Research Statement

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I am an algebraic topologist. My main interests are the theory of topological and spectral operads, their Koszul duality, Goodwillie calculus, and, more recently, the interaction of these topics with genuine equivariant homotopy theory.

1 Background

The following paragraphs briefly outline the topics of *operads*, *equivariant homotopy theory* and *Goodwillie calculus*, which are at the center of my research.

Briefly, an *operad* (introduced by May in [18]) consists of a sequence $\mathcal{O}(n)$ of sets/spaces of "n-ary operations" together with Σ_n-actions and suitable compositions. The main point of operad theory is then the study of the algebras over a fixed operad $\mathcal{O}$, which are objects X (in some appropriate monoidal category $(\mathcal{C}, \otimes)$) together with n-ary operations suitably indexed by $\mathcal{O}(n)$. Indeed, all the classical notions of associative algebra, commutative algebra, Lie algebra, Possion algebra, among others, can then be recovered as the algebras over some fixed operad in an appropriate category. Other prominent examples include the topological E_n-operads, whose algebras are "commutative algebras up to homotopy", and are closely related to the theory of n-loop spaces, which play an important role in both algebraic topology and physics. As a last example, operads also appear in Goodwillie calculus: under certain conditions, the "derivatives of appropriate functors" form an operad.

Equivariant homotopy theory deals with the correct notion of homotopy when in the presence of the action of a group G. For example, a G-equivariant map $f\colon X \to Y$ between spaces with G-actions is considered a "genuine equivariant homotopy equivalence" only if f induces "non-equivariant equivalences" $f^H\colon X^H \to Y^H$ between fixed points for all subgroups $H \leq G$. In this equivariant context, work of Hill, Hopkins and Ravenel on the Kervaire invariant problem has revealed the importance of norms (given a G-object X and G-set A, the associated norm is the tensor product $\otimes_A X = X^{\otimes |A|}$ together with an appropriate mixed G-action) and norm maps (i.e. maps between norms). Furthermore, follow up work of Blumberg and Hill in [1] has shown that there is a close relationship between operads and norms maps. In particular: (i) norm maps can be encoded in terms of G-equivariant operads by looking at certain fixed points $\mathcal{O}(n)^\Gamma$ for special subgroups $\Gamma \leq G \times \Sigma_n$; (ii) general G-equivariant operads contain only some types of norm maps.

Lastly, Goodwillie calculus is a technique originally developed by Goodwillie in [8],[9],[10] to study functors $F\colon \mathcal{C} \to \mathcal{D}$, where $\mathcal{C}, \mathcal{D}$ are either the category Top_* of pointed topological spaces or Sp of spectra. In summary, Goodwillie calculus associates to such a functor a *Taylor tower* of functors

$$F \to \cdots \to P_{n+1}F \to P_nF \to P_{n-1}F \to \cdots \to P_0F \tag{1.1}$$

where P_nF is the so called "universal n-polynomial approximation of F". Additionally, one also has a characterization of the fiber functors $D_nF \to P_nF \to P_{n-1}F$, which in the more interesting case of a functor $F\colon \mathsf{Top}_* \to \mathsf{Top}_*$ have the form

$$D_nF(X) = \Omega^\infty((\partial_n F \wedge (\Sigma^\infty X)^{\wedge n})_{h\Sigma_n})$$

where $\partial_n F$ a spectrum with a Σ_n-action.

2 Past research

2.1 Equivariant operads

Blumberg and Hill's work in [1] hints at important new subtleties that emerge when studying operads in the G-equivariant setting (i.e. operads together with a G-action commuting with all structure).

Non-equivariantly, a map $\mathcal{O} \to \mathcal{O}'$ of operads is deemed a weak equivalence provided that the levelwise maps $\mathcal{O}(n) \to \mathcal{O}'(n)$ are weak equivalences of spaces after forgetting the Σ_n-actions.

On the other hand, in the G-equivariant setting each level $\mathcal{O}(n)$ now has instead a $G \times \Sigma_n$-action. As such, one should at a minimum require a weak equivalence of G-operads $\mathcal{O} \to \mathcal{O}'$ to "induce G-genuine equivalences after forgetting the Σ_n-actions", i.e. to induce equivalences of fixed points $\mathcal{O}(n)^H \to \mathcal{O}'(n)^H$ for subgroups $H \le G$. However, this turns out to be insufficient. As noted in [1], norm maps are encoded by more general fixed point spaces $\mathcal{O}(n)^\Gamma$ where $\Gamma \le G \times \Sigma_n$ is a so called *graph subgroup* (formally, this means $\Gamma \cap \Sigma_n = *$). Therefore, to study G-operads while keeping track of norm data, one must instead work with graph equivalences, i.e. maps $\mathcal{O} \to \mathcal{O}'$ that induce weak equivalences of graph fixed points $\mathcal{O}(n)^\Gamma \to \mathcal{O}'(n)^\Gamma$ for all graph subgroups $\Gamma \le G \times \Sigma_n$.

The study of G-operads thus requires understanding the "interactions between graph fixed points and operadic composition". In trying to capture these, I have discovered (jointly with Peter Bonventre) a category Ω_G of *G-trees* that encodes such interactions, as well as suggests some new models for the homotopy theory of G-operads.

2.1.1 Equivariant dendroidal sets

Moerdijk and Weiss in [19] and Cisinski and Moerdijk in the follow up paper [3] develop a suitable category Ω of trees and, writing $\mathsf{dSet} = \mathsf{Set}^{\Omega^{op}}$, build a model structure on dSet whose fibrant objects are "operads up to homotopy" called ∞-operads. Furthermore, follow up work of Cisinski and Moerdijk in [4], [5] establishes a Quillen equivalence

$$W\colon \mathsf{dSet} \rightleftarrows \mathsf{sOp}\colon hcN, \tag{2.1}$$

where sOp is the category of (colored) simplicial operads.

In [23], I generalized the work in [19] and [3] to obtain the following.

Theorem 2.2. *There is a model structure on dSet^G for which the fibrant objects are the G-∞-operads. Furthermore, there are variant model structures associated to each Blumberg-Hill indexing system.*

Here the notion of G-∞-operads is defined using the category Ω_G of G-trees I discovered. In particular, we note that intuitively G-∞-operads are "operads *with norm maps* up to homotopy" and that, for this reason, Theorem 2.2 can not be obtained by simply applying "formal genuine model structure" constructions (as found in, for example, [27]) to the original non-equivariant Cisinski-Moerdijk result in [3]. Indeed, such "formal genuine model structures" would only lead to "operads *without* norm maps up to homotopy".

2.1.2 Operadic Elmendorf theorem

A classical result of Elmendorf states that there is a Quillen equivalence $\mathsf{Top}^G \simeq \mathsf{Top}^{\mathsf{O}_G^{op}}$. Here O_G is the G-orbit category, formed by the G-sets G/H for each $H \le G$, Top^G is given its "genuine model structure", where weak equivalences are detected by looking at all fixed points, and $\mathsf{Top}^{\mathsf{O}_G^{op}}$ is given its "projective model structure", with weak equivalences detected by looking at each level of the presheaf.

Elmendorf's proof is fairly robust: reasonable conditions on a model category $\mathcal{C}$ allow for analogous Quillen equivalences $\mathcal{C}^G \simeq \mathcal{C}^{\mathsf{O}_G^{op}}$. Indeed, this allows for equivalences $\mathsf{sCat}^G \simeq \mathsf{sCat}^{\mathsf{O}_G^{op}}$ and $\mathsf{sOp}^G \simeq \mathsf{sOp}^{\mathsf{O}_G^{op}}$, where sCat, sOp denote the categories of simplical categories and of simplicial (colored) operads. However, in the operad case sOp^G such an equivalence only works if studying operads *without norm maps.*

To produce the more interesting analogue of Elmendorf's theorem for operads *with* norm maps, one instead needs to replace the category $\mathsf{sOp}^{\mathsf{O}_G^{op}}$ with a more complex one. Informally, the flaw of $\mathsf{sOp}^{\mathsf{O}_G^{op}}$ is that the levels of objects $\underline{\mathcal{O}} \in \mathsf{sOp}^{\mathsf{O}_G^{op}}$ are indexed only by subgroups $H \le G \times \Sigma_n$ such that $H \le G$ rather than all graph subgroups $\Gamma \le G \times \Sigma_n$ such that $\Gamma \cap \Sigma_n = *$.

In [2] (jointly with Peter Bonventre) I define a category sOp_G of what we call *genuine equivariant operads.* Informally, a genuine equivariant operad $\mathcal{P}$ has as levels spaces $\mathcal{P}(\Gamma)$ indexed by graph subgroups together with "restrictions" and "operadic compositions" as prescribed by the category Ω_G of G-trees. We then prove the following.

Theorem 2.3. *The projective model structure (with weak equivalences detected at each level) on sOp_G exists.*

Theorem 2.4. *There is a Quillen equivalence*

$$\mathsf{sOp}_G \rightleftarrows \mathsf{sOp}^G$$

where the category sOp_G of genuine equivariant operads is given the projective model structure and the category sOp^G of (regular) G-equivariant operads is given its "genuine with norms" model structure.

2.1.3 Blumberg-Hill realization conjecture

In general, a G-operad $\mathcal{O}$ needs not have all types of norm maps, i.e. it may be $\mathcal{O}(n)^{\Gamma} = \varnothing$ for some graph subgroups Γ. On the other hand, the collection of those graph subgroups Γ such that $\mathcal{O}(n)^{\Gamma} \neq \varnothing$ is far from arbitrary and, indeed, much of the work in [1] is spent identifying a number of novel and non-obvious closure conditions that such collections must satisfy. Further, Blumberg and Hill coined the term "indexing system" to refer to a collection satisfying those closure conditions.

However, though their work in [1] showed that the types of norm maps present in a G-operad $\mathcal{O}$ do form an indexing system, the converse statement that all indexing systems can be realized from some operad was left as a conjecture (in fact, their conjecture is slightly more precise, asking whether coefficient systems can be realized by special operads that they call N_∞-operads).

As a consequence of Theorem 2.4, we proved the Blumberg-Hill realization conjecture. Briefly, this follows since indexing systems are obviously realized by a genuine equivariant operad in sOp_G.

Corollary 2.5. *Any indexing system is realized by a N_∞-operad.*

We note here that this conjecture has also been concurrently verified by Gutierrez-White in [11] and by Rubin in [25], with each of their approaches having different advantages: Gutierrez-White's model for N_∞-operads is cofibrant while Rubin's model is explicit. Our model, which emerges from the broader framework of genuine equivariant operads, satisfies both of these desiderata.

2.2 Homotopy theory of algebras over operads

2.2.1 Goodwillie calculus for algebras over an operad

The main goal of my thesis work [20],[21] was the study of Goodwillie calculus on the category $\mathsf{Alg}_{\mathcal{O}}$ of algebras over a spectral operad $\mathcal{O}$.

The main results in [20] were a generalization of Goodwillie's original results in [8], [9] and [10] from the setup of functors $\mathcal{C} \to \mathcal{D}$ between either the categories of pointed spaces or spectra to more general categories, as follows.

Theorem 2.6. *Let $\mathcal{C}, \mathcal{D}$ be cofibrantly generated model categories, and $F\colon \mathcal{C} \to \mathcal{D}$ be a homotopy functor. Assume further that $\mathcal{C}$ is either pointed and simplicial or that cofibrations in $\mathcal{C}$ are categorical injections.*

Then there exists a universal n-excisive approximation $P_n F$ to F.

Theorem 2.7. *Assume $\mathcal{C}, \mathcal{D}$ are pointed simplicial model categories, and $F\colon \mathcal{C} \to \mathcal{D}$ is a simplicial n-homogeneous functor. Assume further that: spectra categories $\mathsf{Sp}(\mathcal{C})$, $\mathsf{Sp}(\mathcal{D})$ can be defined, where weak equivalences of spectra are detected by the $\Omega^{\infty-i}$ functors; in $\mathcal{D}$ finite homotopy limits commute with countable directed homotopy colimits.*

Then F can be factored through the spectra categories, i.e., $F \sim \Omega^{\infty} \circ \bar{F} \circ \Sigma^{\infty}$, where $\bar{F}\colon \mathsf{Sp}(\mathcal{C}) \to \mathsf{Sp}(\mathcal{D})$ is itself n-homogeneous.

Applying the Goodwillie calculus set-up above to $\mathsf{Alg}_{\mathcal{O}}(\mathsf{Sp}^{\Sigma})$ I showed the following in [21].

Theorem 2.8. *Let $\mathcal{O}$ be a spectral operad. Then there is a zigzag of Quillen equivalences between $\mathsf{Sp}(\mathsf{Alg}_{\mathcal{O}})$ and $\mathcal{O}(1) - \mathsf{Mod}$. Further, this identifies Σ^{∞} with Topological André-Quillen homology and Ω^{∞} with the trivial algebra functor.*

Theorem 2.9. *The Goodwillie tower of the identity for $Alg_{\mathcal{O}}$ is given by the (left derived) truncation functors $\mathcal{O}_{\leq n} \circ_{\mathcal{O}} (-)$. Further, the n-th derivative is $\mathcal{O}(n)$ itself with its canonical $(\mathcal{O}(1), \mathcal{O}(1)^{\wr n})$-bimodule structure.*

2.2.2 Operad bimodules and André-Quillen filtrations

In joint work with Nick Kuhn in [17], we study the filtration $\mathcal{O}_{\geq n} \circ_{\mathcal{O}} (-)$ dual to the Goodwillie tower in Theorem 2.9. A key observation is that by iterating the filtration (i.e. applying the filtration to itself!) the operad structure leads to extra maps between the filtration levels. Our main results are as follows (where $\mathcal{O}_{\leq n} \circ_{\mathcal{O}} J$ is shortened as J^n for suggestiveness).

Theorem 2.10. *Let $I, J \in \mathsf{Alg}_{\mathcal{O}}(R-\mathsf{Mod})$, and let $f : I \to J^d$ be a morphism in $\mathsf{Alg}_{\mathcal{O}}(R-\mathsf{Mod})$. Then f induces compatible $\mathcal{O}$–algebra maps $f_n : I^n \to J^{dn}$ for all n, and the assignment $f \mapsto f_n$ is functorial and preserves weak equivalences.*

We say that a map $f \in [I, J]_{\mathsf{Alg}}$ has AQ-filtration s if f factors in $ho(\mathsf{Alg}_{\mathcal{O}}(R))$ as the composition of s maps

$$I = I(0) \xrightarrow{f(1)} I(1) \xrightarrow{f(2)} I(2) \to \cdots \to I(s-1) \xrightarrow{f(s)} I(s) = J$$

such that $TQ(f(i))$ is null for each i..

Theorem 2.11. *Let $f \in [I, J]_{\mathsf{Alg}}$ have AQ-filtration s. Then there exists $\tilde{f} \in [I, J^{2^s}]_{\mathsf{Alg}}$ such that*

$$\begin{tikzcd} & J^{2^s} \arrow[d] \\ I \arrow[ur, "\tilde{f}"] \arrow[r, "f"'] & J \end{tikzcd}$$

commutes in $ho(\mathsf{Alg}_{\mathcal{O}}(R))$.

These results are essential for follow up work of Nick Kuhn in [16], studying generalized Hurewicz maps for infinite loopspaces.

2.2.3 Σ-cofibrancy of smash powers of positive spectra

The work in both [21] and [17] crucially relies on the positive model category structure on symmetric spectra Sp^{Σ}, which has long been known to be convenient when studying algebras over an operad in spectra (e.g. [26], [7], [12]).

Informally, this convenience comes from the fact that if X is a positive cofibrant spectrum then the n-fold smash power $X^{\wedge n}$ is "almost Σ_n-cofibrant". However, making this statement precise requires some care and, as it turns out, some wrong formulations made their way into the literature (in fact, my thesis work in [20], [21] originally made use of those formulations, leading to delays in that work).

Filling this gap in the literature was the main goal of [22], where I introduced a notion of lax Σ_n-cofibrancy for n-fold powers of positive spectra which I then proved to have the same key properties as genuine Σ_n-cofibrancy. This was used to prove the main result of [22]: that, under mild cofibrancy conditions, "operadic pushout products"

$$M \circ_{\mathcal{O}} \bar{N} \bigvee_{M \circ_{\mathcal{O}} N} \bar{M} \circ_{\mathcal{O}} N \xrightarrow{f_1 \square^{\circ_{\mathcal{O}}} f_2} \bar{M} \circ_{\mathcal{O}} \bar{N}.$$

are cofibrations, trivial if f_1 or f_2 are.

While technical, this is quite a powerful result, which easily implies all of the following (most of which strengthen previous results in [12], [13]), which were necessary in [21] and [17].

Theorem 2.12. *For $\mathcal{O}$ any operad in Sp^{Σ} there is a projective positive S model structure on $\mathsf{Alg}_{\mathcal{O}}$. Further, $\bar{\mathcal{O}} \circ_{\mathcal{O}} (-) \colon \mathsf{Alg}_{\mathcal{O}} \rightleftarrows \mathsf{Alg}_{\bar{\mathcal{O}}} \colon \mathsf{fgt}$ is a Quillen equivalence when $\mathcal{O} \to \bar{\mathcal{O}}$ is a stable equivalence.*

Theorem 2.13. *Let $\mathcal{O}$ be an operad in Sp^{Σ} which is level cofibrant. Then the forgetful functor $\mathsf{fgt} \colon \mathsf{Alg}_{\mathcal{O}} \to \mathsf{Sp}^{\Sigma}$ sends cofibrations between cofibrant objects to cofibrations between cofibrant objects.*

Theorem 2.14. *For an operad $\mathcal{O}$, right $\mathcal{O}$-module M and left $\mathcal{O}$-module N satisfying mild cofibrancy hypothesis, the bar construction $B_n(M, \mathcal{O}, N) = M \circ \mathcal{O}^{\circ n} \circ N$ is Reedy cofibrant.*

Theorem 2.15. *If A is cofibrant in $\mathsf{Alg}_{\mathcal{O}}$, the functor $\mathsf{Mod}^r_{\mathcal{O}} \xrightarrow{(-)\circ_{\mathcal{O}} A} \mathsf{Sp}^{\Sigma}$ preserves homotopy fiber sequences.*

3 Future research

3.1 Generalize Cisinski-Moerdijk-Weiss

One of the main goals of my project is to complete the generalization of the Cisinski-Moerdijk-Weiss results to the equivariant setup by adapting the work in [4] and [5] and obtaining the following.

Conjecture 3.1. There is a Quillen equivalence

$$W^G \colon \mathsf{dSet}^G \rightleftarrows \mathsf{sOp}^G \colon hcN^G. \tag{3.2}$$

where sOp^G is given its "genuine with norms" model structure.

While the full proof of this result is still work in progress, we list some already established positive results in this direction: (i) the right adjoint hcN^G sends "locally G-graph fibrant" G-operads to G-∞-operads; (ii) in both the case of locally G-graph fibrant G-operads and the case of G-∞-operads the traditional "homotopy operad" construction can be used to build a "genuine equivariant operad" (as in the previous section).

Furthermore, preliminary work in this direction suggests that most of the adaptations necessary to establish this result mirror the adaptations found in [23], indicating that proving this conjecture may be mostly straightforward.

3.2 Boardman-Vogt tensor product of N_∞-operads

One of the main highlights of [1] is the introduction of the notion of N_∞-operads, which are equivariant operads encoding "different degrees of equivariant commutativity", in a way somewhat reminiscent to the classical non-equivariant E_n-operads. In fact, similarly to the classical result that $\mathcal{O} \otimes \mathcal{O}'$ is an E_{n+m}-operad when $\mathcal{O}$ is E_n and $\mathcal{O}'$ is E_m, it is reasonable for formulate a similar conjecture for N_∞-operads.

Conjecture 3.3. For indexing systems $\mathcal{F}$, $\mathcal{F}'$ and $\mathcal{O}$ a $N_{\mathcal{F}}$-operad and $\mathcal{O}'$ a $N_{\mathcal{F}'}$-operad then $\mathcal{O} \otimes \mathcal{O}'$ is a $N_{\mathcal{F} \vee \mathcal{F}'}$-operad where $\mathcal{F} \vee \mathcal{F}'$ denotes the indexing system generated by $\mathcal{F}$ and $\mathcal{F}'$.

Briefly, this conjecture amounts to showing that graph subgroup fixed points $((\mathcal{O} \otimes \mathcal{O}')(n))^\Gamma$ are appropriately either empty or contractible in a way depending on $\mathcal{F}$, $\mathcal{F}'$, Γ. The main challenge is posed by the Boardman-Vogt tensor product $\otimes$, which is a hard operation to study. Nonetheless, using the G-tree language, we have proven that such fixed points are suitably empty or connected, a non obvious claim. The proof of the full result is the subject of current work.

3.3 Composition product description of genuine equivariant operads

The initial description of the category sOp_G of genuine equivariant operads given in [2] is in terms of a certain monad on a category Sym_G of *G-symmetric sequences*, generalizing one the descriptions of (non-equivariant) operads. However, while this description is convenient for the particular technical purposes in [2], in order to discuss the notion of algebras over genuine equivariant operads one needs the following alternate description, which is part of work in progress.

Proposition 3.4. *There is a monoidal structure $\circ$ on Sym_G, called the composition product, such that sOp_G is the category of monoids over $\circ$.*

3.4 Elmendorf theorem for algebras

Since Proposition 3.4 allows for the definition of algebras over a genuine equivariant operad $\mathcal{P} \in \mathsf{sOp}_G$, and in light of Theorem 2.4 it is natural to try to compare algebras over (reqular equivariant operads) and algebras over genuine equivariant operads. I conjecture the following.

Conjecture 3.5. Let $\mathcal{O} \in \mathsf{sOp}^G$ be a Σ-cofibrant operad and $\mathcal{P} \in \mathsf{sOp}_G$ the corresponding genuine equivariant operad (in light of Theorem 2.4). Then there is a Quillen equivalence

$$\mathsf{Alg}_{\mathcal{P}} \rightleftarrows \mathsf{Alg}_{\mathcal{O}}$$

3.5 Genuine equivariant operads in G-symmetric monoidal categories

The original definition of genuine equivariant operad defined in [2] requires working in a symmetric monoidal category that possess so called “diagonal maps”. However, a careful analysis of the definition reveals that one should be able to extend the definition of genuine equivariant operads to a more general context, which we refer to as “G-symmetric monoidal categories”, which are closely related to the notions of symmetric monoidal Mackey functor and equivariant symmetric monoidal category discussed in [15] and the notion of normed symmetric monoidal category discussed in [24], though our notion, which is naturally motivated from the theory of G-trees, does not quite coincide with either of those notions.

3.6 Equivariant Goodwillie calculus on algebras over equivariant operads

In recent work ([6]), Dotto develops a theory of equivariant Goodwillie calculus. A key feature of that work is that, rather than just a tower of functors as in (1.1), G-equivariant calculus produces a tree indexed by finite G-sets F and containing inside it a so called “genuine tower” indexed by the G-sets $n \cdot G$. This is very much reminiscent of the insight that the right notion of weak equivalence for G-operads needs to account for graph subgroups, and we expect the two theories to be closely related.

In particular, we expect Theorem 2.9 to extend directly to the genuine Goodwillie tower of an operad in genuine G-spectra with the notion of weak equivalence between genuine derivatives given by the G-graph stable equivalences of [14].

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