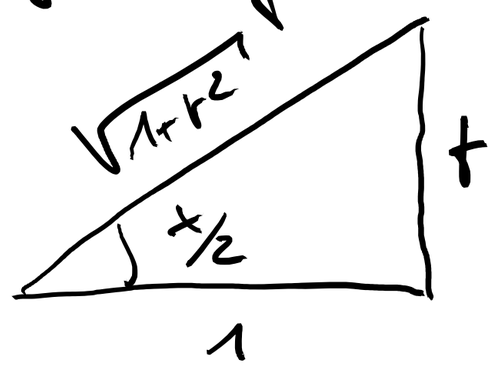
t. formules + sin en cos i.f.v. tg

subst: $\operatorname{tg}\frac{x}{2} = t = \frac{t}{1}$



$$\sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}$$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}}$$

Stel $\frac{x}{2} = \alpha \Rightarrow$

$$\begin{cases} \sin\alpha = \dfrac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} \\ \cos\alpha = \dfrac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} \end{cases}$$

$\sin(2\theta) = 2\sin(\theta)\cdot\cos(\theta)$

$$\sin(x) = 2\,\frac{t}{\sqrt{1+t^2}}\cdot\frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2}$$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$$\cos^2(x) = 1 - \left(\frac{2t}{1+t^2}\right)^2 = \frac{1+t^4+2t^2-4t^2}{(1+t^2)^2} = \frac{1+t^4-2t^2}{(1+t^2)^2} = \frac{(1-t^2)^2}{(1+t^2)^2}$$

$$\Rightarrow \cos(x) = \frac{1-t^2}{1+t^2}$$

$$\operatorname{tg}(x) = \frac{\sin x}{\cos x} = \frac{2t}{1+t^2}\cdot\frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2}$$

$$\frac{dt}{dx} = \left(\operatorname{tg}\left(\frac{x}{2}\right)\right)' = \frac{1}{2}\cdot\frac{1}{\cos^2\left(\frac{x}{2}\right)} \Rightarrow dx = 2\cdot\cos^2\left(\frac{x}{2}\right)\cdot dt$$

$$dx = \frac{2}{1+t^2}\,dt$$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \operatorname{tg} x = \frac{2t}{1-t^2}$$

met $t = \operatorname{tg}\frac{x}{2} \Rightarrow$ stel weer $\alpha = \frac{x}{2} \Rightarrow 2\alpha = x$

$$\sin(2\alpha) = \frac{2\operatorname{tg}\alpha}{1+\operatorname{tg}^2\alpha} \qquad \operatorname{tg}(2\alpha) = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$$

$$\cos(2\alpha) = \frac{1-\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha}$$

$$\Rightarrow \cos(2\alpha) = \frac{1-\frac{\sin^2\alpha}{\cos^2\alpha}}{1+\frac{\sin^2\alpha}{\cos^2\alpha}} = \frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha+\sin^2\alpha} = \cos^2\alpha-\sin^2\alpha$$

$$\Rightarrow \operatorname{tg}(2\alpha) = \frac{2\frac{\sin\alpha}{\cos\alpha}}{1-\frac{\sin^2\alpha}{\cos^2\alpha}} = 2\frac{\sin\alpha}{\cancel{\cos\alpha}} \cdot \frac{\cos^{\cancel{2}}\alpha}{\cos^2\alpha-\sin^2\alpha}$$

$$\operatorname{tg}(2\alpha) = \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha} = \frac{\sin 2\alpha}{\cos 2\alpha}$$