```
Inf • l = eqncirc = x^2 + y^2 == a^2;
            eqnx = x == r Sin[\varphi] + h<sub>2</sub> Sin[\alpha];
            eqny = y == h_1 - r \cos[\varphi] - h_2 \cos[\alpha];
            elim = FullSimplify[Eliminate[{eqnx, eqny}, \{\alpha\}]];
            solve = Thread[FullSimplify[Solve[{eqncirc, elim}, {x, y}]]]
Out[\ \circ\ ] = \left\{ \left\{ x \to \frac{1}{2 r \left( r^2 - 2 r \cos[\varphi] h_1 + h_1^2 - h_2^2 \right) - r \cot[\varphi] \right\} \right\}
                             \sqrt{(-r^2 \sin[\varphi]^2 ((-a^2 + r^2 - 2 r \cos[\varphi] h_1 + h_1^2)^2 - 2 (a^2 + r^2 - 2 r \cos[\varphi] h_1 + h_1^2) h_2^2 + h_2^4))} + Csc[\varphi]
                             h_1 \sqrt{(-r^2 \sin[\varphi]^2 ((-a^2 + r^2 - 2 r \cos[\varphi] h_1 + h_1^2)^2 - 2 (a^2 + r^2 - 2 r \cos[\varphi] h_1 + h_1^2) h_2^2 + h_2^4)))}
                x \to \frac{1}{2 r (r^2 - 2 r \cos[\varphi] h_1 + h_1^2 - h_2^2) + r \cot[\varphi]} (r^2 \sin[\varphi] (a^2 + r^2 - 2 r \cos[\varphi] h_1 + h_1^2 - h_2^2) + r \cot[\varphi]
                             \sqrt{\left(-r^2 \operatorname{Sin}[\varphi]^2 \left(\left(-a^2+r^2-2 \, r \operatorname{Cos}[\varphi] \, h_1+h_1^2\right)^2-2 \left(a^2+r^2-2 \, r \operatorname{Cos}[\varphi] \, h_1+h_1^2\right) h_2^2+h_2^4\right)\right)} - \operatorname{Csc}[\varphi]
                             h_1 \sqrt{\left(-r^2 \operatorname{Sin}[\phi]^2 \left( \left(-a^2+r^2-2 \, r \operatorname{Cos}[\phi] \, h_1+h_1^2 \right)^2-2 \left(a^2+r^2-2 \, r \operatorname{Cos}[\phi] \, h_1+h_1^2 \right) h_2^2+h_2^4 \right)))} \right\},
              \left\{ y \to \frac{1}{2 \left( r^2 - 2 r \cos[\varphi] h_1 + h_1^2 \right)} \left( -r^3 \cos[\varphi] + r^2 \left( 2 + \cos[2 \varphi] \right) h_1 + \frac{1}{2} \left( r^2 - 2 r \cos[\varphi] h_1 + h_1^2 \right) \right) \right\} 
                           h_1(a^2 + h_1^2 - h_2^2) + r Cos[\varphi](-a^2 - 3h_1^2 + h_2^2) -
                          \sqrt{(-r^2 \sin[\varphi]^2 ((-a^2 + r^2 - 2 r \cos[\varphi] h_1 + h_1^2)^2 - 2 (a^2 + r^2 - 2 r \cos[\varphi] h_1 + h_1^2) h_2^2 + h_2^4)))}
                y \rightarrow \frac{1}{2(r^2 - 2 r \cos[\varphi] h_1 + h_1^2)} (-r^3 \cos[\varphi] + r^2 (2 + \cos[2 \varphi]) h_1 +
                          h_1(a^2 + h_1^2 - h_2^2) + r Cos[\varphi](-a^2 - 3h_1^2 + h_2^2) +
                          \sqrt{\left(-r^2 \sin[\varphi]^2 \left(\left(-a^2+r^2-2 r \cos[\varphi] h_1+h_1^2\right)^2-2 \left(a^2+r^2-2 r \cos[\varphi] h_1+h_1^2\right) h_2^2+h_2^4\right))\right)}\right\}}
```

Inf •]:= solve = solve /. Rule → Equal

$$\begin{split} & \int_{\mathbb{R}^{4}} \left\{ \left\{ x == \frac{1}{2 \, r \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)} \, \left(r^{2} \, \text{Sin}[\varphi] \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} - h_{2}^{2} \right) - r \, \text{Cot}[\varphi] \right. \\ & \left. \sqrt{\left(- r^{2} \, \text{Sin}[\varphi]^{2} \, \left(\left(- a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)^{2} - 2 \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right) \, h_{2}^{2} + h_{2}^{4} \right) \right) + \text{Csc}[\varphi]} \\ & \left. h_{1} \, \sqrt{\left(- r^{2} \, \text{Sin}[\varphi]^{2} \, \left(\left(- a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)^{2} - 2 \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right) \, h_{2}^{2} + h_{2}^{4} \right) \right) \right), \\ & x == \frac{1}{2 \, r \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)} \, \left(r^{2} \, \text{Sin}[\varphi] \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} - h_{2}^{2} \right) + r \, \text{Cot}[\varphi]} \right. \\ & \left. \sqrt{\left(- r^{2} \, \text{Sin}[\varphi]^{2} \, \left(\left(- a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)^{2} - 2 \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right) \, h_{2}^{2} + h_{2}^{4} \right) \right) \right\}, \\ & \left\{ y == \frac{1}{2 \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)} \, \left(- r^{3} \, \text{Cos}[\varphi] + r^{2} \, \left(2 + \text{Cos}[2 \, \varphi] \right) \, h_{1} + \frac{1}{2} \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right) \right) \right\}, \\ & y == \frac{1}{2 \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)} \, \left(- r^{3} \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)^{2} - 2 \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right) \, h_{2}^{2} + h_{2}^{4} \right) \right), \\ & y == \frac{1}{2 \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)} \, \left(- r^{3} \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)^{2} - 2 \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right) \, h_{2}^{2} + h_{2}^{4} \right) \right), \\ & y == \frac{1}{2 \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)} \, \left(- r^{3} \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)^{2} - 2 \, \left(a^{2} + r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right) \, h_{1}^{2} + h_{2}^{2} \right) \right), \\ & y = \frac{1}{2 \, \left(r^{2} - 2 \, r \, \text{Cos}[\varphi] \, h_{1} + h_{1}^{2} \right)} \, \left(- r^{3} \, \text{Cos}[\varphi] \, h_{1}^{2} + h_{2}^{2} \right) + r^{2} \,$$

$$\begin{split} & \text{In}(r) \models \text{solve } I. \ \left\{ \left(-a^2 + r^2 - 2 \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right)^2 \to A, \ a^2 + r^2 - 2 \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \to B \right\} \\ & \text{Cos}[\phi] \, h_1 \, \sqrt{-r^2 \, \text{Sin}[\phi]^2 \, \left(A - 2 \, B \, h_2^2 + h_2^4 \right)} \, \left/ \left(2 \, r \, \left(r^2 - 2 \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right) \right), \\ & \text{Sec}[\phi] \, h_1 \, \sqrt{-r^2 \, \text{Sin}[\phi]^2 \, \left(A - 2 \, B \, h_2^2 + h_2^4 \right)} \, \right) / \left(2 \, r \, \left(r^2 - 2 \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right) \right), \\ & \text{Sec}[\phi] \, h_1 \, \sqrt{-r^2 \, \text{Sin}[\phi]^2 \, \left(A - 2 \, B \, h_2^2 + h_2^4 \right)} \, \right) / \left(2 \, r \, \left(r^2 - 2 \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right) \right), \\ & \left\{ y = \left(-r^3 \, \text{Cos}[\phi] + r^2 \, \left(2 + \text{Cos}[2 \, \phi] \right) \, h_1 + h_1 \, \left(a^2 + h_1^2 - h_2^2 \right) + r \, \text{Cos}[\phi] \, \left(-a^2 - 3 \, h_1^2 + h_2^2 \right) - \right. \\ & \sqrt{-r^2 \, \text{Sin}[\phi]^2 \, \left(A - 2 \, B \, h_2^2 + h_2^4 \right)} \, \right) / \left(2 \, \left(r^2 - 2 \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right), \\ & y = \left(-r^3 \, \text{Cos}[\phi] + r^2 \, \left(2 + \text{Cos}[2 \, \phi] \right) \, h_1 + h_1 \, \left(a^2 + h_1^2 - h_2^2 \right) + r \, \text{Cos}[\phi] \, \left(-a^2 - 3 \, h_1^2 + h_2^2 \right) + \right. \\ & \sqrt{-r^2 \, \text{Sin}[\phi]^2 \, \left(A - 2 \, B \, h_2^2 + h_2^4 \right)} \, \right) / \left(2 \, \left(r^2 - 2 \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right) \right) \right\} \\ & \text{Simplify} \left[\left(\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right) + \frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right) \right) \right\} \\ & \text{Simplify} \left[\left(\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + r^2 \, \text{Sin}[\phi] \, \left(B - h_2^2 \right) + r \, \text{Cos}[\phi] \, h_1 + h_1^2 \right) \right) \right\} \right] \\ & \text{Simplify} \left[\left(\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + r^2 \, \text{Sin}[\phi] \, \left(\frac{1}{2} \, h_1^2 + h_2^2 \right) + r \, \text{Cos}[\phi] \, h_1 + r^2 \, \text{Sin}[\phi] \, \left(\frac{1}{2} \, h_1^2 \, h_1^2 \right) \right) \right) \right\} \right] \\ & \text{Simplify} \left[\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \, h_1^2 \right) \right) \right] \\ & \text{Simplify} \left[\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \, h_1^2 \right) \right] \right) \\ & \text{Simplify} \left[\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \, h_1^2 \right) \right] \right) \\ & \text{Simplify} \left[\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \, h_1 + h_1^2 \, h_1^2 \right) \right] \right) \\ & \text{Simplify} \left[\frac{1}{2} \, \left(\frac{1}{2} \, r \, \text{Cos}[\phi] \,$$

$$\begin{split} &\inf_{\|\phi\|_2} \text{ j:= Simplify} \big[\% \text{ $//\cdot$. } \Big\{ r^2 \, Sin[\phi] \, \Big(B - h_2^2 \Big) \to Desc, \text{ } r \, S \, Cot[\phi] - S \, Csc[\phi] \, h_1 \to Besc, \\ &-r \, S \, Cot[\phi] + S \, Csc[\phi] \, h_1 \to -Besc, \text{ } r^2 - 2 \, r \, Cos[\phi] \, h_1 + h_1^2 \to Z \Big\} \Big] \\ &Out[\phi]_2 = \left\{ \left\{ x = = \frac{-Besc + Desc}{2 \, r \, Z} \, , \, x = = \frac{Besc + Desc}{2 \, r \, Z} \right\}, \\ &\left\{ y = = \frac{1}{2 \, r \, Z} \, \left(-S - r^3 \, Cos[\phi] + r^2 \, (2 + Cos[2 \, \phi]) \, h_1 + h_1 \, \left(a^2 + h_1^2 - h_2^2\right) + r \, Cos[\phi] \left(-a^2 - 3 \, h_1^2 + h_2^2\right) \right), \\ &y = = \frac{1}{2 \, r \, Z} \, \left(S - r^3 \, Cos[\phi] + r^2 \, (2 + Cos[2 \, \phi]) \, h_1 + h_1 \, \left(a^2 + h_1^2 - h_2^2\right) + r \, Cos[\phi] \left(-a^2 - 3 \, h_1^2 + h_2^2\right) \right) \Big\} \Big\} \\ &Io[\phi]_2 = \% \, / \cdot \, \left\{ r^2 \, (2 + Cos[2 \, \phi]) \, h_1 + h_1 \, \left(a^2 + h_1^2 - h_2^2\right) + r \, Cos[\phi] \left(-a^2 - 3 \, h_1^2 + h_2^2\right) - r^3 \, Cos[\phi] \to L \right\} \\ &Out[\phi]_3 = \left\{ \left\{ x = \frac{-Besc + Desc}{2 \, r \, Z} \, , \, x = \frac{Besc + Desc}{2 \, r \, Z} \right\}, \, \left\{ y = \frac{L - S}{2 \, Z} \, , \, y = \frac{L + S}{2 \, Z} \right\} \Big\} \end{aligned}$$