

# Goniometrische formules: sinus en afleidingen

Begin met de t-formule voor  $\tan x = \frac{2t}{1-t^2}$  met  $t = \tan\left(\frac{x}{2}\right)$

$$\tan x = \frac{\sin x}{\cos x} = \frac{2t}{1-t^2} \quad + \text{ de noemer voor } \sin \text{ en } \cos \neq \tan$$

$\frac{1+t^2}{1-t^2}$

$$\Rightarrow \tan x = \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}} = \sin x$$
$$\cos x$$

Stel nu  $\alpha = \frac{x}{2} \Rightarrow 2\alpha = x$  en  $\tan\left(\frac{x}{2}\right) = t = \tan(\alpha)$

$$\Rightarrow \sin(2\alpha) = \frac{2t \tan \alpha}{1 + \tan^2 \alpha} \quad \Rightarrow \tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\Rightarrow \cos(2\alpha) = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

Met  $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$

$$\Rightarrow \tan(2\alpha) = \frac{2 \frac{\sin \alpha}{\cos \alpha}}{1 - \frac{\sin^2 \alpha}{\cos^2 \alpha}} = 2 \frac{\sin \alpha}{\cancel{\cos \alpha}} \cdot \frac{\cos^2 \alpha}{\cos^2 \alpha - \sin^2 \alpha}$$

$$\Rightarrow \tan(2\alpha) = \frac{2 \sin \alpha \cos \alpha}{\cos^2 \alpha - \sin^2 \alpha} = \frac{\sin(2\alpha)}{\cos(2\alpha)}$$

$\begin{cases} \sin(2\alpha) = 2 \times \sin \cos, \text{ dan } \cos = 2 \sin \alpha \cos \alpha \\ \cos(2\alpha) = \cos \text{ eerst, kijkt op } \cos^2 \alpha + \sin^2 \alpha = 1 = \cos^2 \alpha - \sin^2 \alpha \end{cases}$

$$\begin{aligned}\sin(2\alpha) &= \sin(\alpha + \alpha) \Rightarrow \sin(a+b) \\ &= 2\sin\alpha\cos\alpha \Rightarrow \sin\alpha\cos\alpha + \sin\alpha\cos\alpha\end{aligned}$$

$$\Rightarrow \sin(a+b) = \underline{\sin a} \cdot \underline{\cos b} + \underline{\sin b} \cdot \underline{\cos a}$$

$$\Rightarrow \sin(a+(-b)) = \sin a \cdot \cos b - \underline{\sin b \cdot \cos a}$$

$$\begin{aligned}\cos(2\alpha) &= \cos(\alpha + \alpha) \Rightarrow \cos(a+b) \\ &= \cos^2\alpha - \sin^2\alpha \Rightarrow \cos\alpha\cos\alpha - \sin\alpha\sin\alpha\end{aligned}$$

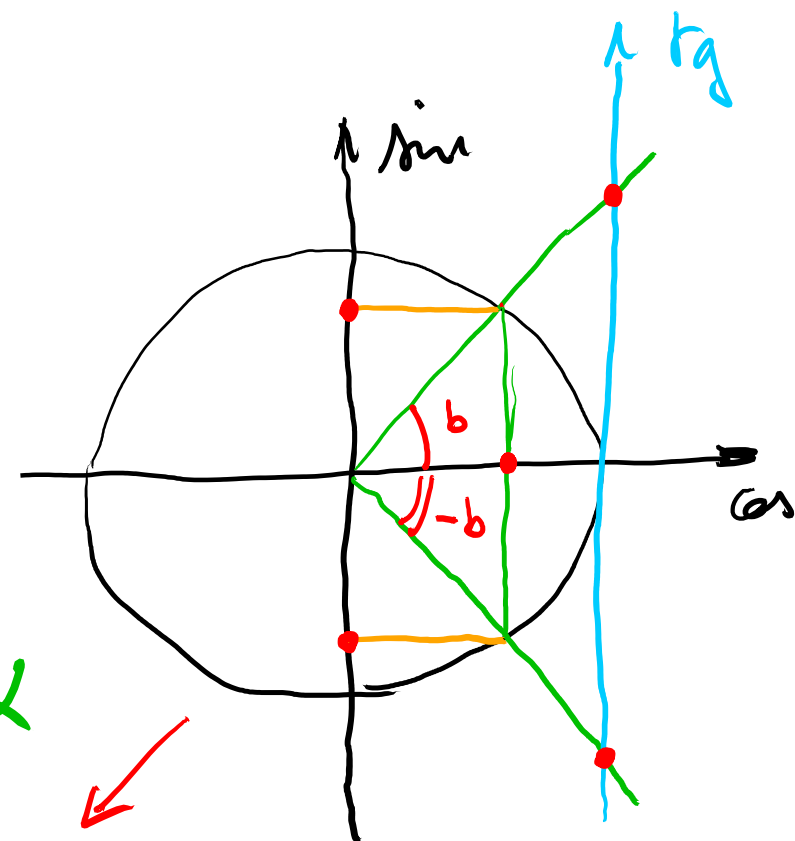
$$\Rightarrow \cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$$

$$\Rightarrow \cos(a+(-b)) = \cos a \cdot \cos b + \sin a \cdot \underline{\sin b}$$

$$\begin{aligned}\tan(2\alpha) &= \tan(\alpha + \alpha) \Rightarrow \tan(a+b) \\ &= \frac{\tan\alpha + \tan\alpha}{1 - \tan\alpha \cdot \tan\alpha}\end{aligned}$$

$$\Rightarrow \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$$

$$\Rightarrow \tan(a+(-b)) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$$



$$\begin{cases} \sin(-b) = -\sin b \\ \cos(-b) = \cos b \\ \tan(-b) = -\tan b \end{cases}$$

# Sinpro en omgekeerde sinpro

A:  $\sin(a+b) = \sin a \cos b + \sin b \cos a$

B:  $\sin(a-b) = \sin a \cos b - \sin b \cos a$

C:  $\cos(a+b) = \cos a \cos b - \sin a \sin b$

D:  $\cos(a-b) = \cos a \cos b + \sin a \sin b$

$$\begin{cases} a+b=x \\ a-b=y \end{cases} \Rightarrow \begin{cases} a = \frac{x+y}{2} \\ b = \frac{x-y}{2} \end{cases}$$

$\Rightarrow A+B$   $\sin(a+b) + \sin(a-b) = 2 \sin a \cos b$   
 $\sin(x) + \sin(y) = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$

$\Rightarrow A-B$   $\sin(a+b) - \sin(a-b) = 2 \sin b \cos a$   
 $\sin(x) - \sin(y) = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right)$

$\Rightarrow C+D$   $\cos(a+b) + \cos(a-b) = 2 \cos a \cos b$   
 $\cos(x) + \cos(y) = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$

$\Rightarrow C-D$   $\cos(a+b) - \cos(a-b) = -2 \sin a \sin b$   
 $\cos(x) - \cos(y) = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

$$\cos a \cdot \cos b = \frac{1}{2} (\cos(a+b) + \cos(a-b))$$

idem voor de anderen!

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha \Rightarrow \cos(2\alpha) + 1 = \cos^2\alpha - \cancel{\sin^2\alpha} + \cos^2\alpha + \cancel{\sin^2\alpha}$$

$$\Rightarrow 2\cos^2\alpha = 1 + \cos(2\alpha) \Rightarrow \cos^2\alpha = \frac{1}{2}(1 + \cos(2\alpha))$$

$$\cos(2\alpha) - 1 = \cancel{\cos^2\alpha} - \sin^2\alpha - (\cancel{\cos^2\alpha} + \sin^2\alpha)$$

$$\Rightarrow 2\sin^2\alpha = 1 - \cos(2\alpha) \Rightarrow \sin^2\alpha = \frac{1}{2}(1 - \cos(2\alpha))$$

Set  $\alpha = \frac{x}{2}$

Wegen  $\rightarrow$  geometrische Einheit  
Wegen!

$$\Rightarrow \cos^2\left(\frac{x}{2}\right) = \frac{1}{2}(1 + \cos x) \Rightarrow \cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\Rightarrow \sin^2\left(\frac{x}{2}\right) = \frac{1}{2}(1 - \cos x) \Rightarrow \sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\Rightarrow \tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$= \sqrt{\frac{1 - \cos x}{1 + \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}} = \sqrt{\frac{1 - \cos^2 x}{(1 + \cos x)^2}}$$

$$= \sqrt{\frac{\sin^2 x}{(1 + \cos x)^2}} = \frac{\sin x}{1 + \cos x} = \tan\left(\frac{x}{2}\right)$$

$$= \sqrt{\frac{1 - \cos x}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x}} = \sqrt{\frac{(1 - \cos x)^2}{1 - \cos^2 x}}$$

$$= \sqrt{\frac{(1 - \cos x)^2}{\sin^2 x}} = \frac{1 - \cos x}{\sin x} = \tan\left(\frac{x}{2}\right)$$