Introduction to Computer Vision

2. Convolutions & filters, template matching, edge detection

02.11.22

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Outline

- **Convolutions recap**
- Correlation & template matching
- Edge detection

Convolutions: some math

Question: what is g for moving average?

$$(f * g)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x - x', y - y')$$

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

This operation & the corresponding matrix are also called filters, kernels, convolutional matrices.

Convolutions: moving average

How does it modify images?

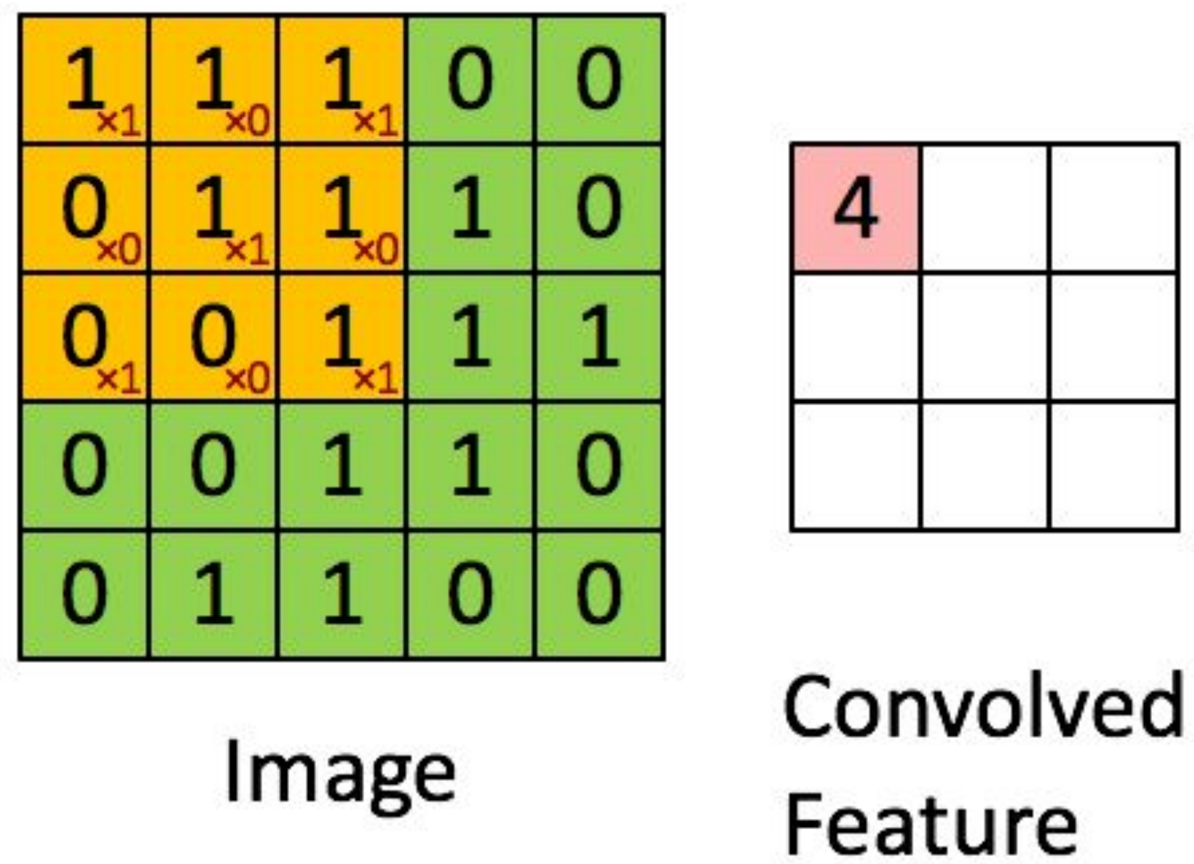
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Original image

Smoothed image

Source: J. Niebles

Convolutions in 2D



Source: [Stanford deep learning tutorial.]()

Outline

- Convolutions recap
- **Correlation & template matching**
- Edge detection

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Convolution vs Correlation

Cross correlation is another important operation we can apply to images:

$$(f \otimes h)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x' - x, y' - y)$$

Is it equivalent to convolution?

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Convolution vs Correlation

Cross correlation is another important operation we can apply to images:

$$(f \otimes h)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x' - x, y' - y)$$

Is it equivalent to convolution? Not exactly!

$$(f * h)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x - x', y - y')$$

See also this blog post:
https://glassboxmedicine.com/2019/07/26/convolution-vs-cross-correlation/

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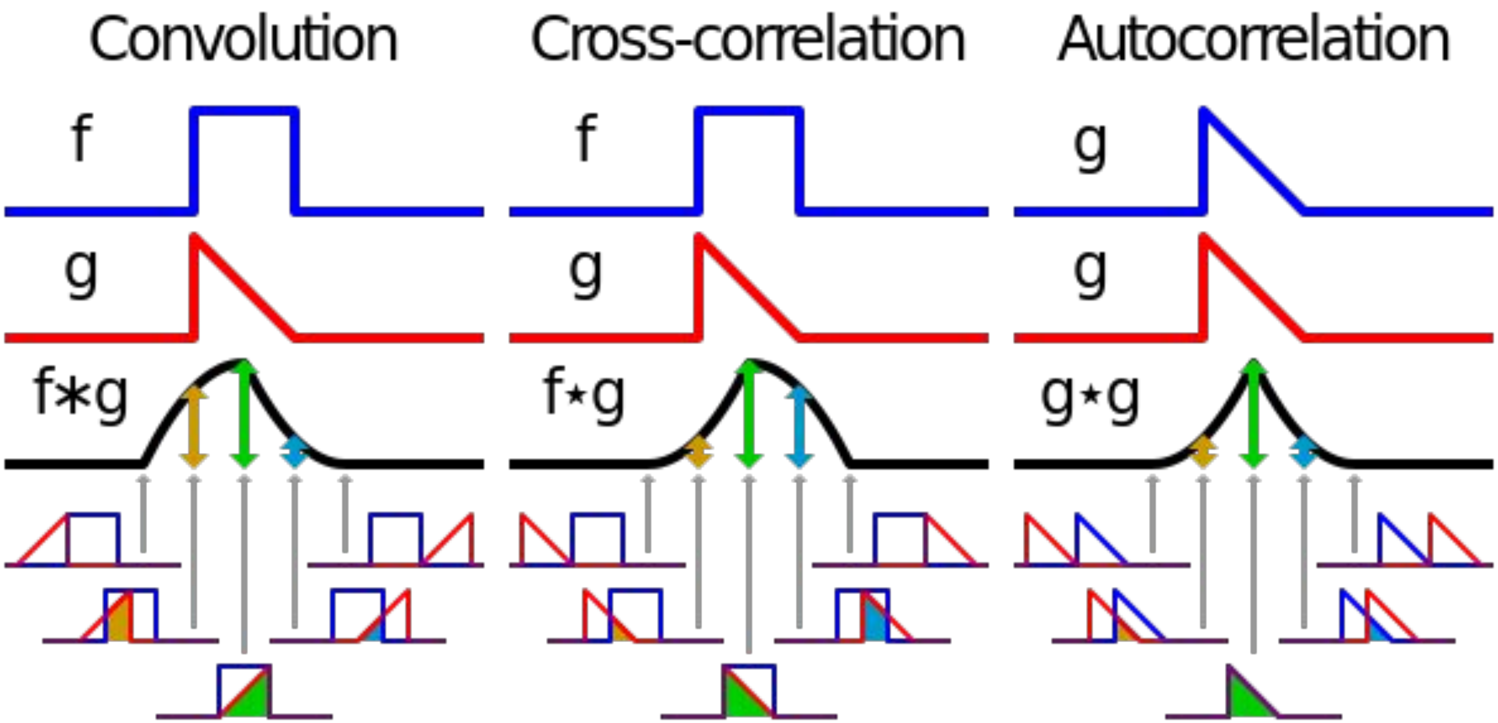
Convolution vs Correlation



Convolution = Cross-correlation with the flipped version of the kernel (along both axes for 2D)

Convolution vs Correlation

```python
def convolve(a, v, mode='full'):
    """
    Returns the discrete, linear convolution of two one-dimensional sequences.

    The convolution operator is often seen in signal processing, where it
    models the effect of a linear time-invariant system on a signal [1]_.  In
    probability theory, the sum of two independent random variables is
    distributed according to the convolution of their individual
    distributions.
    a, v = array(a, copy=False, ndmin=1), array(v, copy=False, ndmin=1)
    if (len(v) > len(a)):
        a, v = v, a
    if len(a) == 0:
        raise ValueError('a cannot be empty')
    if len(v) == 0:
        raise ValueError('v cannot be empty')
    return multiarray.correlate(a, v[::-1], mode)
```

https://github.com/numpy/numpy/blob/b235f9e701e14ed6f6f6dcba885f7986a833743f/numpy/core/numeric.py#L837-L844

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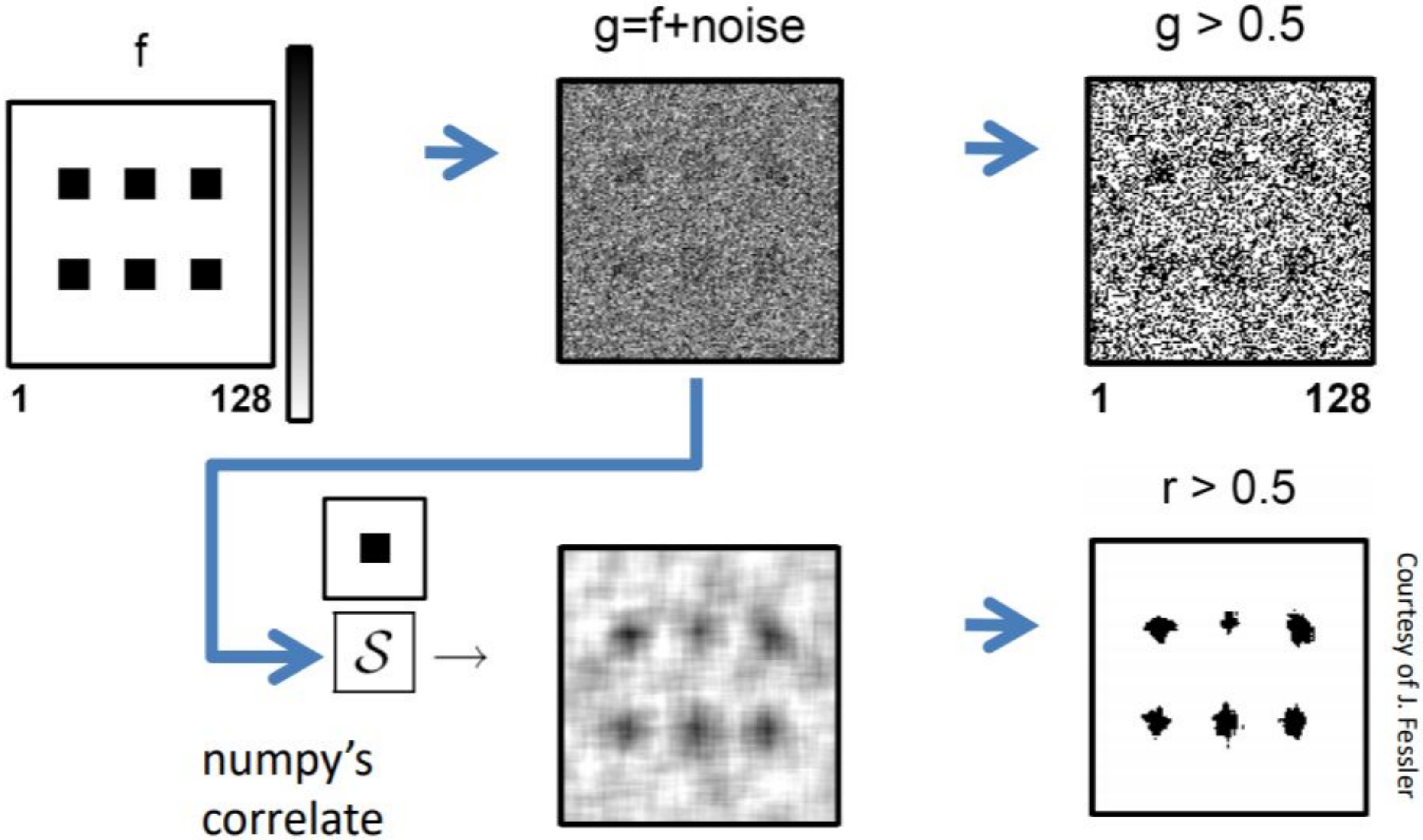
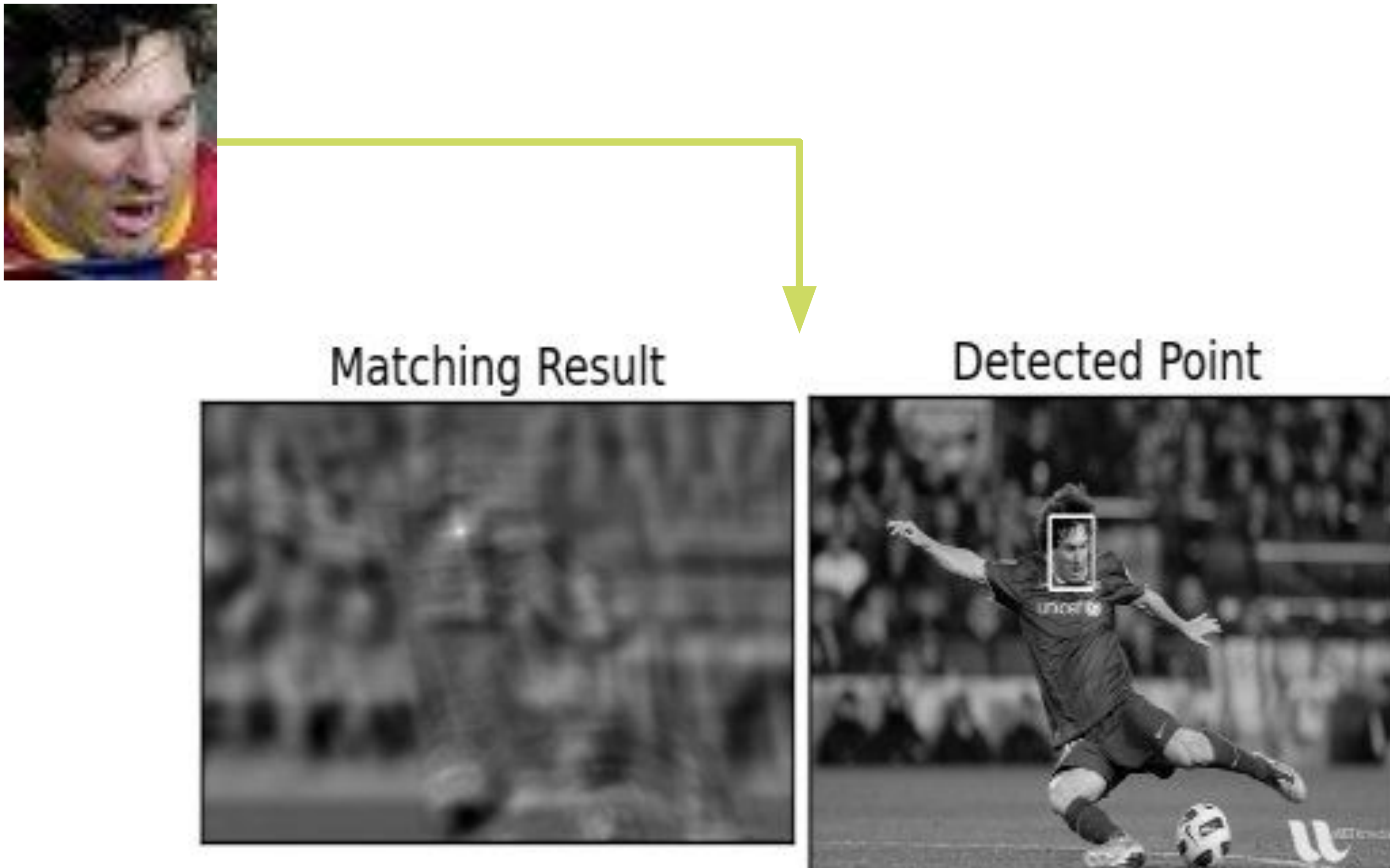
Convolution vs Correlation



Source: J. Niebles

Template matching



https://docs.opencv.org/master/d4/dc6/tutorial_py_template_matching.html

Template matching

How can we estimate that a part of the image is **similar** to the template?

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$$R(x,y) = \sum_{x',y'} (T(x',y') - I(x+x',y+y'))^2$$

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$$R(x,y) = \sum_{x',y'} (T(x',y') \cdot I(x+x', y+y'))$$

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Template matching

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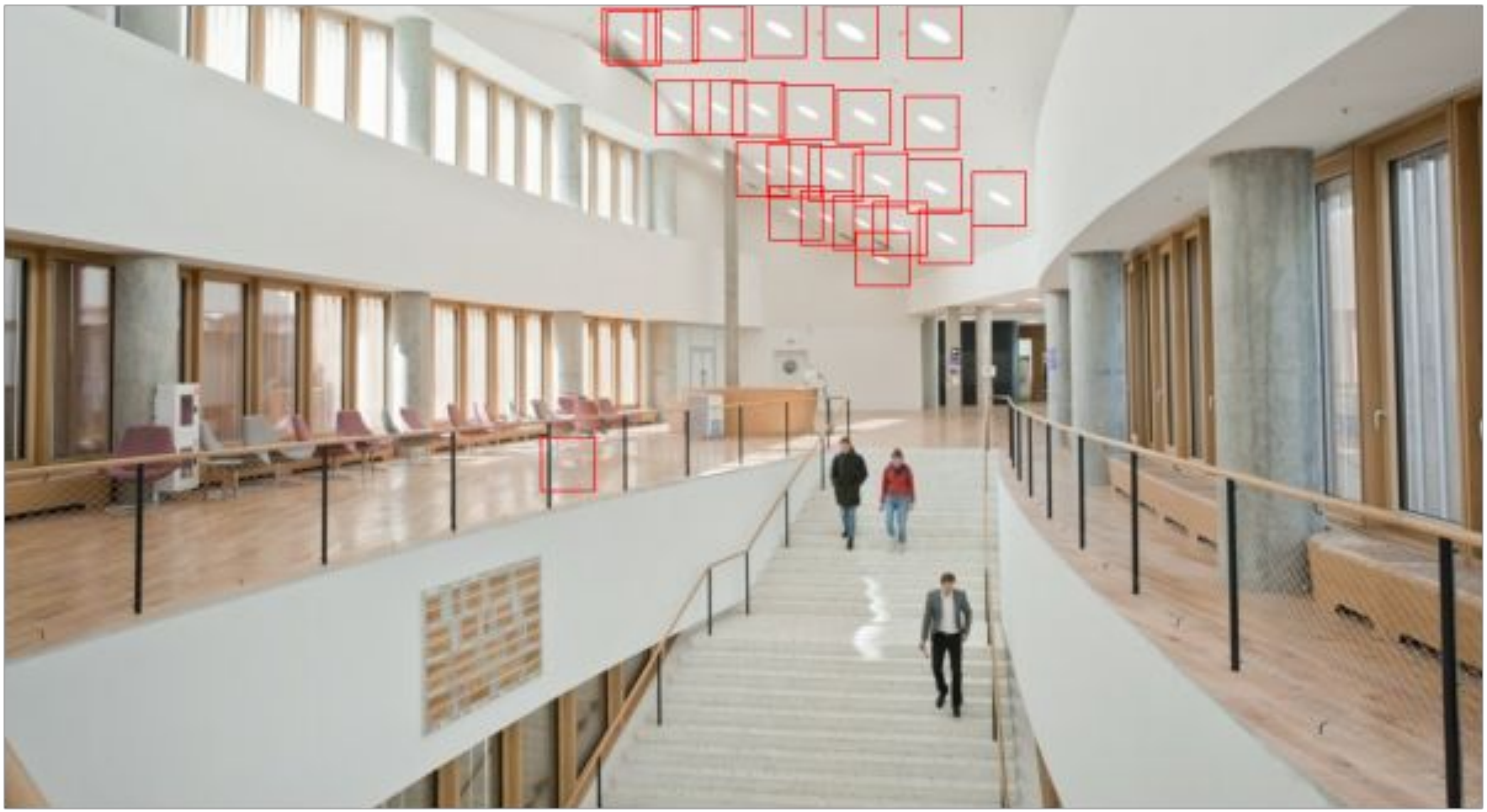
$$R(x,y) = \sum_{x',y'} (T(x',y') - I(x+x',y+y'))^2$$

$$R(x,y) = \sum_{x',y'} (T(x',y') \cdot I(x+x',y+y'))$$

$$R(x,y) = \frac{\sum_{x',y'} (T(x',y') \cdot I(x+x',y+y'))}{\sqrt{\sum_{x',y'} T(x',y')^2 \cdot \sum_{x',y'} I(x+x',y+y')^2}}$$

Template matching

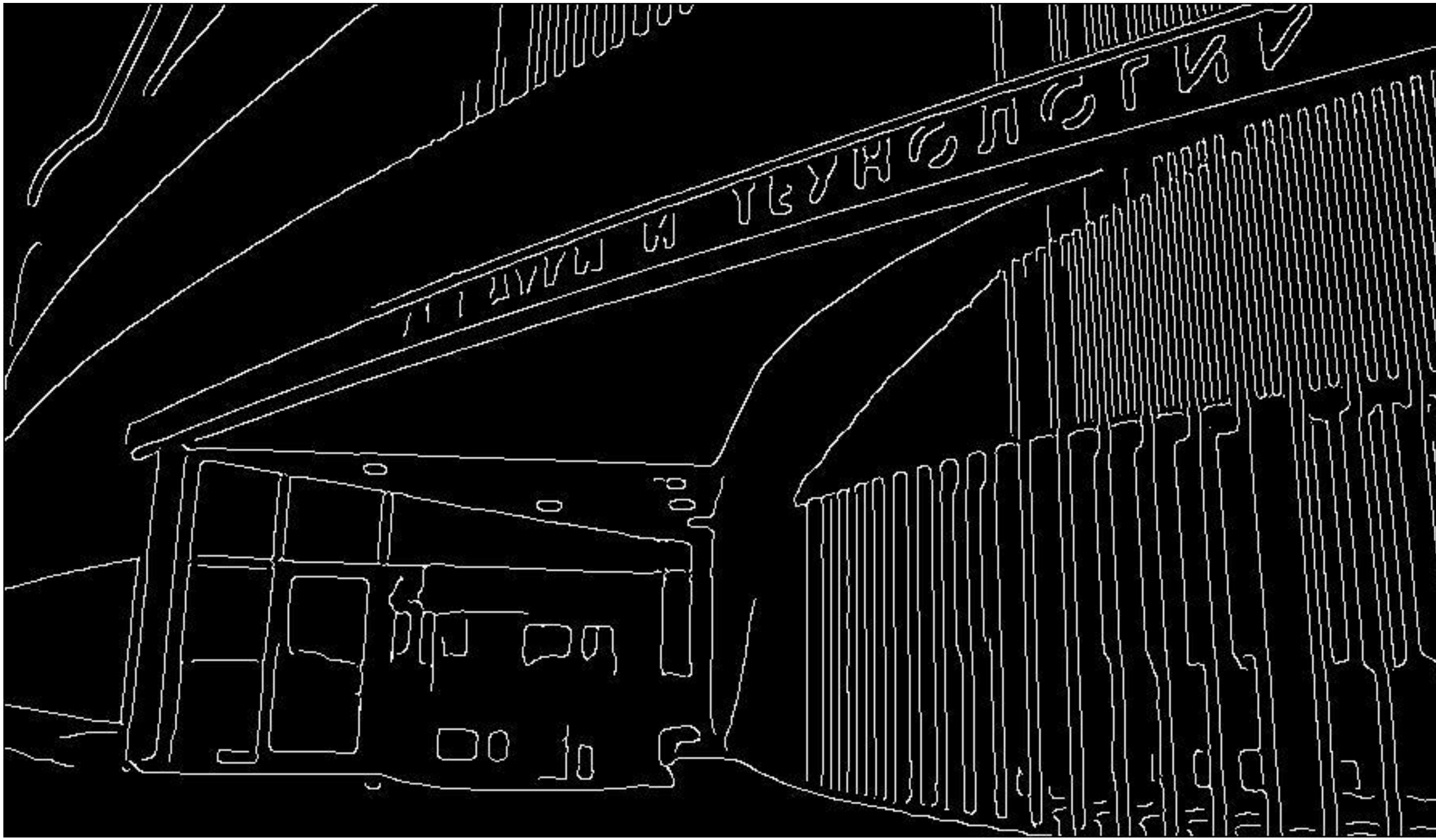
Exercise: count lamps using template matching!



Outline

- Convolutions recap
- Correlation & template matching
- **Edge detection**

Why do we need to detect edges?



Why do we need to detect edges?

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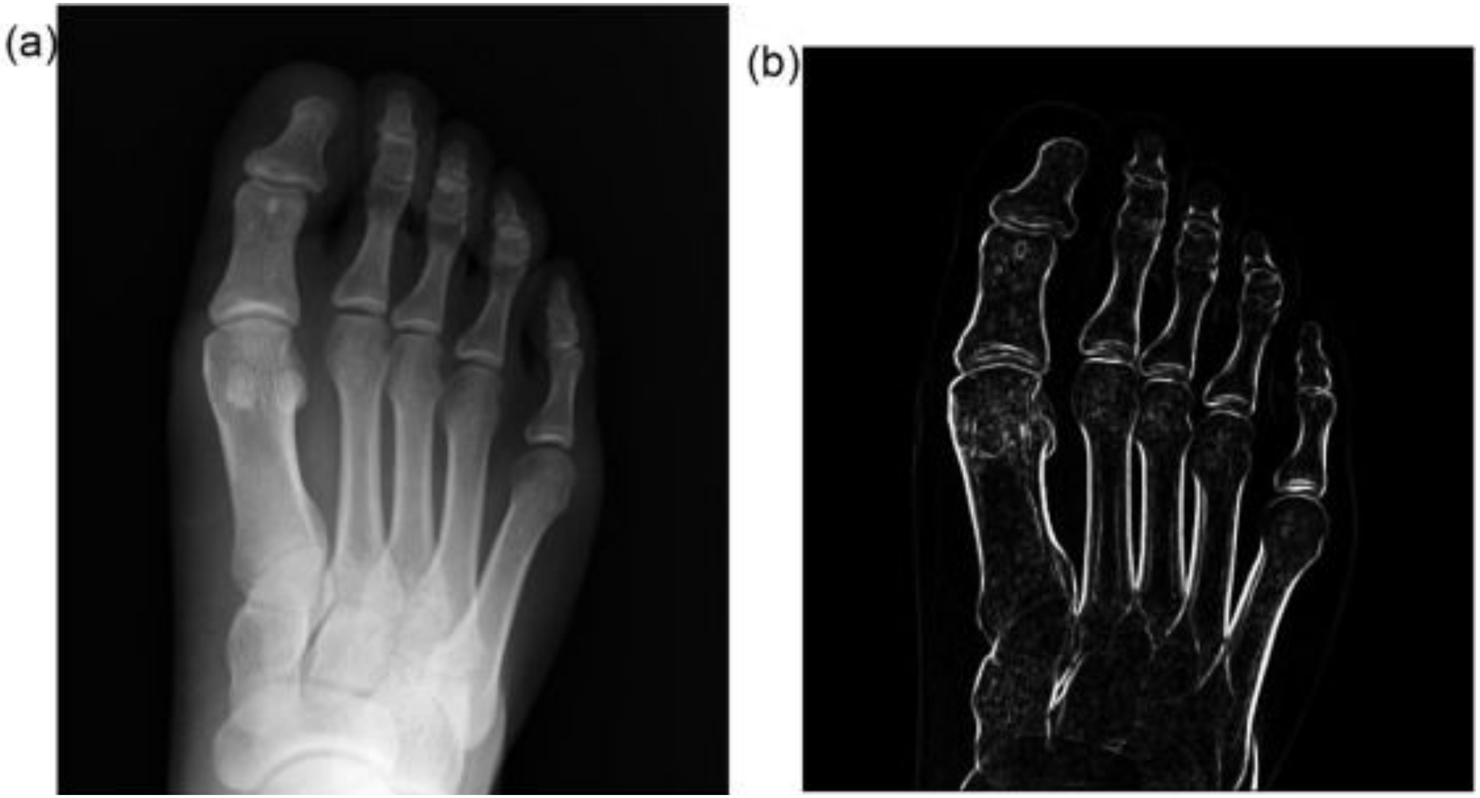

Source: Edge detection in medical images with quasi high-pass filter based on local statistics

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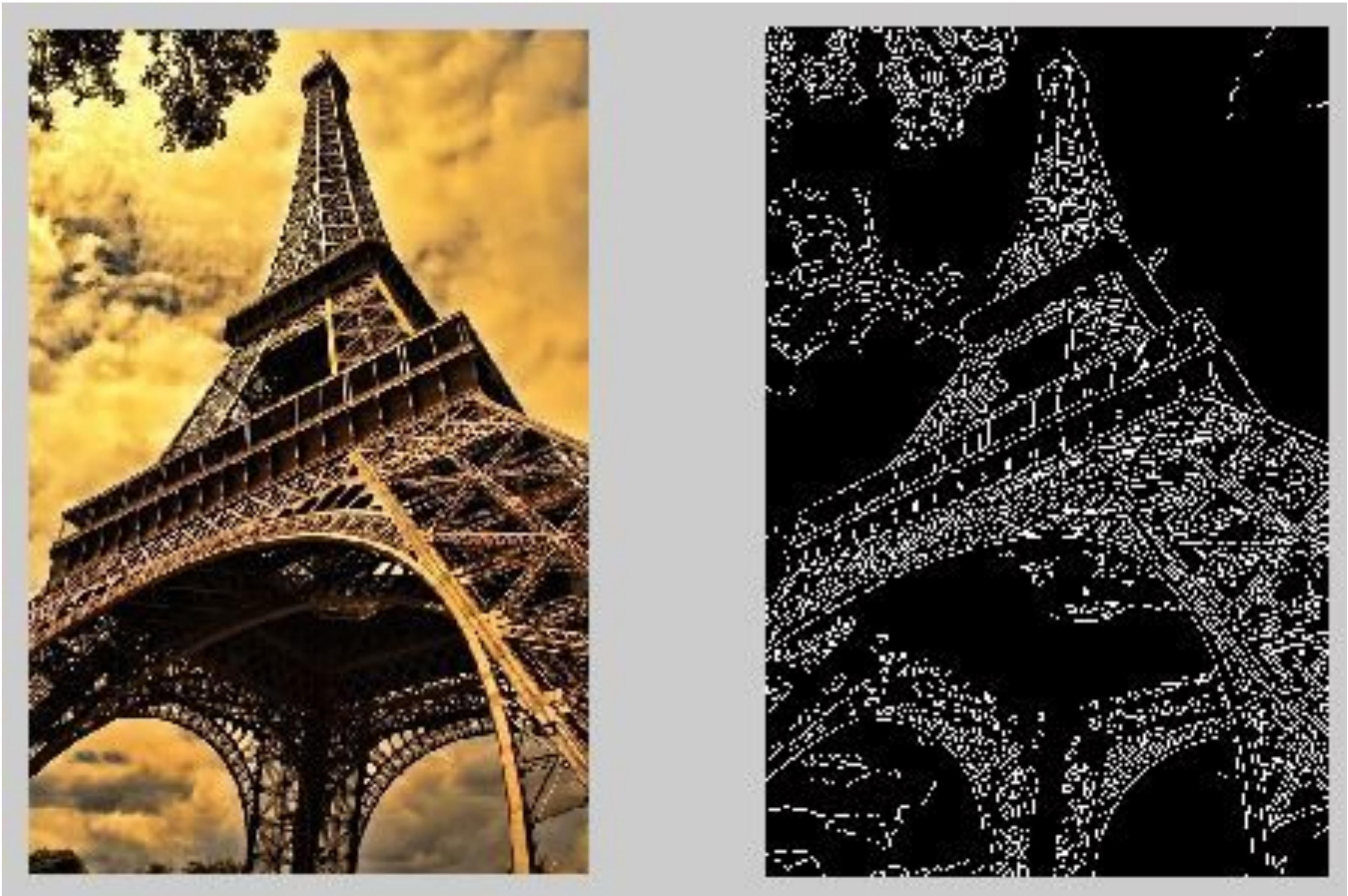
Why do we need to detect edges?



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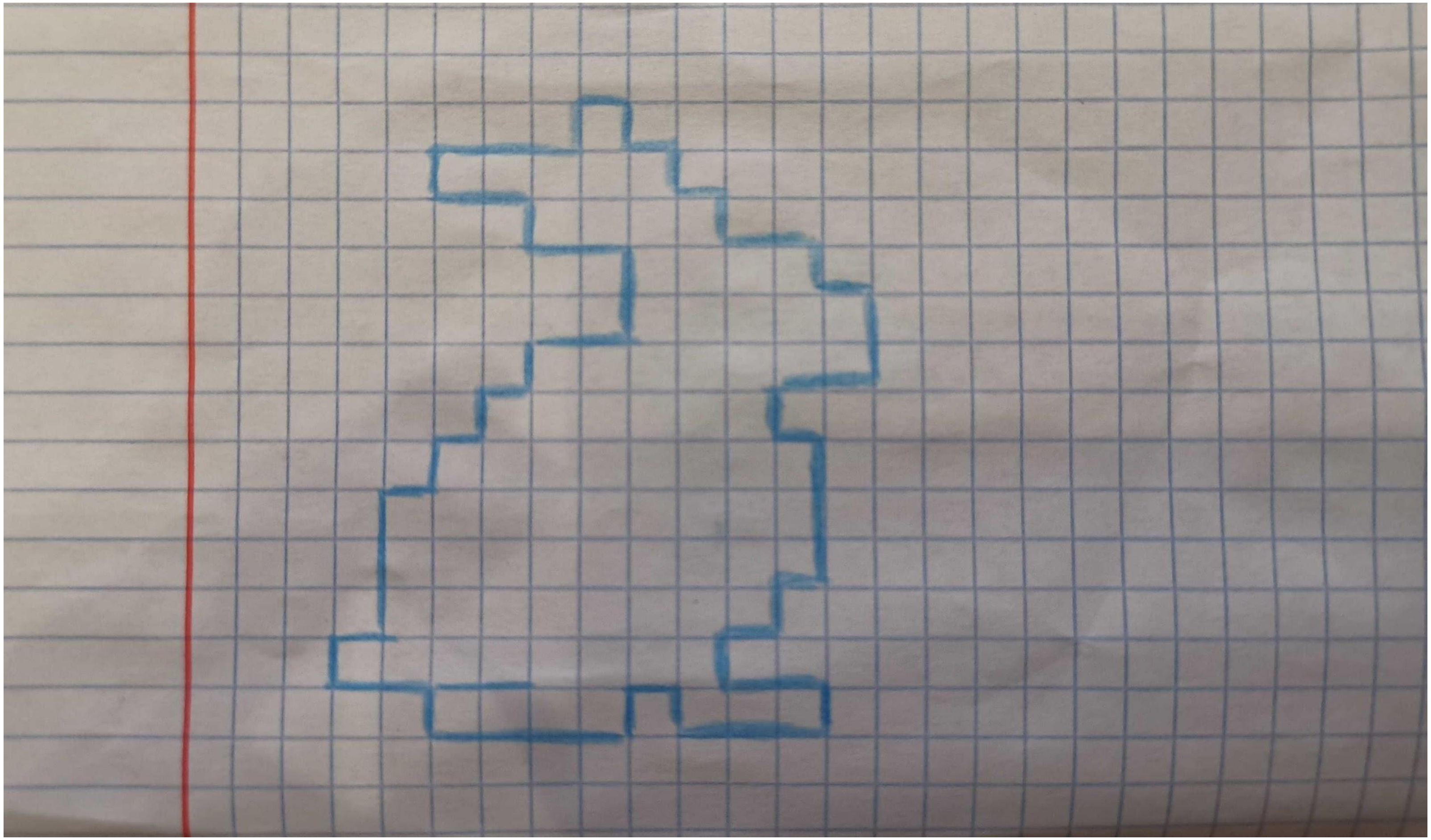

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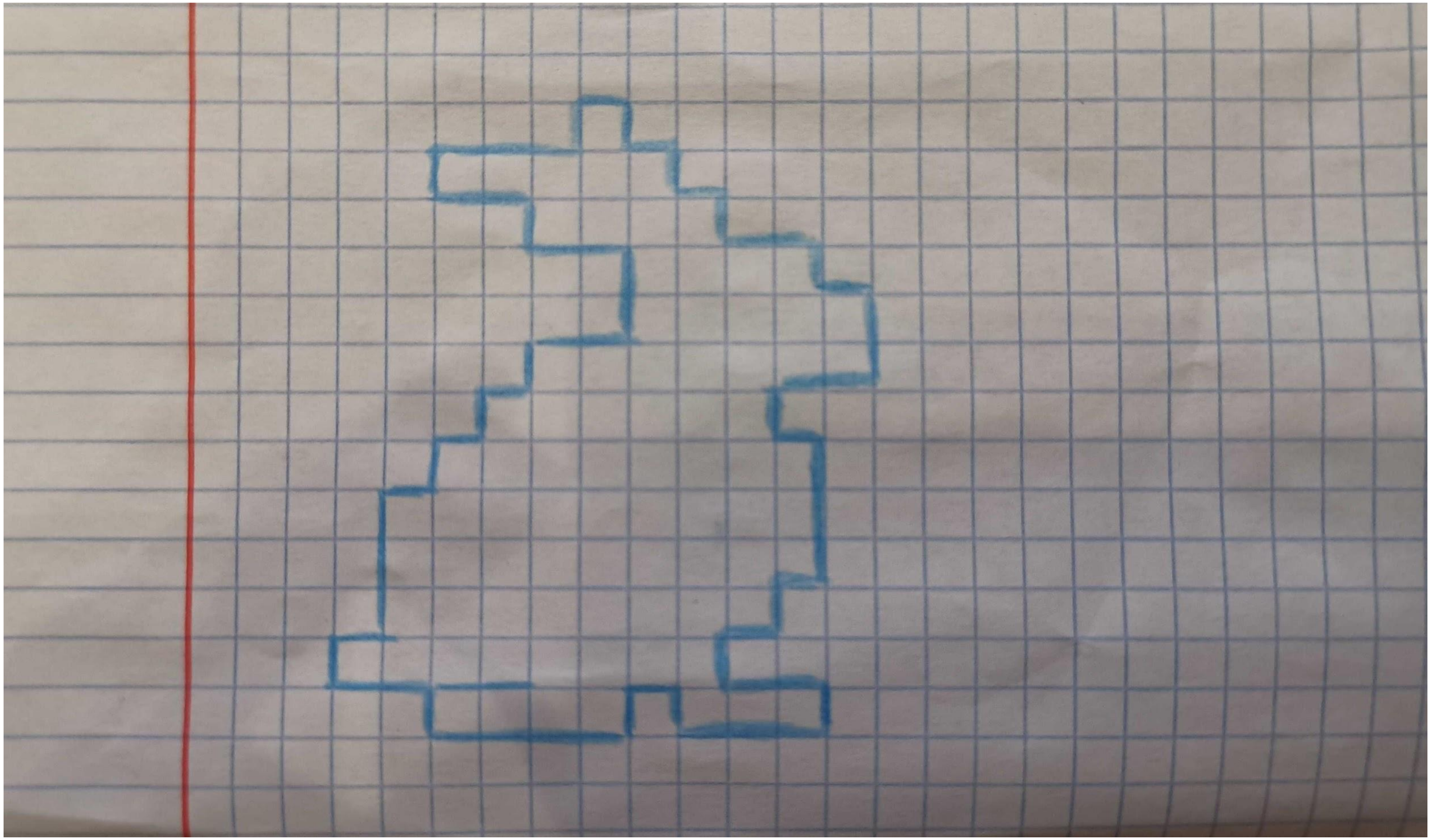
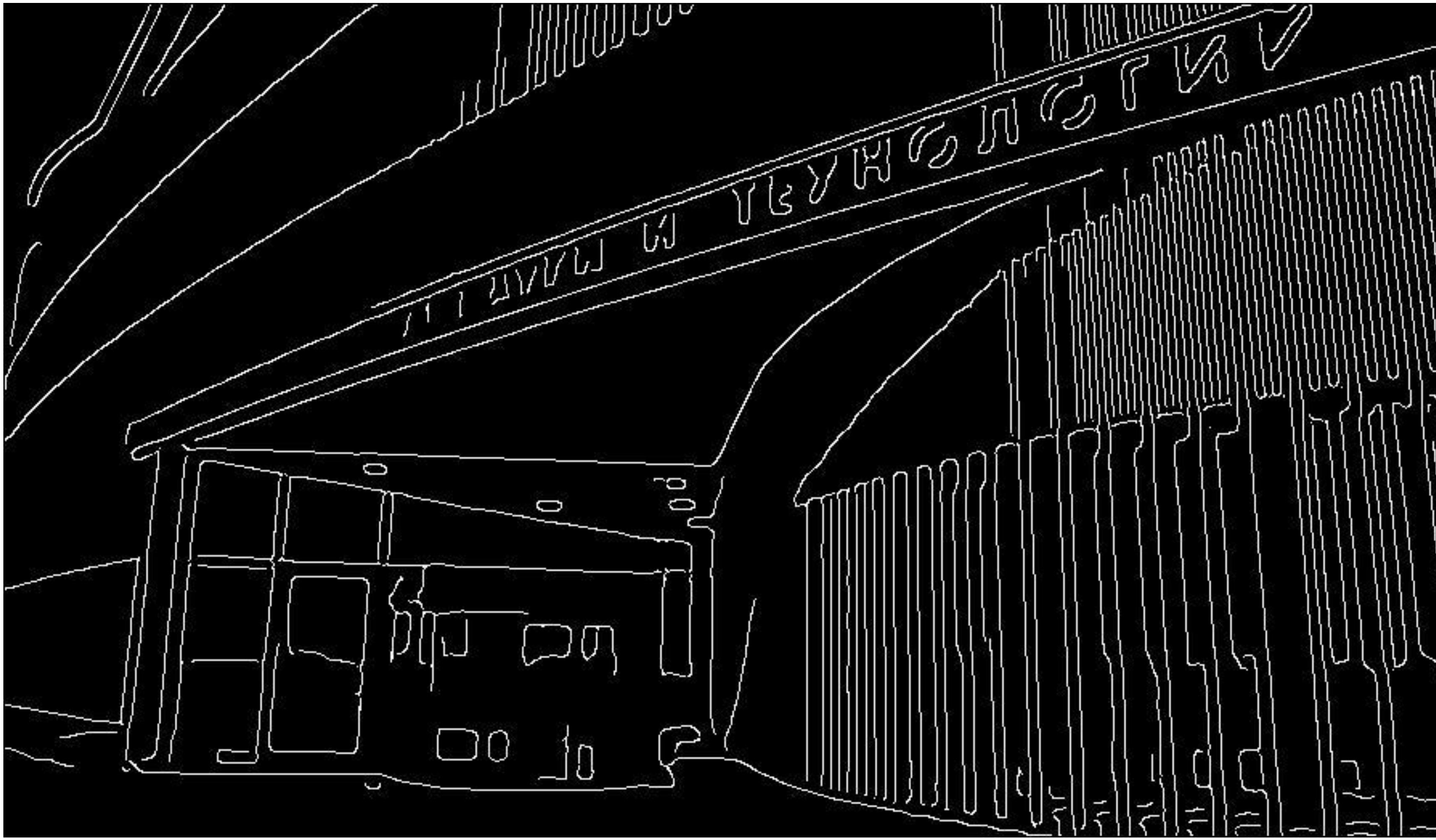


- Edges contain a lot of information about the image
- Generated by discontinuities, edges give a lot of information about separate objects, or their separate parts

Where do edges come from?



Please name 3 “sources” of edges on this photo

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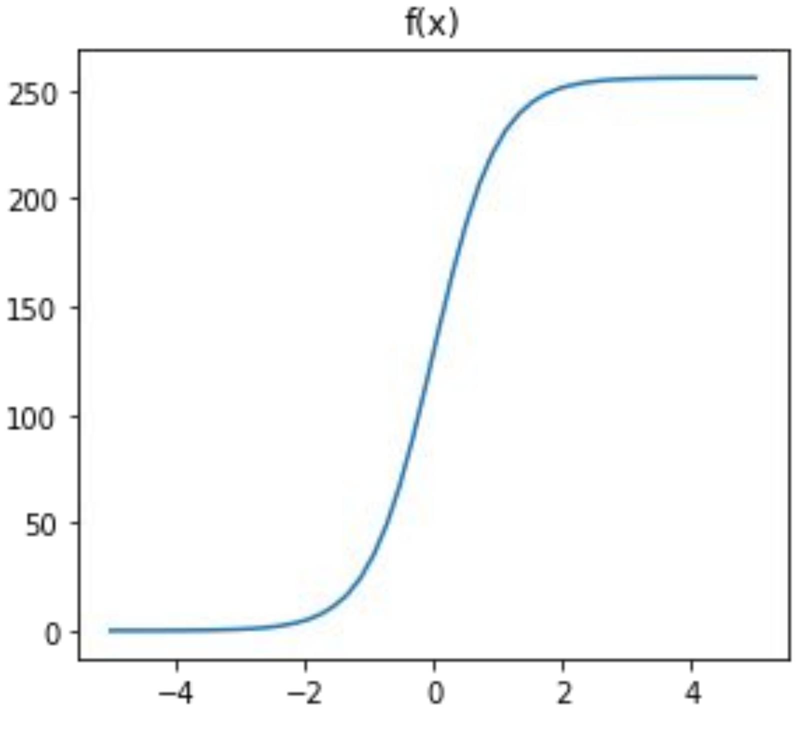
Where do edges come from?

Please name 3 “sources” of edges on this photo

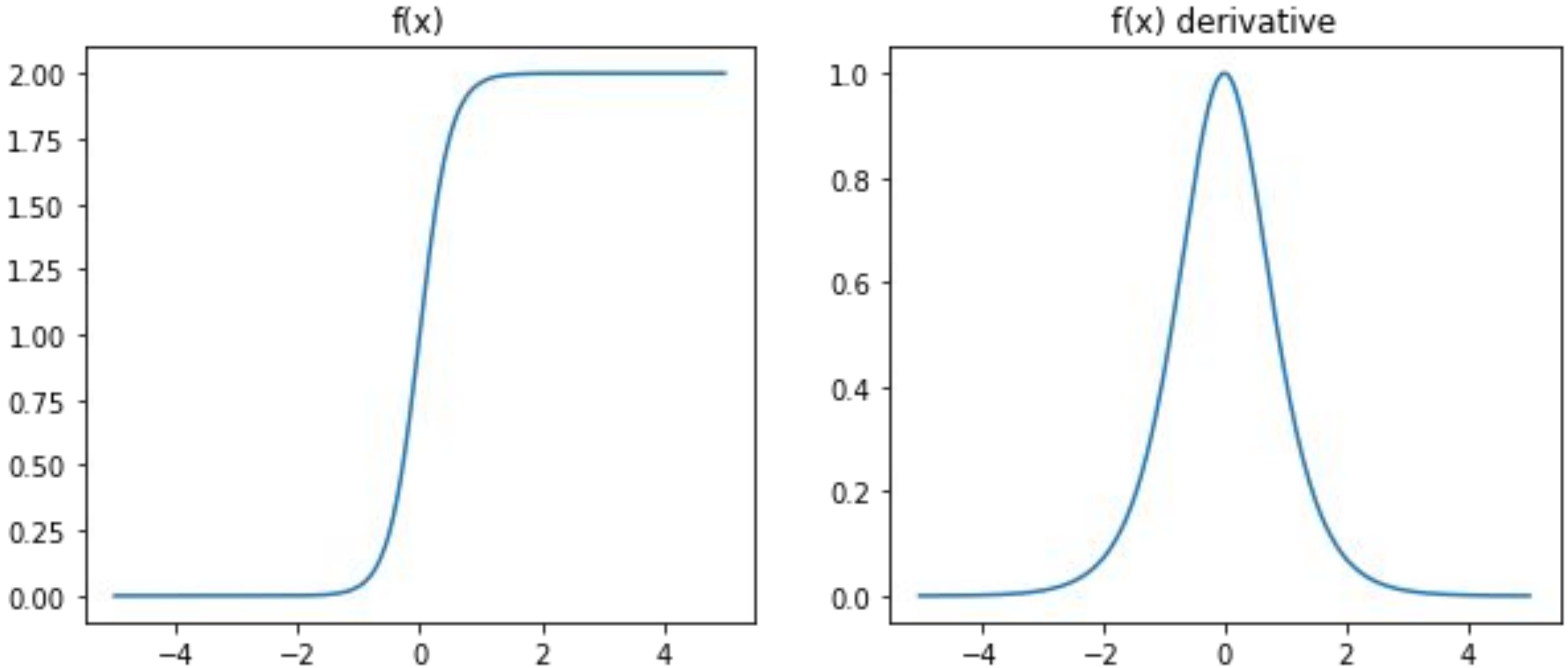
Where do edges come from?

1. Surface / Depth
2. Colors
3. Illumination

How to find discontinuities of a function?



How to find discontinuities of a function?



Derivative is a good way to find edges!

But what can we do with 2D discrete functions?

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Convolutions: examples of filters

How does it work?

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Source: Wikipedia

Convolutions: examples of filters

How does it work?

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Source: Wikipedia

Convolutions: examples of filters

How does it work?

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Source: Wikipedia

Separable convolutions

What is the time complexity of a naive convolution algorithm for an image of shape (n, m) and a kernel of shape (k, l)?

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Separable convolutions

Sometimes kernels can be decomposed into outer product of two vectors:

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Separable convolutions

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In this case we can exploit the associativity of convolution:

$$f \star (v \star w) = f \star (vw) = (f \star v) \star w$$

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Separable convolutions

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$$f \star (v \star w) = f \star (vw) = (f \star v) \star w$$

What is the time complexity of a naive *separable* convolution algorithm for an image of shape (n, m) and a kernel of shape (k, l)?

Sobel filter

The x-derivative of smoothed image

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Gaussian smoothing

Differentiation

Sobel filter

Approximation of differentiation for both directions on the image:

$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

Horizontal changes

$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

Vertical changes

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Sobel filter

Approximation of differentiation for both directions on the image:

$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

Horizontal changes

$$\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

Vertical changes

Gradient magnitude $\mathbf{G} = \sqrt{\mathbf{G}_x{}^2 + \mathbf{G}_y{}^2}$

Gradient direction $\mathbf{\Theta} = \operatorname{arctan}\left(\frac{\mathbf{G}_y}{\mathbf{G}_x}\right)$

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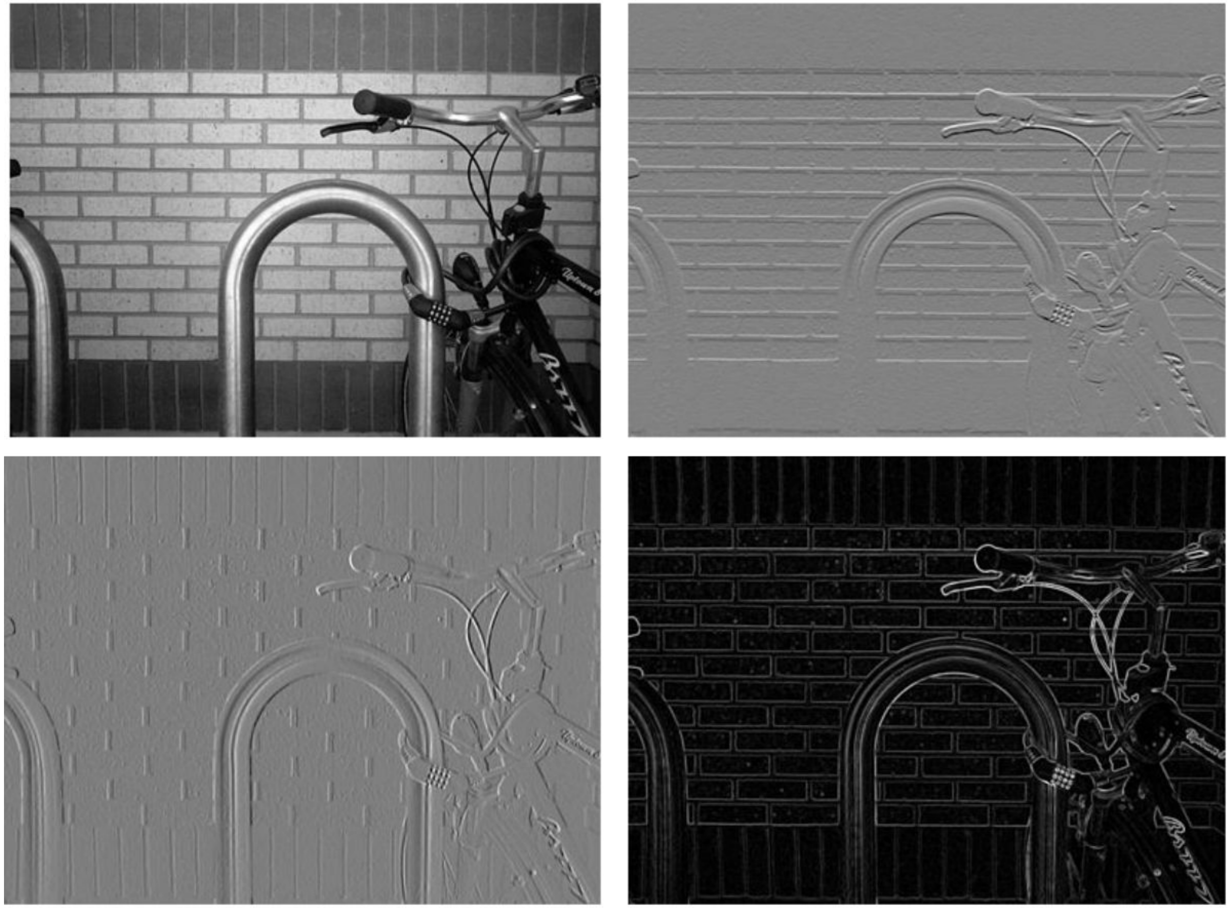
Sobel filter (example)

Grayscale image

y-derivative

G_y



x-derivative

G_x

Gradient magnitude

G

Source: Wikipedia

Edge detection by Sobel

Algorithm:

1. Convolve the image with filters G_X and G_y to estimate image derivatives.

2. Calculate gradient magnitude $\mathbf{G} = \sqrt{{\mathbf{G}_x}^2 + {\mathbf{G}_y}^2}$

Note: these edges aren't binary

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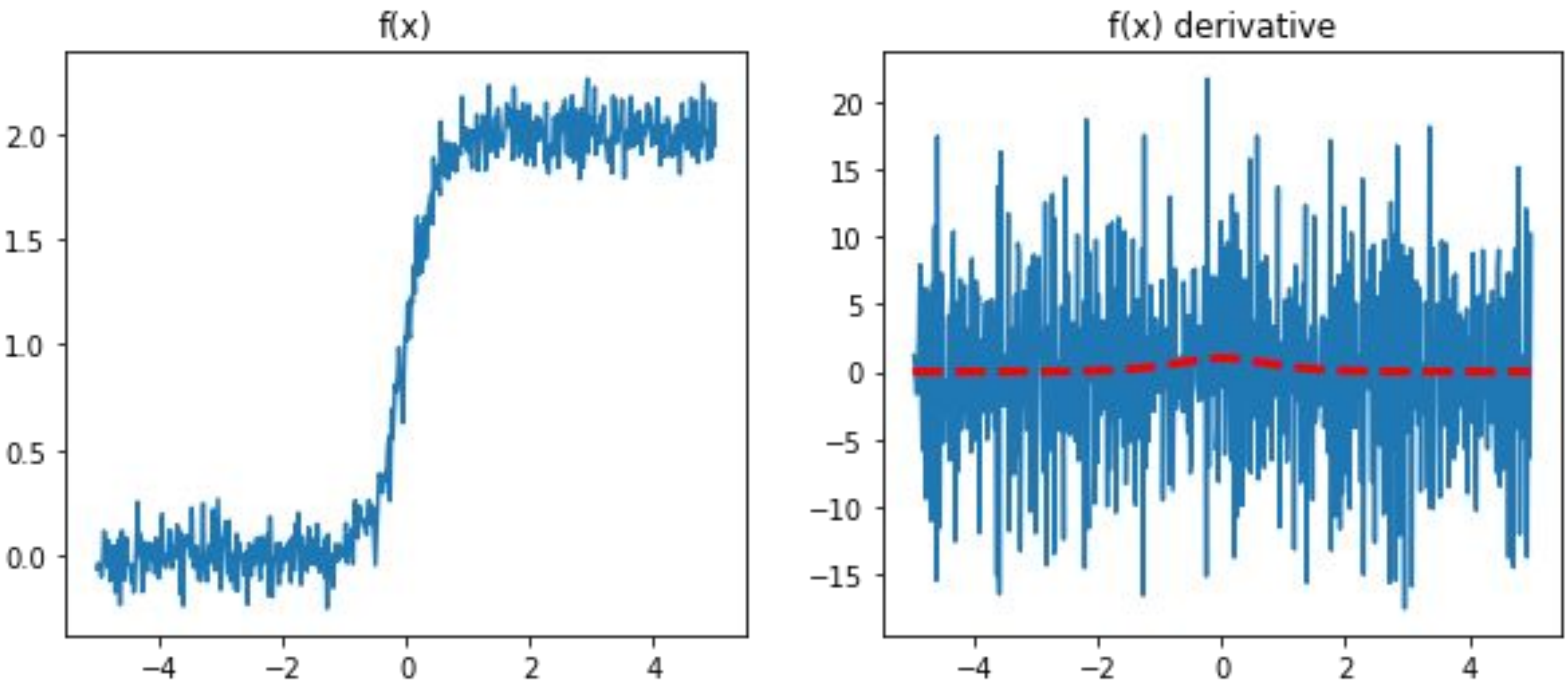
Sobel filter (issues)



Numerical derivatives are sensitive to noise (dashed red line represents the analytical derivative).

What can we do with our image?

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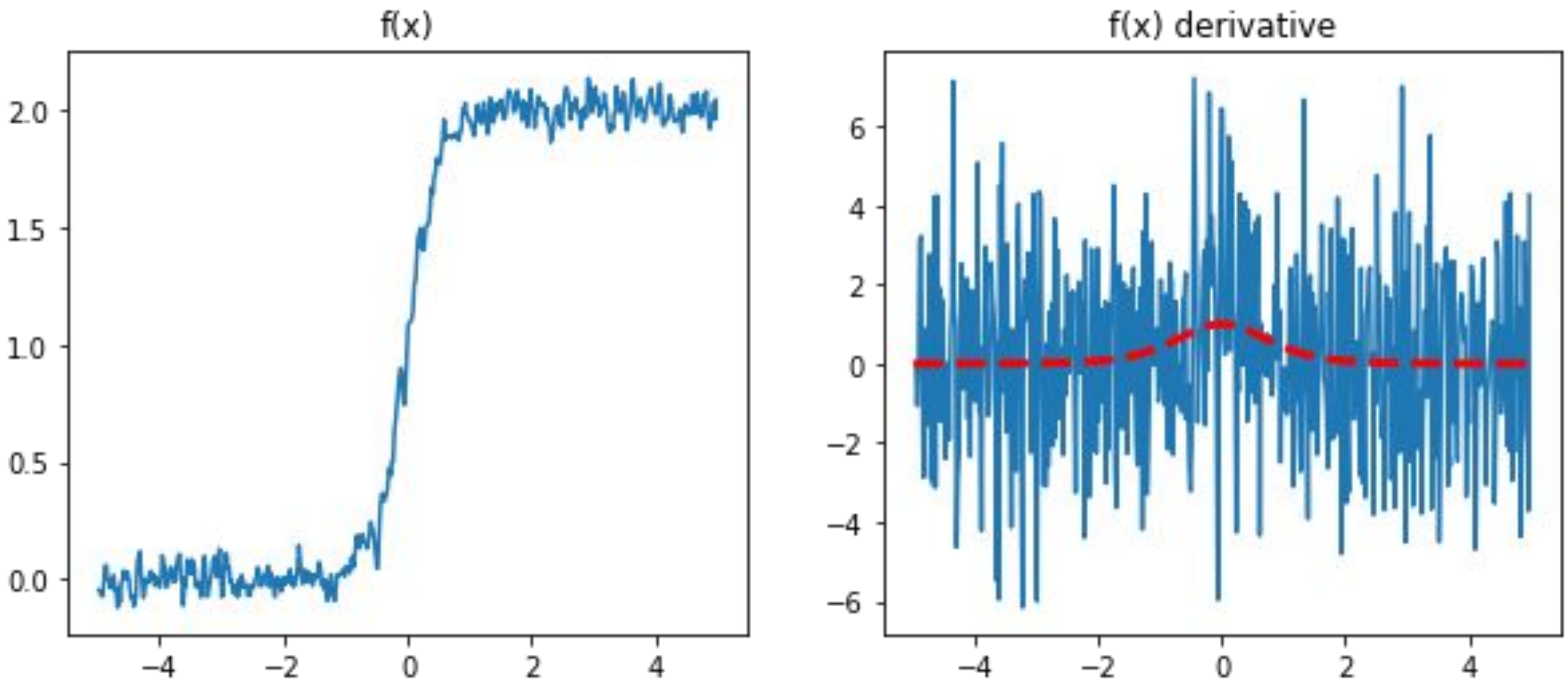
Sobel filter (issues)



Smoothing! For example, moving average with window=3

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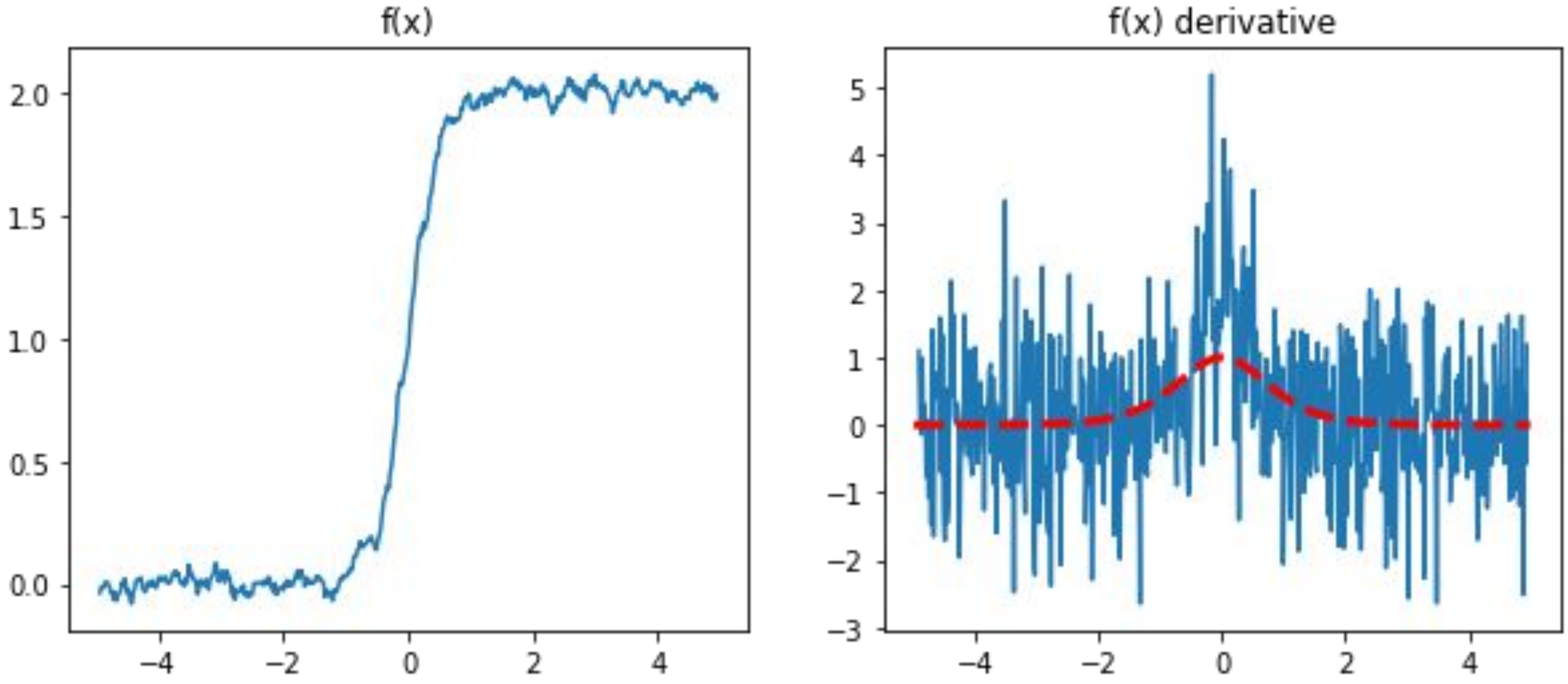
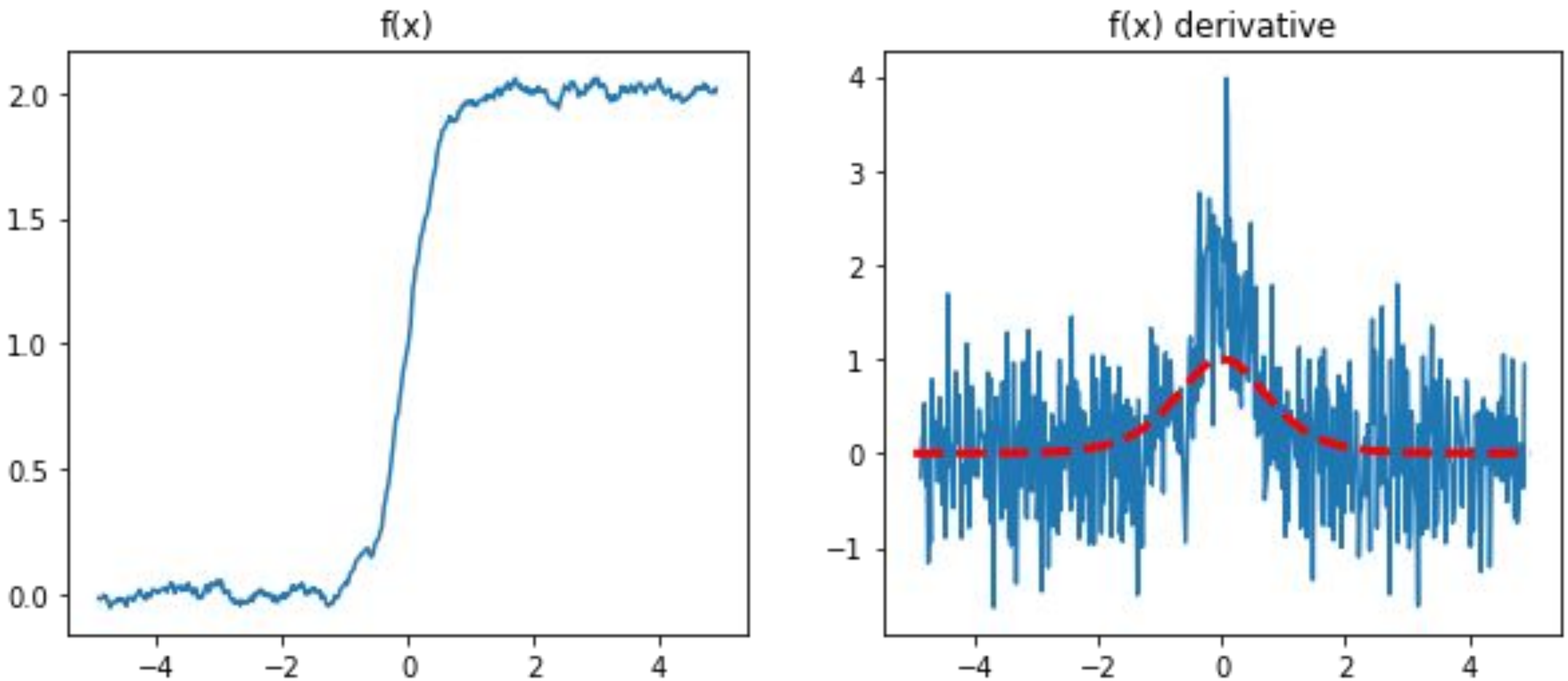
Sobel filter (issues)



Smoothing! For example, moving average with window=7

Sobel filter (issues)



Smoothing! For example, moving average with window=11

Edge detection - desired features

- Detection of edge with low false negative error rate, which means that the detection should accurately catch as many edges shown in the image as possible.

- The edge point should accurately localize on the center of the edge.

- A given edge in the image should only be marked once, and where possible, image noise should not create false edges (low false positive errors rate).

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Canny edge detector - steps

1. Noise suppression (e.g. **gaussian filter**, median filter)

2. Gradient magnitude and direction (e.g. via Sobel filter)

3. **Non-Maximum Suppression**

4. **Hysteresis thresholding and connectivity analysis**

J. Canny, A Computational Approach To Edge Detection, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

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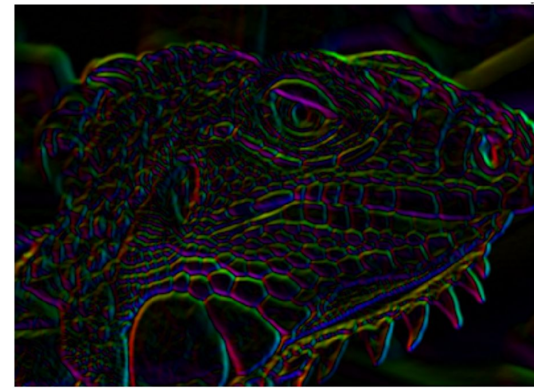
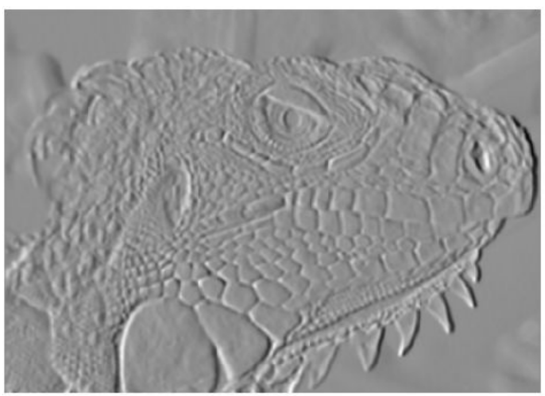
Canny edge detector (step-by-step)

Noise suppression + gradient calculation

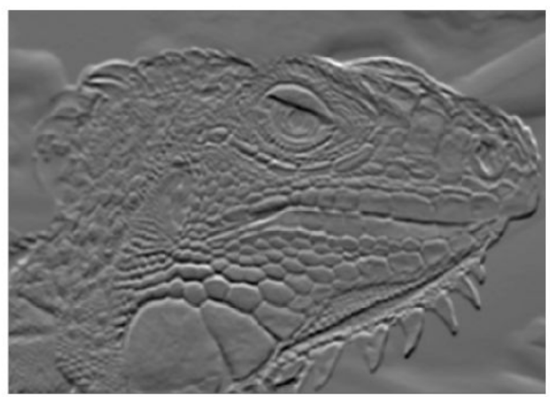
orig. image

gradient orientation





X-Derivative of Gaussian



Y-Derivative of Gaussian

Gradient Magnitude

Source: J. Hayes

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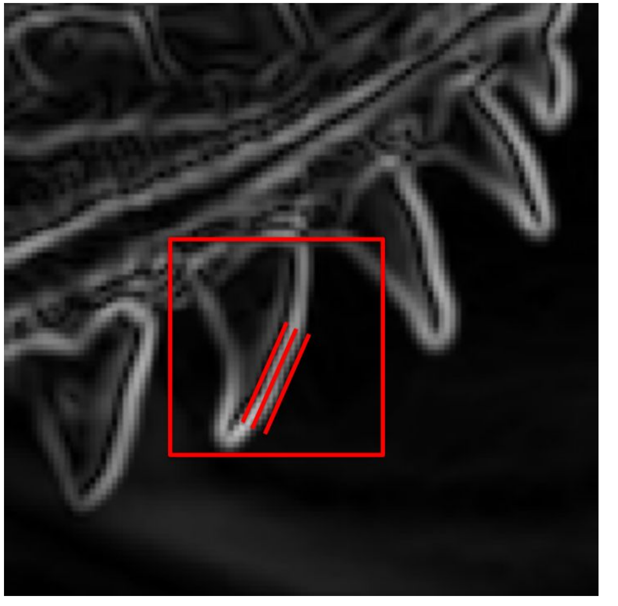
Canny edge detector (step-by-step)

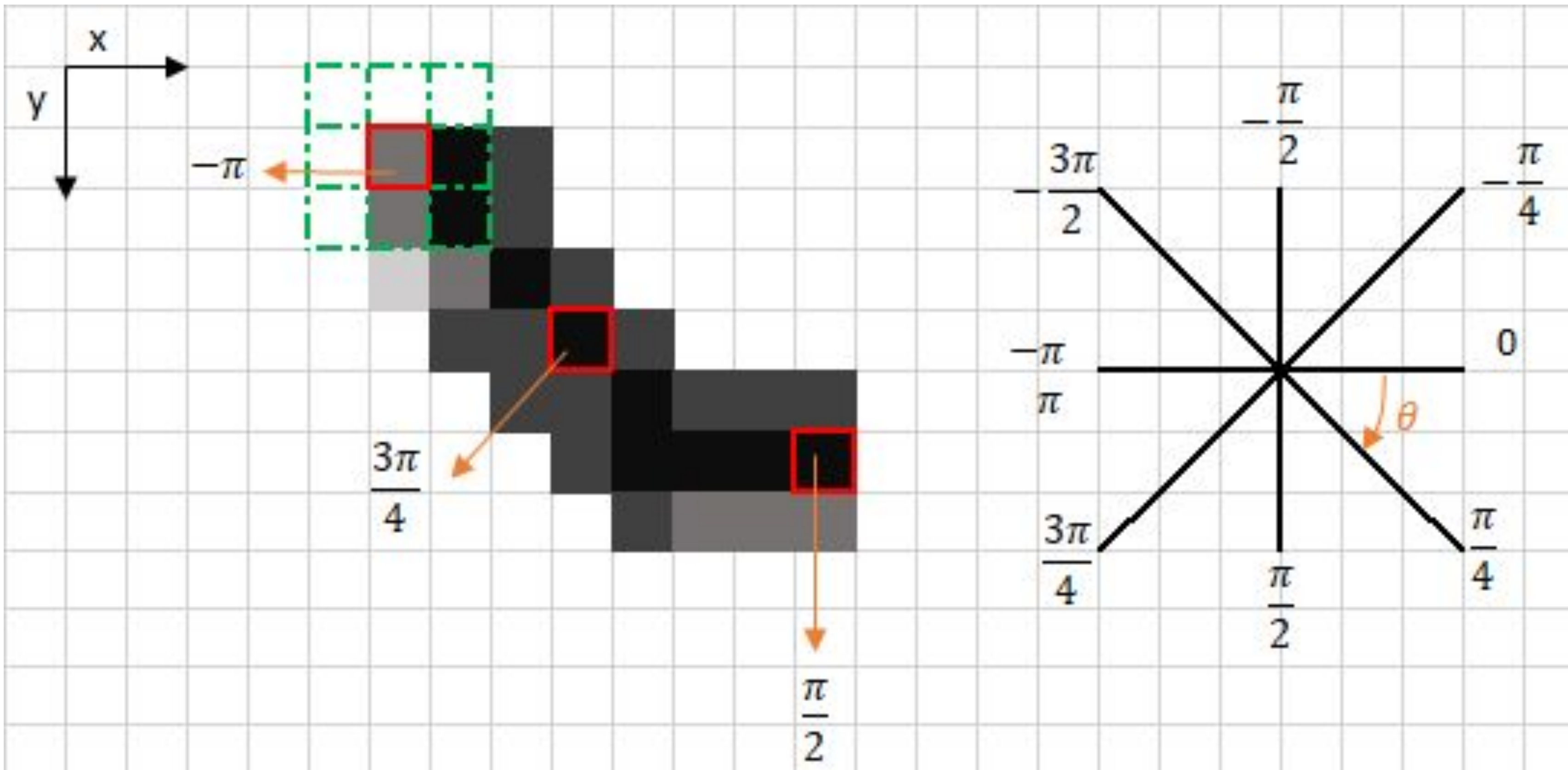
Non-Maximum Suppression

- Edge occurs where gradient reaches a (local) maxima
- Consider only 8 angle directions (e.g. 45º, 90º, 135º, ...)
- Suppress all pixels in each direction which are not maxima



Canny edge detector (step-by-step)

Non-Maximum Suppression



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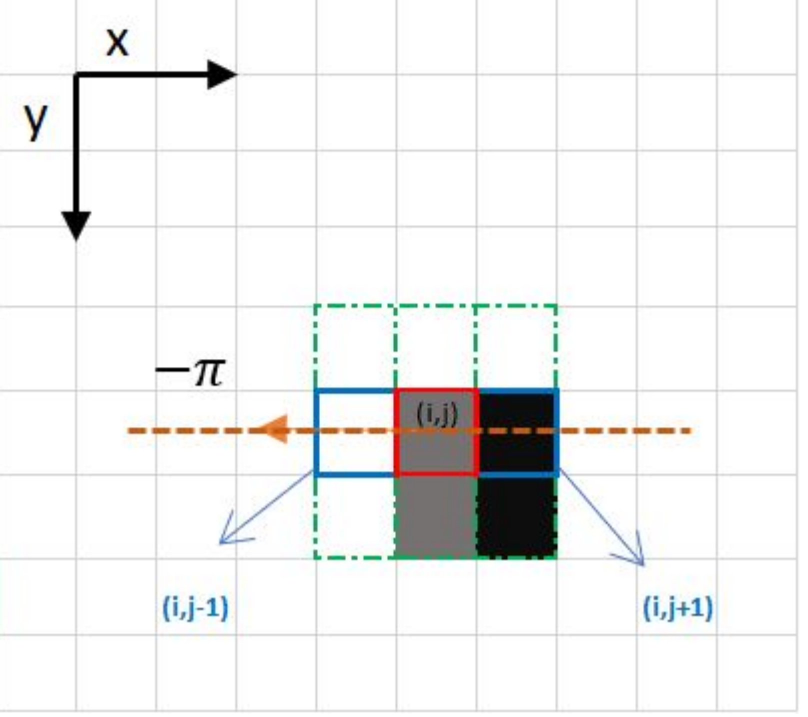
Canny edge detector (step-by-step)

Non-Maximum Suppression

- the pixel **(i, j)** is being processed
- Gradient orientation is *approximately* -π (orange line).
- We consider pixels on the same **gradient direction**: **(i, j-1)** and **(i, j+1)**
- if **(i, j)** is more intense than these two neighbors, then it is kept
- overwise, it is suppressed (set to 0)



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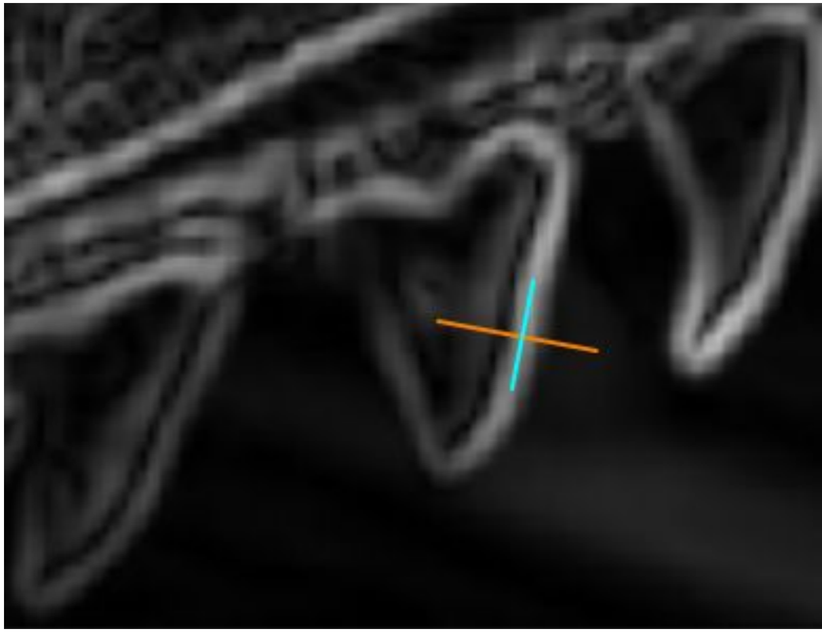

The blue line is aligned with the edge, so we don't want to apply NMS along this direction

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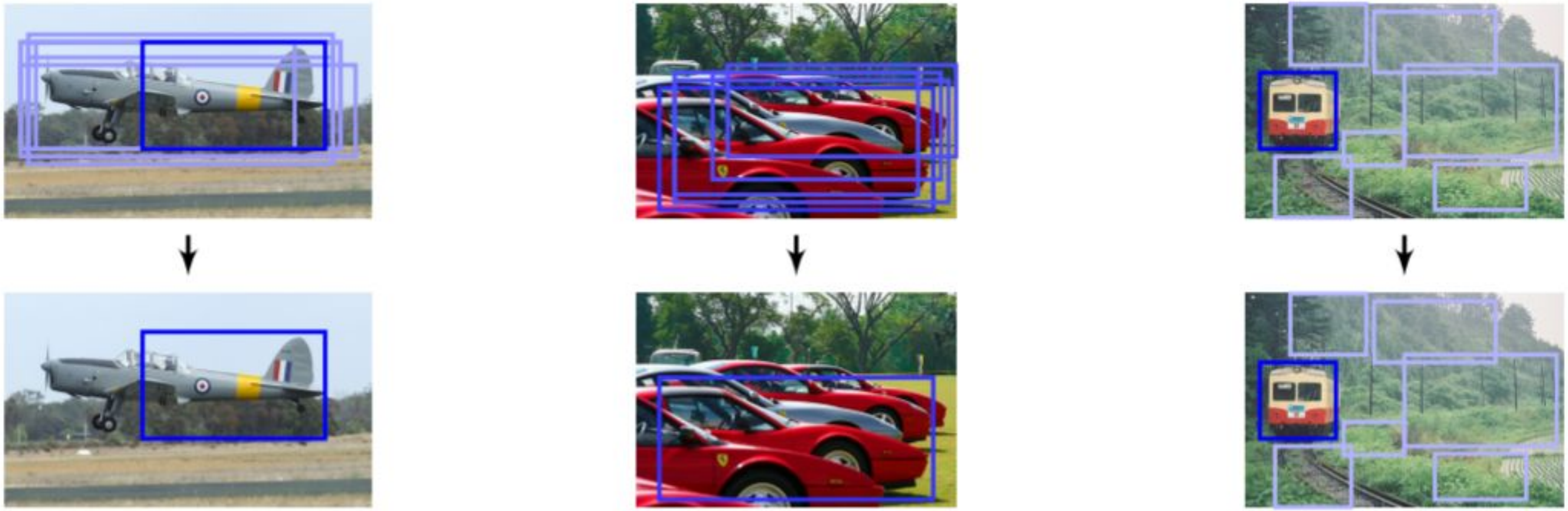
Canny edge detector (step-by-step)

Non-Maximum Suppression - a side note

- NMS (the same idea, but different algorithms) is very useful for object detection



Source: Non-Maximum Suppression for Object Detection by Passing Messages between Windows

Canny edge detector (step-by-step)

Non-Maximum Suppression

Gradient Magnitude > Non-Maximum Suppression

Canny edge detector (step-by-step)

Non-Maximum Suppression

Gradient Magnitude › Non-Maximum Suppression

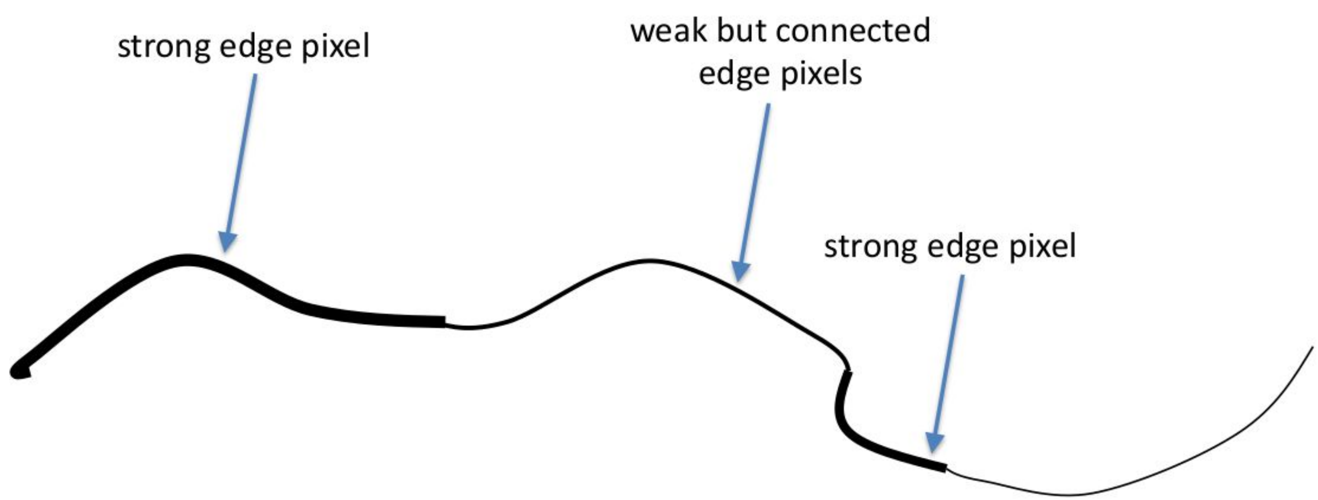
Canny edge detector (step-by-step)

Hysteresis thresholding and connectivity analysis

- Define two thresholds: *Low* and *High*
  - pixel < *Low* :: **not an edge**
  - pixel > *High* :: **strong edge**
  - *Low* < pixel < *High* :: **weak edge**

Canny edge detector (step-by-step)

Hysteresis thresholding and connectivity analysis



- Re-declare **weak edge** as **strong edge** if it is in the same **connected component** with a strong edge
- Re-declare **weak edge** as **not an edge** if it has no strong edges in the connected component

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Canny edge detector (step-by-step)

Non-Maximum Suppression

Non-Maximum Suppression

Hysteresis thresholding

Canny edge detector

Hysteresis thresholding

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Edge detection for a noisy image

Practical exercise: Sobel filter (in jupyter).

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