Genuine equivariant operads

An Elmendorf-Piacenza theorem for operads

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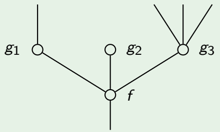

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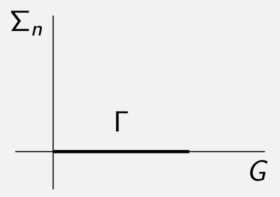

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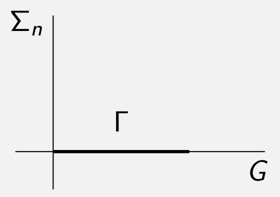

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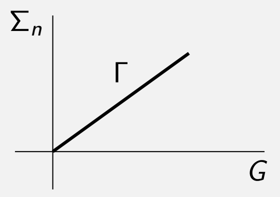

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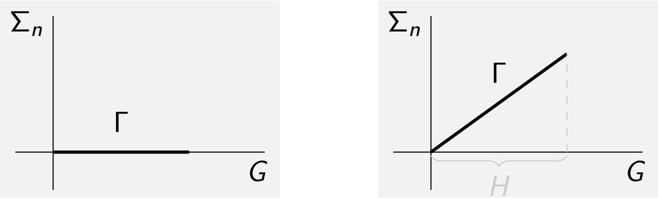

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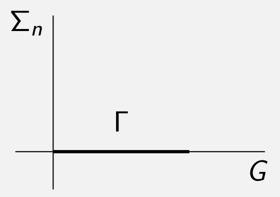

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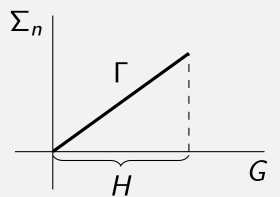

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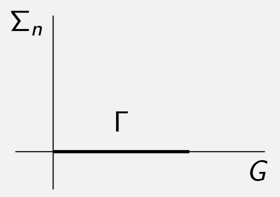

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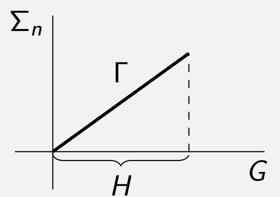

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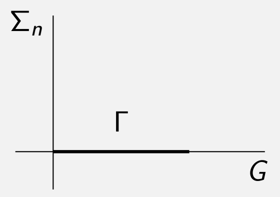

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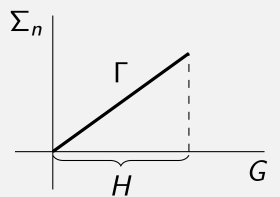

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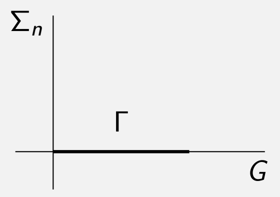

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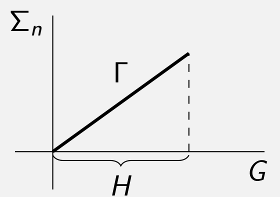

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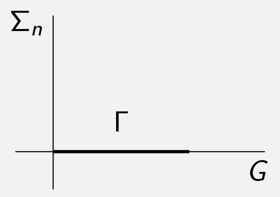

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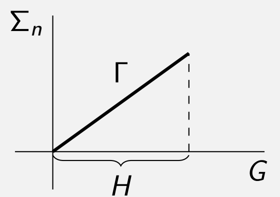

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Hence:

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Weak equivalences of equivariant operads

Upshot: weak equivalences of G-operads should account for norm maps.

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- $\mathcal{F} = \{\mathcal{F}_n\}$ is not arbitrary.
  Must satisfy *indexing system* closure conditions (BH);
- Building N_∞-operads for all indexing systems $\mathcal{F}$ is not trivial.

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Definition

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The dendroidal category Ω

Definition (Moerdijk-Weiss)

A tree diagram T defines a colored operad such that:

- objects are the edges of T;
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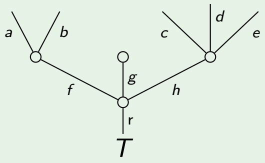

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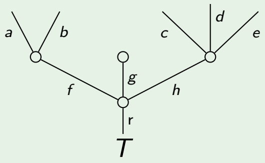

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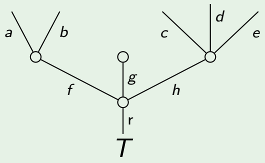

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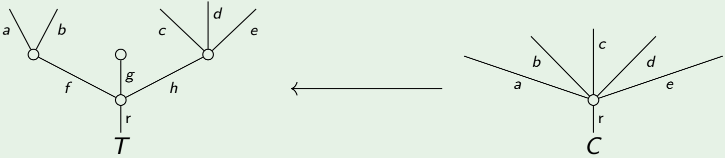

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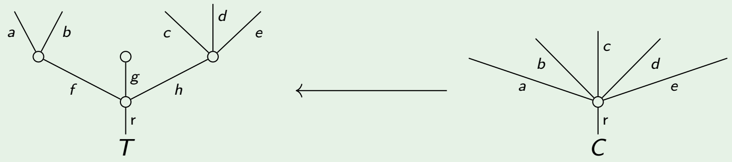

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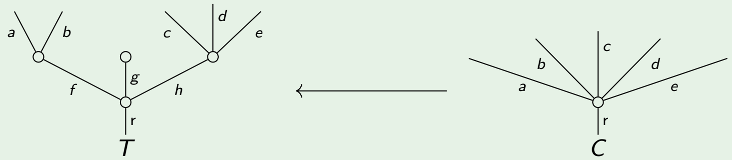

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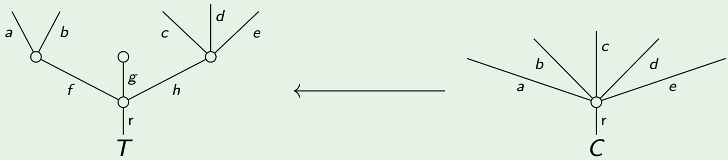

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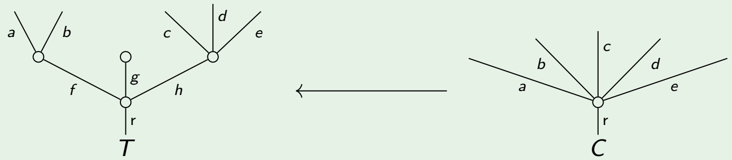

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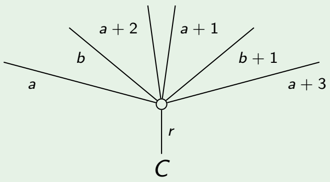

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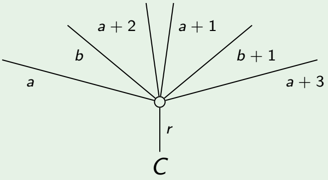

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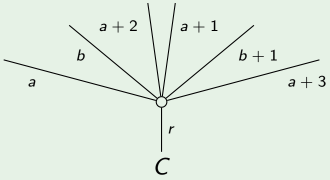

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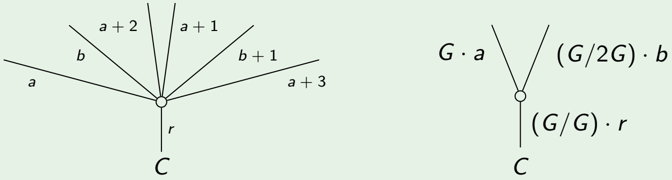

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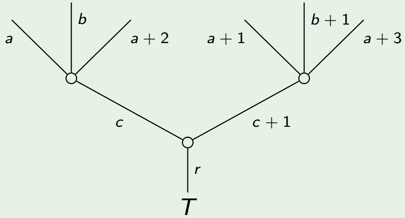

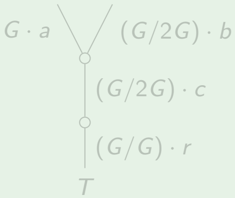

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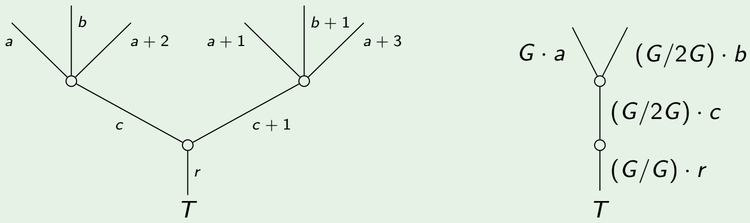

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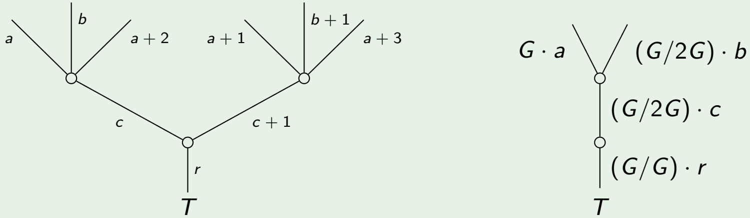

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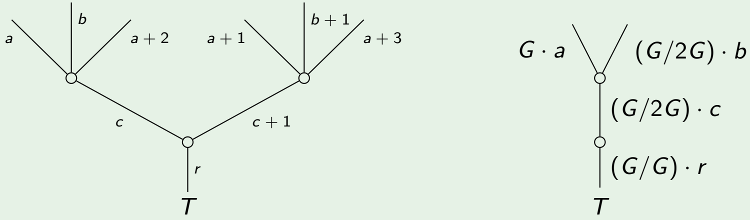

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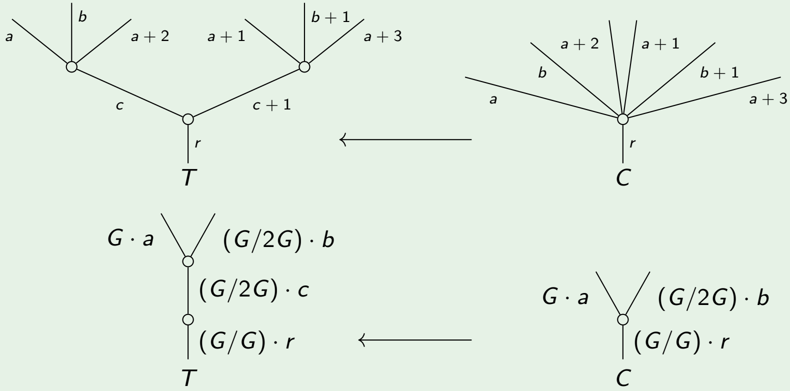

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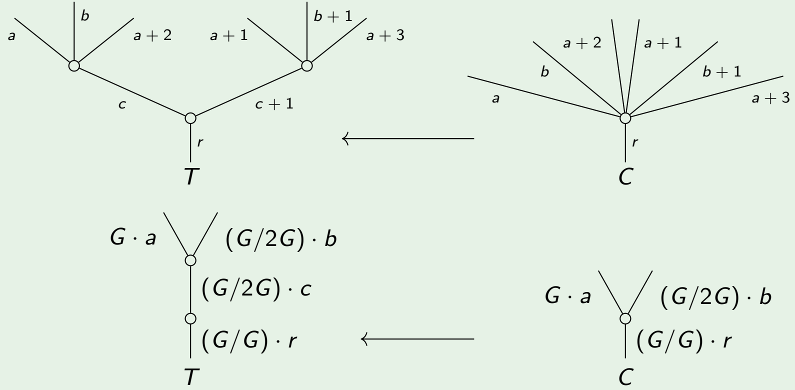

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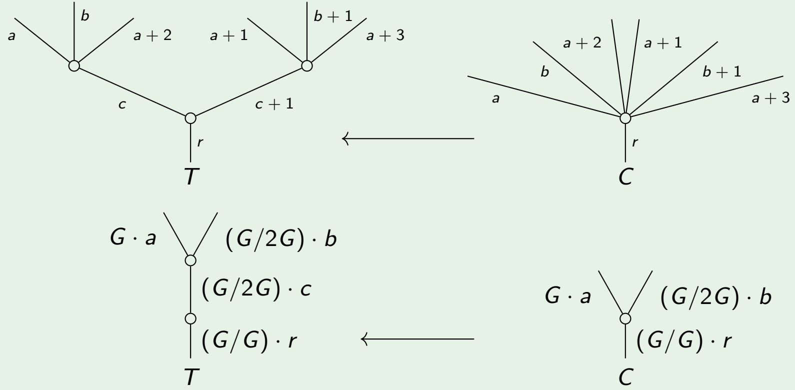

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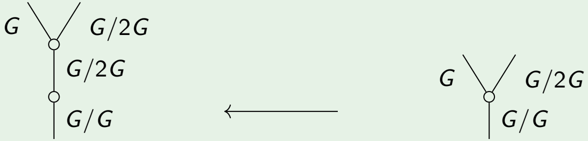

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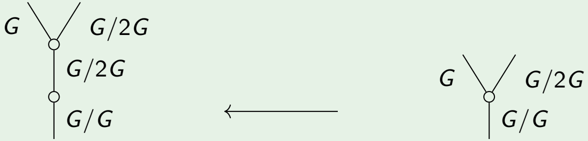

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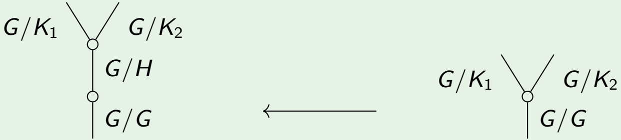

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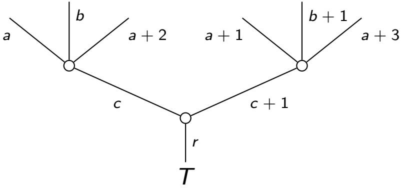

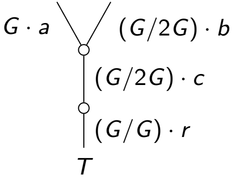

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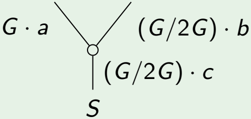

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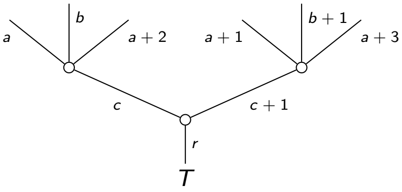

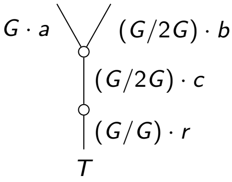

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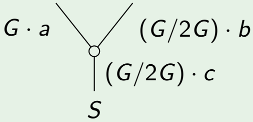

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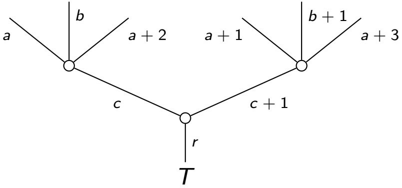
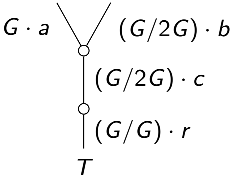
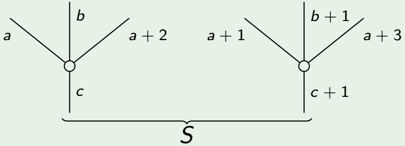
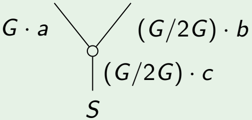




$$\mathrm{Hom}(T, \mathcal{O}) = \mathcal{O}(G/2G)^G \times \mathcal{O}(2G \amalg 2G/2G)^{2G}$$

Example ($G = \mathbb{Z}_{/4}$. Example not in Ω^G)





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Genuine equivariant operads

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A *genuine equivariant operad* $\mathcal{P} \in \mathsf{sOp}_G$ has levels $\mathcal{P}(X)$ for each H-set X.
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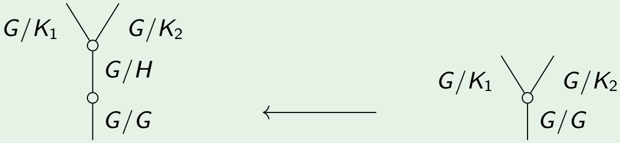
Composition in genuine equivariant operads

Example (general G, $K_1 \leq H \leq G$, $K_2 \leq H \leq G$)



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Hence, a genuine equivariant operad $\mathcal{P} \in \mathrm{Op}_G$ has maps

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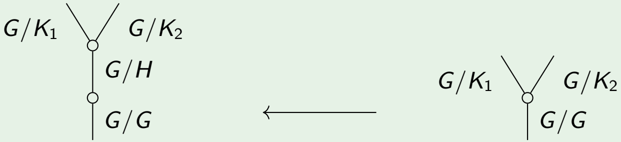

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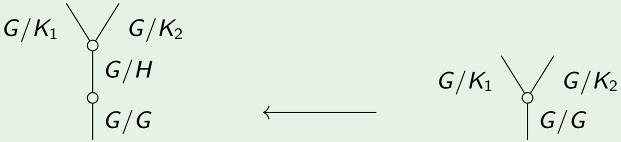

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More generally, compositions in $\mathcal{P} \in \mathsf{Op}_G$ have the form

$$\begin{gathered}\mathcal{P}(H/K_1 \amalg \cdots \amalg H/K_n)\times \\ \times\mathcal{P}(K_1/L_{11} \amalg \cdots \amalg K_1/L_{1m_1}) \times \cdots \times \mathcal{P}(K_n/L_{n1} \amalg \cdots \amalg K_n/L_{nm_n}) \\ \Big\downarrow \\ \mathcal{P}(H/L_{11} \amalg \cdots \amalg H/L_{1m_1} \amalg \cdots \amalg H/L_{n1} \amalg \cdots \amalg H/L_{nm_n})\end{gathered}$$

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Theorem (Bonventre-P.)

There is a Quillen equivalence

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Thanks for listening.

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