Basics of Integrals

December 17, 2017

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1 Explaining Notation

An integral, simply represents the sum of the area of infinitesimally thin rectangles. The *thinness* or width of the rectangle is denoted as dx which represents an infinitesimally small increment in the x direction. Thus the area of each rectangle is $dx \cdot y$ as y of course, represents the height of the rectangle.

The way we used to estimate the area under a graph, was making loads of small rectangles under the curve and add together all the areas; this is imprecise. In order to get better results we need to make the rectangles smaller, thus needing more of them.

1.1 Expressing Yourself

By using integrals we can imagine infinitesimally thin rectangles being added up to give a much more precise answer for the area under the curve. The notation for integrals is just a big 'S' that looks like this: '$\int_a^b$'. You may notice the 'a' and 'b', this can be read as 'The integral of [equation of line] form a to b', with a and b being x coördinates, limiting the area of the graph we're actually calculating.

Now, say we have the equation for the curve:

$$f(x) = 4x^2 - \frac{1}{2}x + 3$$

and we want to find the area under that curve from $x = 4$ to $x = 12$

$$\int_4^{12} f(x)\, dx$$

or, with $f(x)$ expanded

$$\int_4^{12} 4x^2 - \frac{1}{2}x + 3\ dx$$

1.2 Semantics

Let's break this down: ignore the integral sign for now, and lets just focus on one rectangle. We have our equation for the height (the y coördinate, $f(x)$), now we need to multiply that by some infinitesimally small nudge in x to form our infinitesimally thin rectangle, giving us: dx.

d is just notation for an infinitesimally small nudge in a certain direction. This gives us the general equation for any rectangle under the curve: $y \cdot dx$. Moving on to the integral sign, which simply just stands for "sum", because we are summing area of **all** the rectangles between a and b (4 and 12) thusly giving us the total area under the curve $f(x)$ where $4 \leq x \leq 12$.