```
eqncirc = x^2 + y^2 == a^2;
eqnx = x == r Sin[φ] + h2 Sin[α];
eqny = y == h1 - r Cos[φ] - h2 Cos[α];
elim = FullSimplify[Eliminate[{eqnx, eqny}, {α}]];
solve = Thread[FullSimplify[Solve[{eqncirc, elim}, {x, y}]]]
```

$$
\begin{aligned}
\Bigg\{\Bigg\{x \to\ & \frac{1}{2\, r \left(r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)} \Bigg(r^2\, \mathrm{Sin}[\varphi] \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2 - h_2^2\right) - \\
& r\, \mathrm{Cot}[\varphi] \sqrt{-r^2\, \mathrm{Sin}[\varphi]^2 \left(\left(-a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)^2 - 2 \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right) h_2^2 + h_2^4\right)} + \\
& \mathrm{Csc}[\varphi]\, h_1 \sqrt{-r^2\, \mathrm{Sin}[\varphi]^2 \left(\left(-a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)^2 - 2 \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right) h_2^2 + h_2^4\right)}\Bigg), \\
x \to\ & \frac{1}{2\, r \left(r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)} \Bigg(r^2\, \mathrm{Sin}[\varphi] \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2 - h_2^2\right) + \\
& r\, \mathrm{Cot}[\varphi] \sqrt{-r^2\, \mathrm{Sin}[\varphi]^2 \left(\left(-a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)^2 - 2 \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right) h_2^2 + h_2^4\right)} - \\
& \mathrm{Csc}[\varphi]\, h_1 \sqrt{-r^2\, \mathrm{Sin}[\varphi]^2 \left(\left(-a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)^2 - 2 \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right) h_2^2 + h_2^4\right)}\Bigg)\Bigg\}, \\
\Bigg\{y \to\ & \frac{1}{2 \left(r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)} \Bigg(-r^3\, \mathrm{Cos}[\varphi] + r^2 \left(2 + \mathrm{Cos}[2\, \varphi]\right) h_1 + h_1 \left(a^2 + h_1^2 - h_2^2\right) + r\, \mathrm{Cos}[\varphi] \left(-a^2 - 3\, h_1^2 + h_2^2\right) - \\
& \sqrt{-r^2\, \mathrm{Sin}[\varphi]^2 \left(\left(-a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)^2 - 2 \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right) h_2^2 + h_2^4\right)}\Bigg), \\
y \to\ & \frac{1}{2 \left(r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)} \Bigg(-r^3\, \mathrm{Cos}[\varphi] + r^2 \left(2 + \mathrm{Cos}[2\, \varphi]\right) h_1 + h_1 \left(a^2 + h_1^2 - h_2^2\right) + r\, \mathrm{Cos}[\varphi] \left(-a^2 - 3\, h_1^2 + h_2^2\right) + \\
& \sqrt{-r^2\, \mathrm{Sin}[\varphi]^2 \left(\left(-a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right)^2 - 2 \left(a^2 + r^2 - 2\, r\, \mathrm{Cos}[\varphi]\, h_1 + h_1^2\right) h_2^2 + h_2^4\right)}\Bigg)\Bigg\}\Bigg\}
\end{aligned}
$$

```
solve = solve /. Rule → Equal
```

$$\left\{\left\{x == \frac{1}{2\,r\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\left(r^2\,\mathrm{Sin}[\varphi]\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2 - h_2^2\right) - r\,\mathrm{Cot}[\varphi]\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(\left(-a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)^2 - 2\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)\,h_2^2 + h_2^4\right)} + \mathrm{Csc}[\varphi]\,h_1\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(\left(-a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)^2 - 2\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)\,h_2^2 + h_2^4\right)}\right),\right.\right.$$

$$x == \frac{1}{2\,r\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\left(r^2\,\mathrm{Sin}[\varphi]\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2 - h_2^2\right) + r\,\mathrm{Cot}[\varphi]\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(\left(-a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)^2 - 2\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)\,h_2^2 + h_2^4\right)} - \mathrm{Csc}[\varphi]\,h_1\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(\left(-a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)^2 - 2\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)\,h_2^2 + h_2^4\right)}\right)\Big\},$$

$$\Big\{y == \frac{1}{2\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\left(-r^3\,\mathrm{Cos}[\varphi] + r^2\,\left(2 + \mathrm{Cos}[2\,\varphi]\right)\,h_1 + h_1\,\left(a^2 + h_1^2 - h_2^2\right) + r\,\mathrm{Cos}[\varphi]\,\left(-a^2 - 3\,h_1^2 + h_2^2\right) - \sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(\left(-a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)^2 - 2\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)\,h_2^2 + h_2^4\right)}\right),$$

$$\left.\left.y == \frac{1}{2\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\left(-r^3\,\mathrm{Cos}[\varphi] + r^2\,\left(2 + \mathrm{Cos}[2\,\varphi]\right)\,h_1 + h_1\,\left(a^2 + h_1^2 - h_2^2\right) + r\,\mathrm{Cos}[\varphi]\,\left(-a^2 - 3\,h_1^2 + h_2^2\right) + \sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(\left(-a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)^2 - 2\,\left(a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)\,h_2^2 + h_2^4\right)}\right)\right\}\right\}$$

solve /. $\left\{\left(-a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)^2 \to A,\ a^2 + r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2 \to B\right\}$

$$\left\{\left\{x == \frac{r^2\,\mathrm{Sin}[\varphi]\,\left(B - h_2^2\right) - r\,\mathrm{Cot}[\varphi]\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(A - 2\,B\,h_2^2 + h_2^4\right)} + \mathrm{Csc}[\varphi]\,h_1\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(A - 2\,B\,h_2^2 + h_2^4\right)}}{2\,r\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)},\right.\right.$$

$$x == \frac{r^2\,\mathrm{Sin}[\varphi]\,\left(B - h_2^2\right) + r\,\mathrm{Cot}[\varphi]\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(A - 2\,B\,h_2^2 + h_2^4\right)} - \mathrm{Csc}[\varphi]\,h_1\,\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(A - 2\,B\,h_2^2 + h_2^4\right)}}{2\,r\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\Big\},$$

$$\Big\{y == \frac{1}{2\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\left(-r^3\,\mathrm{Cos}[\varphi] + r^2\,\left(2 + \mathrm{Cos}[2\,\varphi]\right)\,h_1 + h_1\,\left(a^2 + h_1^2 - h_2^2\right) + r\,\mathrm{Cos}[\varphi]\,\left(-a^2 - 3\,h_1^2 + h_2^2\right) - \sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(A - 2\,B\,h_2^2 + h_2^4\right)}\right),\ y == \frac{1}{2\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}$$

$$\left.\left.\left(-r^3\,\mathrm{Cos}[\varphi] + r^2\,\left(2 + \mathrm{Cos}[2\,\varphi]\right)\,h_1 + h_1\,\left(a^2 + h_1^2 - h_2^2\right) + r\,\mathrm{Cos}[\varphi]\,\left(-a^2 - 3\,h_1^2 + h_2^2\right) + \sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(A - 2\,B\,h_2^2 + h_2^4\right)}\right)\right\}\right\}$$

Simplify$\left[\%\ /.\ \left\{\sqrt{-r^2\,\mathrm{Sin}[\varphi]^2\,\left(A - 2\,B\,h_2^2 + h_2^4\right)} \to S\right\}\right]$

$$\left\{\left\{x == \frac{-r\,S\,\mathrm{Cot}[\varphi] + S\,\mathrm{Csc}[\varphi]\,h_1 + r^2\,\mathrm{Sin}[\varphi]\,\left(B - h_2^2\right)}{2\,r\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)},\ x == \frac{r\,S\,\mathrm{Cot}[\varphi] - S\,\mathrm{Csc}[\varphi]\,h_1 + r^2\,\mathrm{Sin}[\varphi]\,\left(B - h_2^2\right)}{2\,r\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\right\},\right.$$

$$\Big\{y == \frac{-S - r^3\,\mathrm{Cos}[\varphi] + r^2\,\left(2 + \mathrm{Cos}[2\,\varphi]\right)\,h_1 + h_1\,\left(a^2 + h_1^2 - h_2^2\right) + r\,\mathrm{Cos}[\varphi]\,\left(-a^2 - 3\,h_1^2 + h_2^2\right)}{2\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)},$$

$$\left.\left.y == \frac{S - r^3\,\mathrm{Cos}[\varphi] + r^2\,\left(2 + \mathrm{Cos}[2\,\varphi]\right)\,h_1 + h_1\,\left(a^2 + h_1^2 - h_2^2\right) + r\,\mathrm{Cos}[\varphi]\,\left(-a^2 - 3\,h_1^2 + h_2^2\right)}{2\,\left(r^2 - 2\,r\,\mathrm{Cos}[\varphi]\,h_1 + h_1^2\right)}\right\}\right\}$$

```
Simplify[% //. {r^2 Sin[φ] (B - h2^2) → Desc, r S Cot[φ] - S Csc[φ] h1 -> Besc, -r S Cot[φ] + S Csc[φ] h1 → -Besc,
    r^2 - 2 r Cos[φ] h1 + h1^2 → Z}]
```

$$\left\{\left\{x == \frac{-\text{Besc} + \text{Desc}}{2\, r\, Z},\ x == \frac{\text{Besc} + \text{Desc}}{2\, r\, Z}\right\},\right.$$

$$\left\{y == \frac{-S - r^3 \text{Cos}[\varphi] + r^2\ (2 + \text{Cos}[2\,\varphi])\ h_1 + h_1\ \left(a^2 + h_1^2 - h_2^2\right) + r\ \text{Cos}[\varphi]\ \left(-a^2 - 3\, h_1^2 + h_2^2\right)}{2\, Z},\right.$$

$$\left.\left. y == \frac{S - r^3 \text{Cos}[\varphi] + r^2\ (2 + \text{Cos}[2\,\varphi])\ h_1 + h_1\ \left(a^2 + h_1^2 - h_2^2\right) + r\ \text{Cos}[\varphi]\ \left(-a^2 - 3\, h_1^2 + h_2^2\right)}{2\, Z}\right\}\right\}$$

```
% /. {r^2 (2 + Cos[2 φ]) h1 + h1 (a^2 + h1^2 - h2^2) + r Cos[φ] (-a^2 - 3 h1^2 + h2^2) - r^3 Cos[φ] → L}
```

$$\left\{\left\{x == \frac{-\text{Besc} + \text{Desc}}{2\, r\, Z},\ x == \frac{\text{Besc} + \text{Desc}}{2\, r\, Z}\right\},\ \left\{y == \frac{L - S}{2\, Z},\ y == \frac{L + S}{2\, Z}\right\}\right\}$$