837. New 21 Game

DescriptionHintsSubmissionsDiscussSolution

Alice plays the following game, loosely based on the card game "21".

Alice starts with `0` points, and draws numbers while she has less than `K` points. During each draw, she gains an integer number of points randomly from the range `[1, W]`, where `W` is an integer. Each draw is independent and the outcomes have equal probabilities.

Alice stops drawing numbers when she gets `K` or more points. What is the probability that she has `N` or less points?

Example 1:

```
Input: N = 10, K = 1, W = 10
Output: 1.00000
Explanation:  Alice gets a single card, then stops.
```

Example 2:

```
Input: N = 6, K = 1, W = 10
Output: 0.60000
Explanation:  Alice gets a single card, then stops.
In 6 out of W = 10 possibilities, she is at or below N = 6 points.
```

Example 3:

```
Input: N = 21, K = 17, W = 10
Output: 0.73278
```

Note:

1. `0 <= K <= N <= 10000`
2. `1 <= W <= 10000`
3. Answers will be accepted as correct if they are within `10^-5` of the correct answer.
4. The judging time limit has been reduced for this question.

Seen this question in a real interview before?

Approach #1: Dynamic Programming [Accepted]

Intuition

It is clear that the probability that Alice wins the game is only related to how many points `x` she starts the next draw with, so we can try to formulate an answer in terms of `x`.

Algorithm

Let `f(x)` be the answer when we already have `x` points. Clearly, `f(x) = 1.0` when `K <= x <= N`, and `f(x) = 0.0` when `x > N`.

We can write a recursion,

$$f(x)=\frac{1}{W}\sum_{i=1}^{W} f(x+i)$$

Let dp[i] be f(i), we first let S = min(N-K+1,W), then we update S = S + dp[i] – dp[i+w]

```
class Solution {
public:
    double new21Game(int N, int K, int W) {
        vector<double> dp(N+W+1,0);
        for(int k=K;k<=N;++k)
            dp[k]=1.0;
        double S = min(N-K+1,W);
        for(int k=K-1;k>=0;k--)
        {
            dp[k] = S/W;
            S += dp[k] - dp[k+W];
        }
        return dp[0];
    }
};
```