Units

Ian Ochs

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Maxwell Equations

In Yee algorithm, using Gaussian unit style with $c = 1$, i.e.

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} = -\frac{\partial B}{\partial \tilde{t}} \tag{0.1}$$

$$\nabla \times B = \frac{4\pi}{c} J + \frac{1}{c}\frac{\partial E}{\partial t} = 4\pi \tilde{J} + \frac{\partial E}{\partial \tilde{t}}. \tag{0.2}$$

I chose to keep cm as the length unit, so that the subsequent two Maxwell equations are unaffected:

$$\nabla \cdot E = 4\pi\rho \tag{0.3}$$

$$\nabla \cdot B = 0. \tag{0.4}$$

This means that the time has the unit of (1 cm) / (speed of light in seconds), i.e.

$$1\tilde{s} = \frac{1 \text{ cm}}{2.9979 * 10^{10} \text{ cm/s}} \approx 3.3 \times 10^{-11} \text{ s}. \tag{0.5}$$

Velocity, acceleration, and current density

The velocity $\tilde{v}$ is now normalized automatically to the speed of light, since it is measured in cm / (distance light travels in 1 cm).
We then have the Lorentz acceleration as

$$a = \left(\frac{1}{2.9979 * 10^{10}}\right)^2 \frac{q}{m}\left(E + \tilde{v} \times B\right) \tag{0.6}$$

$$\approx 1.1 \times 10^{-20} \frac{q}{m}\left(E + \tilde{v} \times B\right). \tag{0.7}$$

The small acceleration reflects the timescale separation between light waves and plasma motion.
Finally, current densities should be calculated in statC/cm$^2\tilde{s}$.