Genuine equivariant operads

Peter Bonventre, Luís A. Pereira

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Given $\mathsf{Fin} \wr \mathcal{V} \to \mathcal{V}$ with suitable isomorphisms α, we define the associators by

$$\begin{tikzcd} (A \cdot B) \cdot (C) \arrow[r, "\alpha"] & A \cdot B \cdot C & (A) \cdot (B \cdot C) \arrow[l, "\alpha"'] \end{tikzcd}$$

and the unit morphisms by

$$(A) \cdot () \xrightarrow{\alpha} A \qquad () \cdot (A) \xrightarrow{\alpha} A$$

The associativity "pentagon" axiom follows from

$$\begin{tikzcd}[column sep=small]
 & ((A \cdot B)) \cdot ((C) \cdot (D)) \arrow[dl, "\alpha"'] \arrow[r, equal, "\mathsf{F}\wr\alpha"] & (A \cdot B) \cdot (C \cdot D) \arrow[dd] & ((A) \cdot (B)) \cdot ((C \cdot D)) \arrow[l, equal, "\mathsf{F}\wr\alpha"'] \arrow[dr, "\alpha"] & \\
(A \cdot B) \cdot (C) \cdot (D) \arrow[drr] & & & & (A) \cdot (B) \cdot (C \cdot D) \arrow[dll] \\
((A \cdot B) \cdot (C)) \cdot ((D)) \arrow[u, "\alpha"] \arrow[d, "\mathsf{F}\wr\alpha"'] & & A \cdot B \cdot C \cdot D & & ((A)) \cdot ((B) \cdot (C \cdot D)) \arrow[u, "\alpha"'] \arrow[d, "\mathsf{F}\wr\alpha"] \\
(A \cdot B \cdot C) \cdot (D) \arrow[urr] & & & & (A) \cdot (B \cdot C \cdot D) \arrow[ull] \\
 & ((A) \cdot (B \cdot C)) \cdot ((D)) \arrow[ul, "\mathsf{F}\wr\alpha"] \arrow[r, "\alpha"'] & (A) \cdot (B \cdot C) \cdot (D) \arrow[uu] & ((A)) \cdot ((B \cdot C) \cdot (D)) \arrow[l, "\alpha"] \arrow[ur, "\mathsf{F}\wr\alpha"'] &
\end{tikzcd}$$

The identity axiom is

$$\begin{tikzcd}
((A) \cdot ()) \cdot ((B)) \arrow[r, "\alpha"] \arrow[d, "\mathsf{Fin}\wr\alpha"'] & (A) \cdot () \cdot (B) \arrow[d] & ((A)) \cdot (() \cdot (B)) \arrow[l, "\alpha"'] \arrow[d, "\mathsf{Fin}\wr\alpha"] \\
(A) \cdot (B) \arrow[r, equal] & A \cdot B \arrow[r, equal] & (A) \cdot (B)
\end{tikzcd}$$

The symmetry morphism is defined in the obvious way as $\otimes(\tau)$ for τ the isomorphism $(A, B) \simeq (B, A)$ in $\Sigma_2 \wr \mathcal{V}$.

The inverse law is then simply automatic and the coherence of symmetry with unit is naturality of α, as in the diagram

$$\begin{tikzcd}
(A) \cdot () \arrow[r, "\tau"] \arrow[d, "\alpha"] & () \cdot (A) \arrow[d, "\alpha"] \\
A \arrow[r, "\tau"'] & A
\end{tikzcd}$$

Lastly, associativity coherence is then ensured by the diagram (where the center triangle commutes since it is in the image of $\Sigma_3 \wr \mathcal{V}$)

$$\begin{tikzcd}
 & & (A)\cdot(B\cdot C) \arrow[dl, "\alpha"'] \arrow[rr, "\tau"] & & (A)\cdot(C\cdot B) \arrow[dr, "\alpha"] & & \\
 & A\cdot B\cdot C \arrow[rrrr, "\tau"] \arrow[ddrr, "\tau"'] & & & & A\cdot C\cdot B \arrow[ddll, "\tau"] & \\
(A\cdot B)\cdot(C) \arrow[ur, "\alpha"] \arrow[dr, "\tau"'] & & & & & & (A\cdot C)\cdot(B) \arrow[ul, "\alpha"'] \arrow[dl, "\tau"] \\
 & (B\cdot A)\cdot(C) \arrow[rr, "\alpha"'] & & B\cdot A\cdot C & & (B)\cdot(A\cdot C) \arrow[ll, "\alpha"] &
\end{tikzcd}$$