

MAD Assignment 2

Ask Jensen

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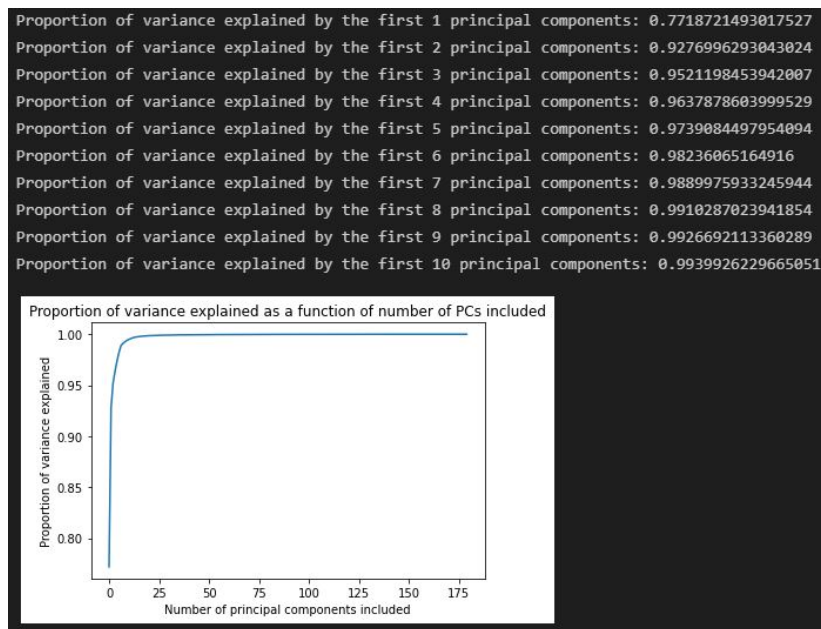
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1 Problem 1

1.1 (a)

```
1 import numpy.matlib
2
3 def pca(data):
4     # Creating "clone" of matrix
5     data_cent = np.full_like(data,0)
6
7     # Iterate the matrix subtracting the mean diatom
8     # from each row
9     for i in range(780):
10         data_cent[:,i] = diatoms[:,i] - mean_diatom
11
12     # Create the covariance matrix
13     cov_matrix = np.cov(data_cent)
14
15     # Calculate the eigenvectors and eigenvalues
16     PCevals, PCevecs = np.linalg.eigh(cov_matrix)
17
18
19     # linalg.eigh returns the vectors and values
20     # in the wrong order.
21     # Np.flip will reverse the order so it is correct and
22     # corresponding to the exercise requirements
23     PCevals = np.flip(PCevals)
24     PCevecs = np.flip(PCevecs, axis=1)
25     return PCevals, PCevecs, data_cent
26
27 PCevals, PCevecs, data_cent = pca(diatoms)
```



Figur 1: Values for the first 10 proportions of variance, and the corosponding graph

1.2 (b)

```

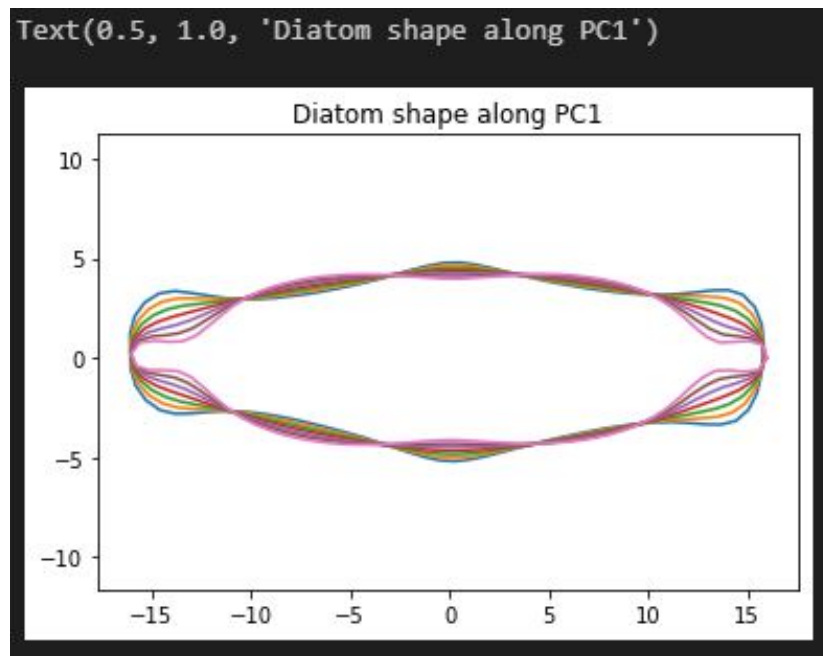
1      # gets the fourth eigenvector
2      e4 = PCevecs[:, 3]
3      # gets the fourth eigenvalue
4      lambda4 = PCevals[3]
5      # In case the naming std is confusing --
6      # the eigenvalues have a statistical interpretation
7      # print(std4)
8      std4 = np.sqrt(lambda4)
9
10     # Makes matrix filled with zeros
11     diatoms_along_pc = np.zeros((7, 180))
12
13     # Iterates the length of the matrix
14     # For each row, add the mean diatom with added
15     # values
16     for i in range(7):
17         diatoms_along_pc[i] = mean_diatom + ( e4 * std4 * (i-3))
18
19     # Plotting each diatom
20     for i in range(7):

```

```

21     plot_diatom(diatoms_along_pc[i])
22
23     plt.title('Diatom shape along PC1')

```



Figur 2: Plotted diatom

2 Problem 2

3 Problem 2

assesses the given claim $E[(X - \mu)^4] \geq \sigma^4$

X has the mean μ and the variance σ^2 which can be rewritten as $\sigma^4 = (\text{Var}(X))^2$

$E[(X - \mu)^4]$ can be rewritten as $E(g(x))$ where $g(x) = (X - \mu)^4$

It is possible to Jensen's inequality if $g''(x) \geq 0$ $g(x) = (X - \mu)^4$ $g'(x) = 4(X - \mu)^3$ $g''(x) = 12(X - \mu)^2$ $g(x)$ is convex, since the second derivative of the function is quadratic. Hence it will always be greater than zero.

4 Problem 3

4.1 (a)

5 Problem 4

5.1 (a)