MAD Assignment 1

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26. november 2021

Indhold

1 Problem 1

the exercise is to compute the partial derivations of the following three problems each containing a different function.

1.1 (a)

Considering the function, it is fairly straight forward to compute the derivative with respect to x and y. Both derivations will be done using the power rule: $x^n = nx^{n-1}$

$$f(x,y) = x^4y^3 + x^5 - e^y$$

Answer:
calculating $\frac{\partial}{\partial x}$

$$f'_x = 4x^3y^3 + 5x^4$$

calculating $\frac{\partial}{\partial y}$

$$f'_y = x^4 3y^2 - e^y$$

It is worth mentioning, that the exponent (e^y) will derive to itself even though the power function states otherwise.

1.2 (b)

This function is bit harder to compute. I will be using the power rule: $x^n = nx^{n-1}$ and the chain rule: $f(g(x)) = f'(g(x)) \cdot g'(x)$

$$f(x,y) = \frac{1}{\sqrt{x^3 + xy + y^2}}$$

I will start by deriving the innter function with respect to x and y since these derivations are going to be used later on in the calculations.

calculating $\frac{\partial}{\partial x}$

$$f'_x = 3x^2 + y$$

calculating $\frac{\partial}{\partial y}$

$$f'_y = x + 2y$$

Answer:
I will rewrite the function using simple fraction rules in order to make the function a bit easier to work with.

$$f(x,y) = (x^3 + xy + y^2)^{-\frac{1}{2}}$$

applying the power rule

$$f(x,y) = -\frac{1}{2}(x^3 + xy + y^2)^{-\frac{3}{2}}$$

applying the chain rule

$$f(x,y) = -\frac{1}{2}(x^3 + xy + y^2)^{-\frac{3}{2}} \cdot (3x^2 + y)$$

calculating $\frac{\partial}{\partial x}$

$$f'_x = -\frac{3x^2 + y}{2(x^3 + xy + y^2)^{\frac{3}{2}}}$$

I will not be showing the full calculation for f'_y since it derives trivially in the same was as f'_x, with the only exception being that instead of using the chain rule with the derivation of x you now use the derivative of y thus I will only show the last two steps

applying the chain rule

$$f(x,y) = -\frac{1}{2}(x^3 + xy + y^2)^{-\frac{3}{2}} \cdot (x^4 3y^2 - e^y)$$

calculating $\frac{\partial}{\partial x}$

$$f'_x = -\frac{x + 2y}{2(x^3 + xy + y^2)^{\frac{3}{2}}}$$

1.3 (c)

I order to compute this function I will be using the power rule: $x^n = nx^{n-1}$ and the quotient rule: $(\frac{f(x)}{g(x)})' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$$f(x,y) = \frac{x^3 + y^2}{x + y}$$

Answer:
I will start by finding the derivations for f(x) and f(y) aswell for g(x) and g(y). these calulations are done using the power rule. Having these calculating done, it's a simple matter of substituting with the "new"expressions in the qoutient rule
$f'_x \cdot g(x) = 3x^2(x + y)$
$f'_y \cdot g(y) = 2y(x + y)$
$f(x) \cdot g'_x = 1 \cdot (x^3 + y^2)$
$f(y) \cdot g'_y = 1 \cdot (x^3 + y^2)$

$$f'_x = \frac{3x^2(x + y) - (x^3 + y^2)}{(x + y)^2}$$

$$f'_y = \frac{2y(x + y)) - (x^3 + y^2)}{(x + y)^2}$$

2 Problem 2

In order to solve these exercises I will make use of *Tabel 1.4* in the book "A first couse in machine learning"

2.1 (a)

$\overrightarrow{x}^T \overline{x}^T$

2.2 (b)

2.3 (c)

3 Problem 3

3.1 (a)

3.2 (b)

3.3 (c)

3.4 (d)

4 Problem 4