Supplement: Errors in single pixel laser photography emerging from spatial size limits in the bucket detector

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This supplemental material contains details about the algorithm used for the simulation and the derivation of the discrete versions of equations (1-8) of the main article.

1. Simulation procedure

The experiment is simulated in the following manner: Arbitrary light fields are created using $\sqrt{N} \times \sqrt{N} = 32 \times 32$ pixel images ($N = 1024$) which are chosen as amplitude and phase distributions, A and ϕ, respectively. The images are taken from the CIFAR-10 database [1] or created using Zernike polynomials on the same grid.
An arbitrary field x is constructed with: $u = A\exp\bigl(i\phi\bigr)$ on a grid size of 32×32 pixels that is upscaled to a $M \times M = 384 \times 384$ grid at the SLM with Nearest-Neighbour interpolation. Additionally, the amplitude A is multiplied with a Gaussian function to mimic the Gaussian beam profile in the experiment. Hence, the field is represented in the discrete grid as $u_{n,m}$.
The Hadamard basis $\Phi = H$ is a matrix with $N \times N$ elements. Each row (size $1 \times N$) of that matrix is an orthogonal vector element of the basis which is implemented in the same grid as the images. So the vector is reshaped into

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a $\sqrt{N} \times \sqrt{N} = 32 \times 32$ grid and then resized to the $M \times M = 384 \times 384$px grid of the SLM. The Matrix is represented by $\Phi_{n,m} = H_{n.m}$.

Single pixel photography experiments that use a basis containing negative elements commonly split the basis matrix into a positive Φ^+ and negative Φ^- so that each vector element is sampled twice and then the results are subtracted.

The field and the vector elements l of the basis are both encoded at the SLM with the elementwise product $\Phi_l^{\pm} \odot u$, which on the $M \times M$ grid at the SLM is simply $\Phi_{n,m}^{l,\pm} u_{n,m}$. The focusing lens projects the Fourier Transform at the focal plane, so that the field is proportional to $\mathcal{FT}\{\Phi^{l,\pm} \odot u\}_{k_X,k_Y}$ discretized via fast fourier transform to $\text{fft}\{\Phi_{n,m}^{l,\pm} u_{n,m}\}_{k_X,k_Y}$. The intensity is proportional to $|\text{fft}\{\Phi_{n,m}^{l,\pm} u_{n,m}\}_{k_X,k_Y}|^2$, which can be integrated by the bucket detector over the full $M \times M$ frequency grid or a subset of that grid:

$$y_l^{\pm} = \sum_{kx,ky}^{M} |\text{fft}\{\Phi_{n,m}^{l,\pm} u_{n,m}\}_{k_X,k_Y}|^2 \tag{1}$$

This are the $\pm$ measurements for a single vector element l.
Ideally the photodiode will collect the full spectrum, so that there is no power loss. The different detector sizes are simulated by limiting the frequencies in the sums.
The fft of a single projection l reads:

$$\text{fft}\{\Phi_{n,m}^{l,\pm} \odot u_{n,m}\}_{k_X,k_Y} = \sum_{n,m=0}^{M-1} \Phi_{n,m}^{l,\pm} u_{n,m} \exp\left(-2\pi i (k_X n + k_Y m)/M\right) \ , \tag{2}$$

Notice that the spatial integrals of the Fourier transform that represent the field at the focus and its complex conjugate are done independently, so we use two sets of spatial coordinates (n, m) and (n', m') for the focused field and its complex conjugate, respectively. Thus, the intensity at the focal plane yields:

$$\begin{aligned} |\text{fft}\{\Phi_{n,m}^{l,\pm} \odot u_{n,m}\}_{k_X,k_Y}|^2 = & \sum_{n,m,n',m'=0}^{M-1} \Phi_{n,m}^{l,\pm} u_{n,m} \Phi_{n',m'}^{l,\pm *} u_{n',m'}^{*} \cdot \\ & \exp\left(-2\pi i \left(k_X(n-n') + k_Y(m-m')\right)/M\right) \ . \end{aligned} \tag{3}$$

Going back to the discrete integrated spectrum and using $u_{n,m} = A_{n,m} \exp\left(i\phi_{n,m}\right)$:

$$y_l^{\pm} = \Bigg| \sum_{k_X, k_Y} \sum_{n,m,n',m'=0}^{M-1} \Phi_{n,m}^{l,\pm} \Phi_{n',m'}^{l,\pm *} A_{n,m} A_{n',m'} \exp\left(i(\phi_{n,m} - \phi_{n',m'})\right) \cdot \\ \exp\left(-2\pi i \left(k_X(n-n') + k_Y(m-m')\right)/M\right) \Bigg| \, . \quad (4)$$

The sum over k_X and k_Y is a geometrical series. Evaluating over the full spectrum, the series yields:

$$\sum_{k_X=1}^{M} e^{-2\pi i k_X (n-n')/M} = \frac{1 - \exp\left(-2\pi i (n-n')\right)}{1 - \exp\left(-2\pi i (n-n')/M\right)} \, . \quad (5)$$

This expression is only non-zero for $n = n'$, where it reduces to the value M. So we can write it in terms of the Kronecker delta function as $M\delta_{n,n'}$.In this way

$$y_l^{\pm} = M^2 \sum_{n,m=0}^{M-1} \left(\Phi_{n,m}^{l,\pm} A_{n,m}\right)^2 \, . \quad (6)$$

For the case, that a subset of the spectrum is integrated, the geometric series has arbitrary borders then we use eq. (4) adjusting the range of k_X and k_Y. As a consequence, the phase cross-talk given by $\exp\left(i(\phi_{n,m} - \phi_{n',m'})\right)$ does not disappear, resulting in phase induced artifacts in the reconstructed amplitude. Note, that this expression is independent of the used sampling basis (e.g. Gaussian, Hadamard). Furthermore, in the case of the canonical basis, no crosstalk appears, as the canonical basis only contains diagonal elements.

Algorithm 1 Simulation procedure. Implementation can be found in a data repository [2]

$N \leftarrow 1024$
$M \leftarrow 384$
$y \leftarrow \text{Array}(N)$
$H \leftarrow \text{Hadamard}(N)$
$H^+ \leftarrow (H+1)/2$ ▷ Split basis into positive and negative part
$H^- \leftarrow -1 \times (M-1)/2$
$A \leftarrow \text{resize2d}(M, M)$ ▷ Resize Amplitude and phase to 384 × 384.
$\Phi \leftarrow \text{resize2d}(M, M)$
$u \leftarrow g \times A \times \exp(1i \times \phi)$ ▷ Amplitude image A, phase image ϕ

for $it = 1 : N$ **do**
  $\Phi^+ \leftarrow \text{reshape2d}(H^+(it))$ ▷ Reshape a row of measurement matrix into a 2d vector
  $\Phi^- \leftarrow \text{reshape2d}(H^-(it))$
  $y^+ \leftarrow \text{abs}(\text{FFT2}(\Phi^+ \times u)) \wedge 2$ ▷ Sample postive and negative part separately
  $y^- \leftarrow \text{abs}(\text{FFT2}(\Phi^- \times u)) \wedge 2$
  $y(it) = \text{sum}(y^+) - \text{sum}(y^-)$ ▷ Integrate over (parts of) the spectrum
end for

$x = \text{Matmul}(H, y)$ ▷ Reconstruct the image

References

[1] Krizhevsky, A., Hinton, G., 2009. Learning multiple layers of features from tiny images. Technical Report 0. University of Toronto. Toronto, Ontario.

[2] Scheidt, D., Quinto-Su, P., 2024. Supplementary data and code for 'errors in single pixel photography emerging from light collection limits by the bucket detector'. https://github.com/Denbo313/Errors-in-single-pixel-photography-emerging-from-light-collection-limits-by-the-bucket-detector.