MAD Assignment 2

Ask Jensen

5. december 2021

Indhold

1 Problem 1

1.1 (a)

$$\mathscr{L} = \frac{1}{N}\sum_{n=1}^{N} \alpha_n(\mathbf{w}^T\mathbf{x}_n - t_n)^2$$

$$= \frac{1}{N}(\mathbf{Xw} - t)^T A(\mathbf{Xw} - t)$$

$$= \frac{1}{N}(\mathbf{Xw})^T - t^T(A\mathbf{Xw} - At)$$

$$\frac{1}{N}(\mathbf{Xw})^T A\mathbf{Xw} - \frac{1}{N}(\mathbf{Xw})^T At - \frac{1}{N}A\mathbf{Xw}t^T + \frac{1}{N}Att^T$$

$$= \frac{1}{N}\mathbf{w}^T\mathbf{X}^T A\mathbf{Xw} - \frac{2}{N}\mathbf{w}^T\mathbf{X}^T At + \frac{1}{N}t^T At$$

using case 4 and case 1 from the table 1.14 when differentiating I get:

$$\frac{\partial\mathscr{L}}{\partial w} = 2\mathbf{X}^T\mathbf{AXw} - 2\mathbf{X}^T\mathbf{A}t = 0$$

+2 and -2 cancel out leaving me with

$$\mathbf{X}^T\mathbf{AXw} - \mathbf{X}^T\mathbf{A}t = 0$$

$$\mathbf{X}^T\mathbf{AXw} = \mathbf{X}^T\mathbf{A}t$$

multiplying both sides with the identity matrix

$$\mathbf{Iw} = (\mathbf{X}^T\mathbf{AX})^{-1}\mathbf{X}^T\mathbf{A}t$$

Multiplying the vector **w** with the Identity matrix, will simply return the vector **w** thus the result is

$$\hat{\mathbf{w}} = (\mathbf{X}^T\mathbf{AX})^{-1}\mathbf{X}^T\mathbf{A}t$$

2 Problem 2

3 Problem 3

3.1 (a)

The a PDF is the derivative of a CDF.
The function $e^{-\beta x^\alpha}$ is a composition of f(g(x)). Thus the after applying the chainrule

and deriving where $g(x) = -\beta x^{\alpha}$:

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx}(-\beta x^{\alpha} \cdot e^{g(x)}) = -\beta\alpha x^{\alpha-1} \cdot e^{-\beta x^{\alpha}}$$

Thus the PDF is:

$$f(x) \begin{cases} -\beta\alpha x^{\alpha-1} \cdot e^{-\beta x^{\alpha}} & \text{if} \quad x > 0 \\ 0 & \text{otherwise} \end{cases}$$

3.2 (b)

In this question there are two subquestions.

1. What is the probability that the chip works longer than four years?
2. What is the probability that the chip stops working in the time interval [5; 10] years?

I'm given two values for α and β stubstituting these values into the original function I have:

$$f(x) = 1 - e^{-\frac{1}{4}x^2} = 1 - e^{-\frac{x^2}{4}}$$

To answer the first question I can simply calculate f(5). Since I'm asked to answer the probability for the chip to live *more* than four years. It is important to note that when the value = 1 the chip is dead. Otherwise, the chip would become better over time. $f(5) = 0.998069$ This means, that after 5 years, the chip is almost certanly dead.

To calculate the probability that the chip will die somewhere between the interval [5; 10] I can do it like so:

$$f(10) - f(4) = 0.0183156$$

This means that the chip has $\approx 1.83\%$ chance of surviving. So to make this more readble I can say:

$$1 - (f(10) - f(4)) \approx 98.16\% \quad \text{chance of death}$$

in the interval from 5 to 10 years.

3.3 (c)

Finding the median of the function. I can simply set it equal to $\frac{1}{2}$ and solve it for x.

$$1 - e^{-\beta x^{\alpha}} = \frac{1}{2}$$

simple rewriting

$$1 - \frac{1}{2} = e^{-\beta x^{\alpha}}$$

multiplying with $ln()$ to remove the exponent

$$ln(\frac{1}{2}) = -\beta x^{\alpha}$$

dividing with $-\beta$

$$\frac{ln(\frac{1}{2})}{-\beta} = x^{\alpha}$$

$ln(\frac{1}{2})$ is the same as $-ln(2)$ so rewriting and removing the minuses

$$\frac{ln(2)}{\beta} = x^{\alpha}$$

taking $ln(x^{\alpha})$ to move the α

$$\ln(\frac{ln(2)}{\beta}) = \alpha \cdot ln(x)$$

moving alpha

$$\frac{\ln(\frac{ln(2)}{\beta})}{\alpha} = ln(x)$$

We already know that in order to remove the exponenet, we can use ln. so to remove ln I can add back the exponent, which gives me the answer
Answer:

$$e^{\frac{\ln(\frac{ln(2)}{\beta})}{\alpha}} = x$$

4 Problem 4