
Units

EMOOPIC

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MAXWELL EQUATIONS

In Yee algorithm, using Gaussian unit style with $c = 1$, i.e.

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t} = -\frac{\partial B}{\partial \tilde{t}} \quad (0.1)$$

$$\nabla \times B = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial E}{\partial t} = 4\pi \tilde{J} + \frac{\partial E}{\partial \tilde{t}}. \quad (0.2)$$

I chose to keep cm as the length unit, so that the subsequent two Maxwell equations are unaffected:

$$\nabla \cdot E = 4\pi \rho \quad (0.3)$$

$$\nabla \cdot B = 0. \quad (0.4)$$

This means that the time has the unit of (1 cm) / (speed of light in seconds), i.e.

$$1\tilde{s} = \frac{1 \text{ cm}}{c_0} \approx 3.3 \times 10^{-11} \text{ s}, \quad (0.5)$$

where $c_0 \approx 2.9979 \times 10^{10}$ is the speed of light in unit of cm/s.

VELOCITY, ACCELERATION, AND CURRENT DENSITY

The velocity \tilde{v} is now normalized automatically to the speed of light, since it is measured in cm / (distance light travels in 1 cm). Namely,

$$\tilde{v} = \frac{v}{c_0 \text{ cm/s}}. \quad (0.6)$$

The non-relativistic thermal velocity v_{th} , defined as the mean of the magnitude of the velocity, is then

$$\tilde{v}_{th} = \frac{v_{th}}{c} = \sqrt{\frac{8T}{\pi m c^2}} \quad (0.7)$$

$$\approx 2.232 \times 10^{-3} \sqrt{\frac{T/1\text{eV}}{m/m_e}}. \quad (0.8)$$

We then have the Lorentz acceleration as

$$a = \frac{1}{c_0^2} \frac{q}{m} (E + \tilde{v} \times B) \quad (0.9)$$

$$\approx 1.1 \times 10^{-21} \frac{q}{m} (E + \tilde{v} \times B). \quad (0.10)$$

The small acceleration reflects the timescale separation between light waves and plasma motion.

Finally, current densities should be calculated in $\text{statC}/\text{cm}^2 \tilde{s}$.

CHARGE, MASS, AND FIELD

It is convenient to normalize charge q to electron charge e , and normalize mass m to electron mass m_e . Namely,

$$\tilde{q} = \frac{q}{e}, \quad \tilde{m} = \frac{m}{m_e}. \quad (0.11)$$

Recall the value

$$\frac{e}{m_e} \approx 5.2728 \times 10^{17} \text{statC/g}. \quad (0.12)$$

Having normalized charge and mass, it is convenient to normalize the electric field E to kilo statV/cm (KstatV/cm), and normalize the magnetic field B to kilo Gauss (KG). Namely,

$$\tilde{E} = \frac{E}{\text{KstatV/cm}}, \quad \tilde{B} = \frac{B}{\text{KG}}. \quad (0.13)$$

It is useful to note

$$\frac{1 \cdot \text{KstatV/cm}}{c_0 \cdot \text{KG}} = 1. \quad (0.14)$$

With such normalization, there is no large or small coefficient in the Newton's equation with Lorentz force. Explicitly, we have

$$\tilde{a} = \frac{a}{\text{cm} \cdot \tilde{s}^{-2}} \quad (0.15)$$

$$= \frac{\tilde{q}}{\tilde{m}} (\tilde{E} + \tilde{v} \times \tilde{B}) \times \frac{e \cdot \text{Kstat/cm}}{m_e \cdot \text{cm} \cdot \tilde{s}^{-2}} \quad (0.16)$$

$$\approx 0.58668774 \frac{\tilde{q}}{\tilde{m}} (\tilde{E} + \tilde{v} \times \tilde{B}). \quad (0.17)$$

To get a sense of how large the electric and magnetic fields are, it is useful to note

$$\tilde{E} = 1 \leftrightarrow E \approx 299.79 \text{KV/cm}, \quad (0.18)$$

$$\tilde{B} = 1 \leftrightarrow B = 0.1 \text{T}, \quad (0.19)$$

where KV/cm stands for kilo volte/cm, and T stands for tesla. Notice that a constant field $\tilde{E} = 1$ is a very large electric field, which can accelerate an electron to relativistic velocity within $1 \tilde{s}$. In comparison, a constant field $\tilde{B} = 1$ is moderate magnetic field that can be easily produced in laboratory.