MAD Assignment 2

Ask Jensen

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Indhold

1 Problem 1

1.1 (a)

$$\mathscr{L} = \frac{1}{N}\sum_{n=1}^{N} \alpha_n(\mathbf{w}^T\mathbf{x}_n - t_n)^2$$

$$= \frac{1}{N}(\mathbf{Xw} - t)^T A(\mathbf{Xw} - t)$$

$$= \frac{1}{N}(\mathbf{Xw})^T - t^T(A\mathbf{Xw} - At)$$

$$\frac{1}{N}(\mathbf{Xw})^T A\mathbf{Xw} - \frac{1}{N}(\mathbf{Xw})^T At - \frac{1}{N}A\mathbf{Xw}t^T + \frac{1}{N}Att^T$$

$$= \frac{1}{N}\mathbf{w}^T\mathbf{X}^T A\mathbf{Xw} - \frac{2}{N}\mathbf{w}^T\mathbf{X}^T At + \frac{1}{N}t^T At$$

using case 4 and case 1 from the table 1.14 when differentiating I get:

$$\frac{\partial \mathscr{L}}{\partial w} = 2\mathbf{X}^T\mathbf{AXw} - 2\mathbf{X}^T\mathbf{A}t = 0$$

+2 and -2 cancel out leaving me with

$$\mathbf{X}^T\mathbf{AXw} - \mathbf{X}^T\mathbf{A}t = 0$$

$$\mathbf{X}^T\mathbf{AXw} = \mathbf{X}^T\mathbf{A}t$$

multiplying both sides with the identity matrix

$$\mathbf{Iw} = (\mathbf{X}^T\mathbf{AX})^{-1}\mathbf{X}^T\mathbf{A}t$$

Multiplying the vector **w** with the Identity matrix, will simply return the vector **w** thus the result is

$$\hat{\mathbf{w}} = (\mathbf{X}^T\mathbf{AX})^{-1}\mathbf{X}^T\mathbf{A}t$$

2 Problem 2

3 Problem 3

3.1 (a)

The a PDF is the derivative of a CDF.
The function $e^{-\beta x^{\alpha}}$ is a composition of f(g(x)). Thus the after applying the chainrule

and deriving where $g(x) = -\beta x^{\alpha}$:

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx}(-\beta x^{\alpha} \cdot e^{g(x)}) = -\beta\alpha x^{\alpha-1} \cdot e^{-\beta x^{\alpha}}$$

Thus the PDF is:

$$f(x) \begin{cases} -\beta\alpha x^{\alpha-1} \cdot e^{-\beta x^{\alpha}} & \text{if} \quad x > 0 \\ 0 & \text{otherwise} \end{cases}$$

3.2 (b)

In this question there are two subquestions.

1. What is the probability that the chip works longer than four years?
2. What is the probability that the chip stops working in the time interval [5; 10] years?

I'm given two values for α and β stubstituting these values into the original function I have:

$$f(x) = 1 - e^{-\frac{1}{4}x^2} = 1 - e^{-\frac{x^2}{4}}$$

To answer the first question I can simply calculate f(5). Since I'm asked to answer the probability for the chip to live *more* than four years

3.3 (c)

4 Problem 4

5 Problem 5