1

$$B = \frac{30}{10} \longrightarrow 3$$

$$C = \frac{100}{20} = \frac{50}{10}$$

$$G = \frac{190}{50} = \frac{38}{10}$$

$$J = \frac{35}{15} = \frac{70}{30} = \frac{23,33}{10} \longrightarrow 4$$

$$P = \frac{190}{30} = \frac{63,33}{10} \longrightarrow 1$$

$$S = \frac{210}{40} = \frac{52,5}{10} \longrightarrow 2$$

PSBJ D

② $ax^2+bx+c=0 \Rightarrow$ wortels $(x_1,x_2) \in \mathbb{R}$

? Welke vorm heeft wortels $\left(-\frac{1}{x_1}, -\frac{1}{x_2}\right)$?

$$a(x-x_1)(x-x_2) = a\left[x^2 - x_2x - x_1x + x_1x_2\right]$$

$$= ax^2 + \underbrace{a(-x_1-x_2)}_{b}x + \underbrace{ax_1x_2}_{c}$$

$S: -\frac{b}{a} = x_1+x_2$ $\quad$ $P: \frac{c}{a} = x_1x_2$

$$\begin{cases} S = -\frac{1}{x_1} - \frac{1}{x_2} = \frac{-x_1-x_2}{x_1x_2} = \frac{\frac{b}{a}}{\frac{c}{a}} = \frac{b}{a}\cdot\frac{a}{c} = \frac{b}{c} \\ P = \frac{1}{x_1x_2} = \frac{a}{c} \end{cases}$$

$$\to x^2 - \frac{b}{c}x + \frac{a}{c} = 0 \Rightarrow cx^2 - bx + a = 0$$

D

③ $(x^2+x+1)^{\cancel{2}} = -\cancel{(x^2+x+1)}\left(x^2+x-\frac{1}{2}\right)$

$\hookrightarrow D = 1^2 - 4.1.1 < 0$

↳ geen reële wortels

↳ functie altijd > 0

↳ je mag delen

$\Rightarrow (\times 2) \Rightarrow 2x^2+2x+2 = -2x^2-2x+1$

$\Rightarrow 4x^2+4x+1=0$

$\hookrightarrow D = 4^2-4.4.1 = 16-16 = 0$

$\Rightarrow$ 1 wortel $-\frac{4}{2.4} = -\frac{1}{2}$ A

④ $x^2+y^2-16x-12y+75=0$

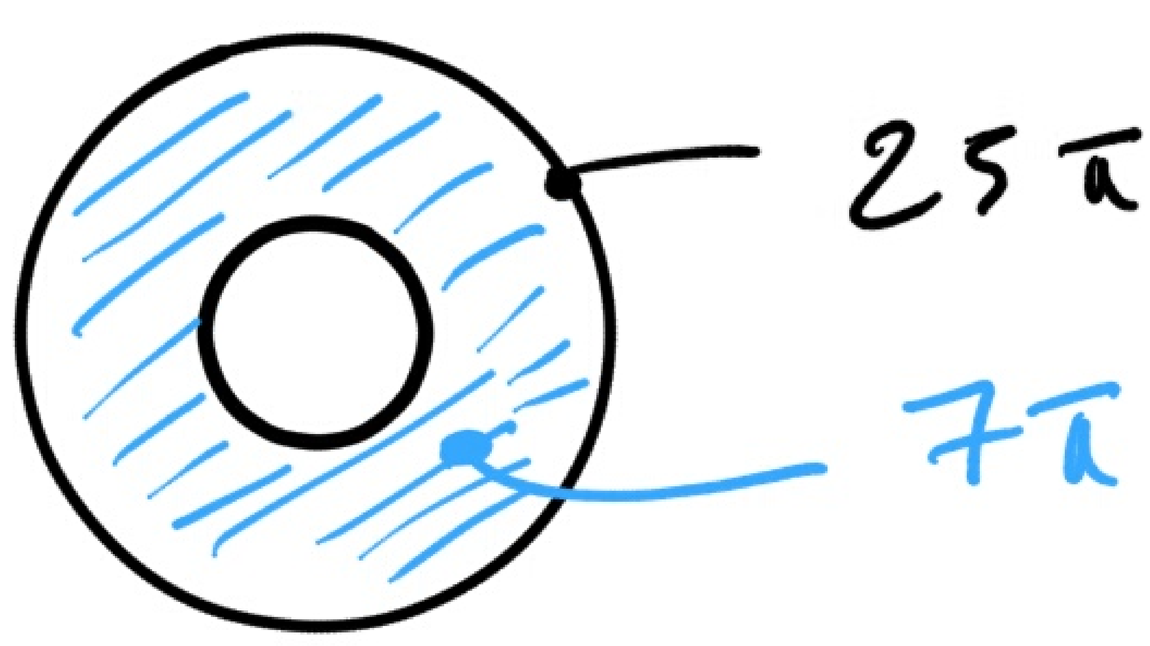
$$\begin{cases} x^2-16x = x^2-16x+64-64 = (x-8)^2-64 \\ y^2-12y = y^2-12y+36-36 = (y-6)^2-36 \end{cases}$$

$(x-8)^2+(y-6)^2-64-36+75=0$

$(x-8)^2+(y-6)^2=25 \Rightarrow \begin{cases} \text{middelpunt} = (8,6) \\ \text{straal} = \sqrt{25} = 5 \end{cases}$

$\downarrow$

opp $= \pi r^2 = 25\pi$



opp kleine cirkel:

$25\pi - 7\pi = \underbrace{18}\pi$

$\hookrightarrow r^2 = 18$

Vergelijking kleine cirkel:

$(x-8)^2+(y-6)^2=18$

A: $(4-8)^2+(8-6)^2 = 16+4$

B: $(6-8)^2+(7-6)^2 = 4+1$

C: $(10-8)^2+(10-6)^2 = 4+16$

D: $(11-8)^2+(9-6)^2 = \boxed{9+9=18}$

D

5 $(\sin x + \cos x)^2$

$= \underline{\sin^2 x + \cos^2 x} + \underline{2\sin x \cos x}$

$= \underline{1} + \sin(2x)$

$\Rightarrow \underline{6} + \sin(30) + \sin(60) + \sin(90)$

$+ \sin(120) + \sin(150) + \sin(180)$

$= 6 + \frac{1}{2} + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} + \frac{1}{2} + 0$

$= 6 + 2 + \sqrt{3} = 8 + \sqrt{3}$ D

⑥ $f(x) = \dfrac{1}{x-1} - 2x + 1$? snijpunt asymptoten

$$\Rightarrow \frac{1 - 2x(x-1) + x - 1}{x-1} = \frac{-2x^2 + 3x}{x-1}$$

VA: noemer $= 0$ $\Rightarrow$ functie kan daar niet!

$$\Rightarrow x - 1 = 0 \Rightarrow \boxed{x = 1}$$

HA: $\dfrac{-2x^2+3x}{x-1}$ $\begin{array}{l} \to \text{graad} = 2 \\ \to \text{graad} = 1 \end{array} \Bigg\} \Rightarrow$ geen HA!

SA: Deling uitvoeren, maar dat is wat gegeven was!

$$f(x) = \frac{1}{x-1} - 2x + 1$$

$$\lim_{x \to \infty} \frac{1}{x-1} = \lim_{x \to \infty} \frac{1/x}{1 - 1/x} = \frac{0}{1} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x-1} = 0$$

Allebei $0 \Rightarrow$ OK!

$$\Rightarrow \text{vgl SA} = \boxed{y = -2x + 1}$$

Snijpunt $\to x = 1 \to y = -2.1 + 1$
$= -1$

$\to (1, -1)$ C

⑦ $f(x) = 2x - \ln(2x)$ met $x > 0$

? rico raaklijn die door $(0,0)$ gaat ?

vgl rechte door $(0,0) \Rightarrow y = m \cdot x$

vgl rechte door punt $(a, f(a))$

$$y - f(a) = f'(a)(x-a)$$

met punt $(a, f(a))$ het punt waar de raaklijn aan de functie raakt.

$$f'(x) = 2 - \frac{2}{2x} = 2 - \frac{1}{x}$$

$$\Rightarrow y - 2a + \ln(2a) = \left(2 - \frac{1}{a}\right)(x-a)$$

$$y - \cancel{2a} + \ln(2a) = 2x - \cancel{2a} - \frac{x}{a} + 1$$

$$\Rightarrow y = 2x + \frac{x}{a} + 1 - \ln(2a)$$

↳ in punt $(0,0)$

$$0 = 0 + 0 + 1 - \ln(2a)$$

$$\Rightarrow \ln(2a) = 1 \Rightarrow e^{\ln(2a)} = e^1$$

$$2a = e \Rightarrow \boxed{a = \frac{e}{2}}$$

$$\Rightarrow \text{rico} \Rightarrow f'(a) = f'\left(\frac{e}{2}\right) = 2 - \frac{2}{e}$$

A

$$= \frac{2e - 2}{e}$$

7 OF

raaklijn door $(0,0) \Rightarrow y = y' \cdot x$

$$\frac{y}{x} = y'$$

$$\Rightarrow \frac{2x - \ln(2x)}{x} = 2 - \frac{1}{x}$$

$$\cancel{2x} - \ln(2x) = \cancel{2x} - 1$$

$$\Rightarrow \ln(2x) = 1 \Rightarrow e^{\ln(2x)} = e^{1}$$

$$2x = e \Rightarrow x = \frac{e}{2}$$

$$\Rightarrow \text{rico } f'\left(\frac{e}{2}\right) = 2 - \frac{2}{e} = \frac{2e-2}{e}$$

A

⑧ $f(x) = 3x^3 + 3x^2 - 6x$

? opp onder de x-as en tussen

$x = -3$ en $x = 1$?

⇒ nulpunten : $3x(x^2 + x - 2)$

0

$$\frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

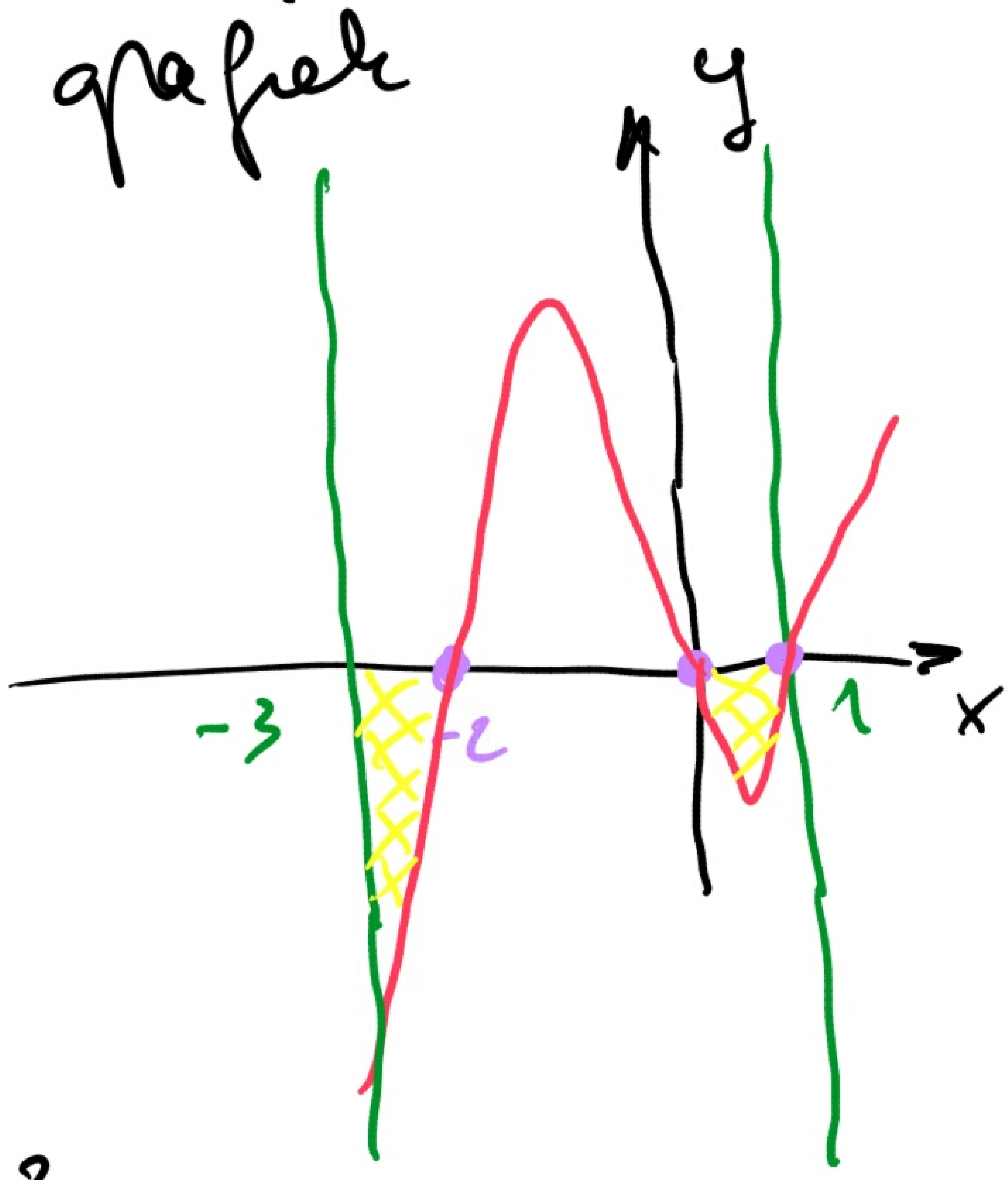
$$-\frac{1}{2} \pm \frac{\sqrt{9}}{2} = -\frac{1}{2} \pm \frac{3}{2}$$

$2/2 = 1$

$-4/2 = -2$

$x = 0, \; x = 1, \; x = -2$

⇒ grafiek



$x = 1/2$

$3 \cdot \frac{1}{8} + 3 \cdot \frac{1}{4} - \frac{6}{2} < 0$

$x = -1$

$3(-1)^3 + 3(-1)^2 + 6$

$-3 + 3 + 6 > 0$

$$\int_{-3}^{-2} (3x^3 + 3x^2 - 6x)\,dx + \int_{0}^{1} (3x^3 + 3x^2 - 6x)\,dx$$

$$\left[\frac{3}{4}x^4 + x^3 - 3x^2\right]\Bigg|_{-3}^{-2} + \left[\frac{3}{4}x^4 + x^3 - 3x^2\right]\Bigg|_{0}^{1}$$

⑧ $\left[\frac{3}{4}(-2)^4+(-2)^3-3(-2)^2\right]$

$-\left[\frac{3}{4}(-3)^4+(-3)^3-3(-3)^2\right]$

$+\left[\frac{3}{4}+1-3\right]-0$

$=\left[\frac{3}{4}16-8-12\right]-\left[\frac{243}{4}-27-27\right]$

$+\left[\frac{3}{4}-2\right]$

$=\left[12-8-12\right]-\left[\frac{243}{4}-\frac{108}{4}-\frac{108}{4}\right]+\left[\frac{3}{4}-2\right]$

$=\left[-8\right]-\left[\frac{27}{4}\right]+\left[\frac{3}{4}-2\right]$

$=-8-2-\frac{27}{4}+\frac{3}{4}=-10-\frac{24}{4}$

$=-10-6=\boxed{-16}$

$\Rightarrow$ opp $=16$ C

9 3M 7V → kies 4 personen, minstens 1 M!

? Hoeveel keuzes? → Combinatie / geen volgorde \ geen herhaling

⇒ 10 personen → 4 kiezen

$$C_{10}^{4} = \frac{10!}{4!\,(10-4)!} = \frac{10 \cdot \cancel{9}^{3} \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5}}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2}} = 210$$

⇒ Alle keuzes met 4 vrouwen vallen af, want minstens 1 man!

$$C_{7}^{4} = \frac{7!}{4!\,(7-4)!} = \frac{7 \cdot \cancel{6} \cdot 5}{\cancel{3} \cdot \cancel{2}} = 35$$

⇒ Mogelijke keuzes ⇒ $210 - 35 = \boxed{175}$

D

9 OF

1M+3V of 2M+2V of 3M+1V

$C_3^1 \cdot C_7^3 + C_3^2 \cdot C_7^2 + 1 \cdot 7$

$$\frac{3!}{1!(3-1)!} \cdot \frac{7!}{3!(7-3)!} + \frac{3!}{2!(3-2)!} \cdot \frac{7!}{2!(7-2)!} + 7$$

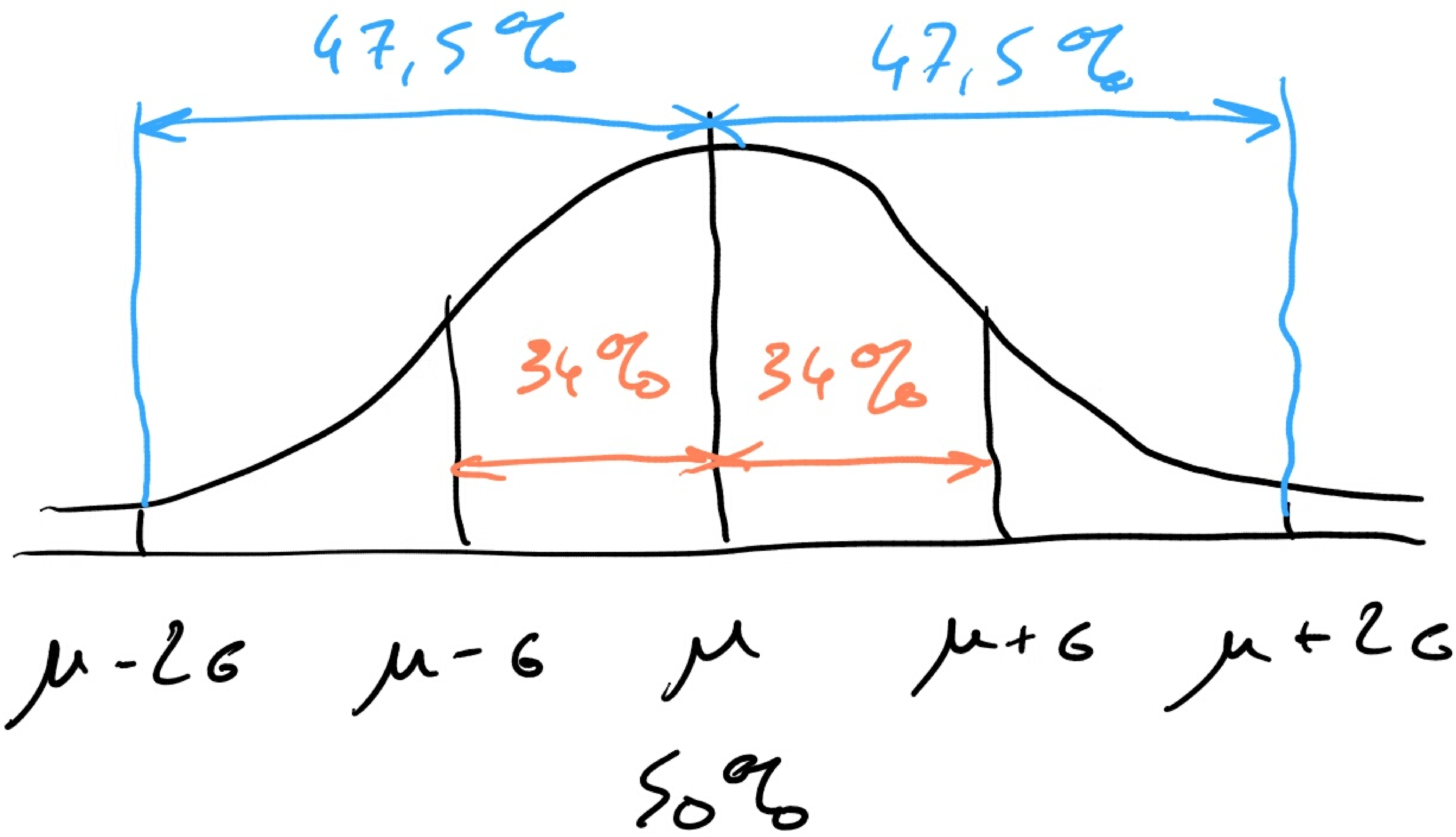
$$\frac{3 \cdot \cancel{2}}{\cancel{2}} \cdot \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2}} + \frac{3}{1} \cdot \frac{7 \cdot \cancel{6}^{3}}{\cancel{2}} + 7$$

$3 \cdot 35 + 63 + 7 = 105 + 63 + 7$

$= \boxed{175}$

D

10



$\mu = 161$ cm, $\sigma = 6$ cm, 200000 ✓

? hoeveel tussen 149 cm en 167 cm ?

$$\begin{cases} 149 = 161 - 12 = \mu - 2\sigma & 47{,}5\% \\ 167 = 161 + 6 = \mu + \sigma & 34\% \end{cases}$$

$\Rightarrow$ totaal: $47{,}5 + 34 = 81{,}5\,\%$

$$200\,00\cancel{0} \cdot \frac{81{,}5}{1\cancel{00}} = \boxed{163\,000}$$

C