)}\left(c_1 \cos(\mu(t_p)) + c_2 \sin(\mu(t_p))\right) + e^{\lambda(t_p)}\left(-\mu c_1 \sin(\mu(t_p)) + c_2 \mu \cos(\mu(t_p))\right)$$

$y(t_{pc}) = P_c \quad y'(t_{pc}) = V \rightarrow$ velocidad final
caida libre

$$y(t_{pc}) = c_1 + \frac{mg}{k} + y_0 - y_c$$

$$y'(t_{pc}) = \lambda(c_1) + c_2\mu$$

$$y(t_{pc}) = P_c \rightarrow P_c = c_1 + \frac{mg}{k} + y_0 - y_c$$

$$\underline{c_1 = P_c - \frac{mg}{k} - y_0 + y_c}$$

$$y'(t_{pc}) = V$$

$$V = \lambda c_1 + c_2\mu$$

$$\underline{c_2 = \frac{V - \lambda c_1}{\mu}}$$